Local damage assessment of a building using Support Vector Machine

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Abstract

A damage detection method utilizing the Support Vector Machine (SVM) is proposed. The SVM is a powerful pattern recognition tool applicable to complicated classification problems. Modal frequencies of a structure are used for pattern recognition in the proposed method. Typically, only two vibration sensors detecting a single input and a single output for a structural system can easily determine modal frequencies. For training SVMs the relationship between changes normalised by original modal frequencies, before suffering any damage, is utilized. The SVM trained for single damage was also found to be effective for detecting damage in multiple stories. The SVM based damage assessment is able to identify damage qualitatively as well as quantitatively.

1 Introduction

Many approaches for structural health monitoring (SHM) have been proposed for the purpose of maintenance cost reduction or a performance guarantee of the civil and the building structures. Generally, it is not difficult to obtain the detailed structural health information of a structure if sufficient sensors are installed. However, the cost available for a structural health monitoring system may not be rationalized due to the system. For example, it would be difficult to find any economic rationale to introduce the system that measures acceleration of each floor. The purpose of this research is to propose a damage assessment system that can obtain the detailed damage information by installing the minimum number of sensors. The system is optimized to minimize the possibility of incorrect judgements. The SVM is effectively utilized in the system to achieve this purpose.
2 Sensitivity of natural frequency to damage

Generally speaking, a natural frequency change associated with a certain mode does not provide the spatial information of structural damage. However, multiple natural frequency changes can provide the information on the location of damaged stories. For a multi-mass shear system, the equilibrium equation for an undamped structure is given by

\[
\left( - \omega_r^2 [M] + [K] \right) \phi_r = \{0\}
\]  

(1)

where \( r = 1, 2, \ldots, N \), \([M]\) and \([K]\) = mass and stiffness matrices, respectively, and \( \{ \phi_r \} = r\)-th mode shape corresponding to the natural frequency \( \omega_r \) and is normalized to be unit-mass mode shapes, i.e., \( \{ \phi_r \}^T [M] \{ \phi_r \} = 1 \).

The sensitivity coefficient of the \( r \)-th natural frequency in terms of \( k_{ij} \) is defined by the derivative of Eq.1 with respect to \( k_{ij} \)[3].

\[
\frac{\partial \omega_r}{\partial k_{ij}} = \frac{1}{2\omega_r} \{ \phi_r \}^T \frac{\partial [K]}{\partial k_{ij}} \{ \phi_r \}
\]  

(2)

If we take into consideration the symmetry of stiffness matrix, the sensitivity coefficients of the natural frequencies can be rewritten as

\[
\frac{\partial \omega_r}{\partial k_{ij}} = \begin{cases} 
\frac{1}{\omega_r} \phi_{ir} \phi_{jr}, & i \neq j \\
\frac{1}{2\omega_r} \phi_{ir}^2, & i = j 
\end{cases}
\]  

(3)

Where \( \{ \phi_r \} \) is \( r \)-th component of the \( r \)-th mode. This equation for natural frequencies can be expanded using Taylor’s series. The resulting series taking only the first order terms represents the change in natural frequency as

\[
\Delta \omega_r = \sum_{i=1}^{N} \sum_{l=1}^{N} \frac{\partial \omega_r}{\partial k_{ij}} \Delta k_{ij}
\]  

(4)

For the multi-mass shear system, when the \( i \)-th story stiffness is reduced, only \( k_{ii}, k_{(i-1)(i-1)}, k_{(i-1)i} \) and \( k_{(i-2)i} \) are affected in the stiffness matrix. Hence, Eq.4 is simplified into

\[
\frac{\Delta \omega_r}{\omega_r} = \frac{\Delta k_{ij}}{2\omega_r^2} (\phi_{ir} - \phi_{(i-1)r})^2
\]  

(5)

The above relation will be used for forming feature vectors for SVM based damage diagnosis.
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3 Support Vector Machine

The Support Vector Machine (SVM) is a mechanical learning system introduced by Vapnik and co-workers that uses a hypothesis space of linear functions in a high dimensional feature space [1, 2]. The simplest model of SVM is the so-called Linear SVM (LSVM) shown in Fig. 2. It works only for data which are linearly separable in the original feature space, and hence cannot be applicable to many real-world problems. In the early 1990s, nonlinear classification in the same procedure as LSVM became possible by introducing nonlinear functions called a Kernel function, without being conscious of actual map space. The technique extended to nonlinear feature spaces is called Nonlinear SVM (NSVM) shown in Fig. 3. In what follows, we assume a training sample $S$ consisting of vectors $x_i \in \mathbb{R}^p$ with $i = 1, \ldots, N$.

$$S = \{(x_1, y_1), \ldots, (x_N, y_N)\}$$

(6)

Each vector $x_i$ belongs to either of two classes and thus is given a label $y_i \in \{-1, 1\}$. The pair of $(w, b)$ defines a hyperplane of equation

$$(w \cdot x) + b = 0$$

(7)

named separating hyperplane (see Fig. 1).

We now need to establish the optimal separating hyperplane (OSH) that divides $S$ leaving all the points of the same class on the same side while maximizing the margin which is the distance of the closest point of $S$ (see Fig. 2). This closest vector $x_i$ is the support vector.

![Figure 1: Separating hyperplane.](image1)

![Figure 2: OSH (the dashed lines define the margin).](image2)

For the reasons mentioned above, the OSH $(w', b')$ can be determined by solving an optimization problem defined by
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\[
\begin{align*}
\text{minimize} & \quad \text{margin } d(w) = \frac{1}{2} (w \cdot w'), \\
\text{subject to} & \quad y_i ((w \cdot x_i) + b') \geq 1, \quad i = 1, 2, ..., N
\end{align*}
\] (8)

The resulting SVM is called Hard Margin SVM. However, applicable problems are limited. In order to relax the situation, we have to allow for a small number of misclassified feature vectors. The previous analysis (Eq.8) is generalized by introducing \( N \) nonnegative variables \( \xi = (\xi_1, \xi_2, ..., \xi_N) \) such that

\[
\begin{align*}
\text{minimize} & \quad \text{margin } d(w) = -\frac{1}{2} (w \cdot w') + C \sum \xi_i, \\
\text{subject to} & \quad y_i ((w \cdot x_i) + b') \geq 1 - \xi_i, \quad i = 1, 2, ..., N, \quad \xi \geq 0.
\end{align*}
\] (9)

This SVM defined Eq.9 is called Soft Margin SVM. The purpose of the extraterm of the \( C \sum \xi_i \), where the sum is for \( i = 1, 2, ..., N \) is to keep under control the number of misclassified vectors. The parameter \( C \) can be regarded as a regularization parameter. The OSH tends to maximize the minimum distance \( 1/w \) for small \( C \), and minimize the number of misclassified vectors for large \( C \).

Thus we have described the case of linear decision surfaces. To allow much more general decision surfaces, one can first nonlinearly transform a set of original feature vectors \( x_i \) into a high-dimensional feature space by a map \( \Phi : x_i \mapsto z_i \) and then do a linear separation there. It requires the computation of inner products \( (\Phi(x) \cdot \Phi(x_i)) \) in a high-dimensional space. We are interested in cases where these expensive calculations can be reduced significantly by using a Kernel function which satisfies the Mercer’s theorem such that

\[
(\Phi(x) \cdot \Phi(x_i)) = K(x, x_i)
\] (10)

The polynomial kernel and the Gaussian kernel are typical kernel functions. Each kernel function is shown in the following formula.

\[
\begin{align*}
\text{Polynomial Kernel} & \quad K(x, x_i) = (x, x_i)^d, \\
\text{Gaussian Kernel} & \quad K(x, x_i) = \frac{\exp(-\|x - x_i\|^2)}{\sigma}
\end{align*}
\] (11, 12)

![Non-linear SVM](image-url)

Figure 3: Non-linear SVM.
4 Damage detection in a building using SVM

4.1 Feature vectors

We define the $i$-th vector of natural frequency change associated with the $i$-th story stiffness change as

$$\{\gamma_i\} = \left[\frac{\Delta \omega_{i1}}{\omega_1}, \frac{\Delta \omega_{i2}}{\omega_2}, \ldots, \frac{\Delta \omega_{in}}{\omega_N}\right] (i = 1, 2, \ldots, N)$$

(13)

where $\Delta \omega_n$ is $i$-th component of the $r$-th natural frequency. The vectors have typical patterns for each story damage. In this study, the above vectors of natural frequency change are used as feature vectors.

Under the assumption that the 5-mass shear system model consists of the same stiffness and the same mass at each story, feature vectors are shown in Fig.5. The stiffness of each story is reduced to 99%, 98%, 90%, 70%, and 50% of the original stiffness for simulating damages. From Fig.5, we can recognize the candidate the feature vectors are promising for damage detection.

4.2 SVM for 5-story building

The number of classes that should be recognized by the SVM is 5, 5$^{th}$ story damaged, 4$^{th}$ story damaged, 3$^{rd}$ story damaged, 2$^{nd}$ story damaged and 1$^{st}$ story damaged. As the feature vectors are complicated enough application of the LSVM is not possible. The polynomial Kernel that allows high separation of accuracy is chose as the kernel function. In order to build optimal NSVMs,
a) It is based on the classification which divides the each class and the other into 2 classes.

b) Training data for SVM should carry out parallel movement of the whole so that it might become 0 about each minimum rate of change in the rate-of-change in Fig.4.

c) Slack variables are introduced as Soft Margin SVM Consequently, in this pattern recognition, 5 NSVMs are built, and the parameters which should be determined for each NSVM are \( d \) as in Eq.12, and \( C \) as in Eq.9. These parameters are chosen to minimize the misclassified data. \( l-o-o \) (leave-one-out bounds) represents the probability that the data does not exist in a margin \( 0 < f < 1 \). Thus, after being able to classify the data into two classes by the boundary of \( f = 0 \) (correctness=100%), discernment accuracy becomes good as \( l-o-o \) is close to 100%.

### 4.3 Performance evaluation

In the following, indicates damaged, indicates it may be damaged, and indicates undamaged. By seeing outputs of SVM1~SVM5 (as showed Fig.5) to a certain data is seen, it judges where damage is.

The outputs of 5 NSVMs are shown in Fig.6. Let us begin the analysis by considering the output from 5 NSVMs, No.1 represents undamaged. And No.2–6 represent damage in 1\(^{\text{st}}\) story. In the same way as No.2–6, they show that No.7–11, No.12–16, No.17–21 and No.21–25 have damage respectively in 2\(^{\text{nd}}\) story, 3\(^{\text{rd}}\) story, 4\(^{\text{th}}\) story and 5\(^{\text{th}}\) story. Thus, about only 1 story damage detection by the simulation, it was able to identify the damage story in which stiffness reduced to 10% to 50%. Furthermore, it turns out that stiffness reduction and outputs of SVM have a linear relation in Fig.6. So, we evaluated the outputs of SVM quantitatively. Estimated value of stiffness reduction is computed by inputting outputs of SVM into the approximation formula in Fig.6.

![Figure 6: 1 story damage detection by the simulation data.](image-url)
4.4 Verification for multiple damages

Furthermore, verification with the same said of data in case 2 stories have damage was performed. The damage stories (3rd story and 5th story) are fixed and the combination when changing a stiffness value from 90% to 50%. The output result of data is shown in Fig.7 when 3rd story and 5th story are damaged as an example. About 2 stories damage detection to use SVM trained by data when only 1 story is damaged, 2 stories with damage were detectable.

And we evaluated the outputs of SVM quantitatively. The exact value and the estimated value computed from the approximation formula are shown in Fig.8. Although there is some error, the stiffness reduction could be identified in case 2 stories have damage.

Figure 7: Damage detection for multiple damages at 3rd and 5th stories.

Figure 8: Exact value and estimated value (damage in 3rd story and 5th story).
5 Experimental verification

In order to record experiment data, damage was imitated by using board springs (thickness of 1.5mm) thinner than origin (2.5mm) for 2 pillars of one side shown as Fig.10. We measured the acceleration in the experiment equipment as shown in Fig.9. And natural frequency was computed.

Fig.11 is the verification result using the experiment data. The vertical axis shows the rate of stiffness reduction by each SVM. In Fig.11, No.2–No.6 show the result in case only 1 story stiffness is reduced to 60%, and No.7–No.16 are in case of 2 stories damaged. It turns out that accuracy has fallen a little compared with the simulation result. In this experiment, although stiffness is reduced 40%, it can be said that this stiffness value is close to a limit as a range of perturbation in formulization.

![Figure 9: Experiment equipment.](image1)

![Figure 10: The damaged story from the top](image2)

Table 1: Data number of experiment data.

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Figure 11: Stiffness reduction computed from outputs of SVMs.
6 Concluding remarks

It was shown that damage identification is possible using the index of natural frequency change with comparatively easy calculation as input data of SVM. In the case of a construction structure, natural frequency is computed easily by the data from only 2 sensors in the highest story and the ground. Moreover, since the damage on two or more stories is detectable by learning the data in the case of only 1 story damaged, we think that the application range of this technique is wide. As a future subject, in order to raise discernment accuracy in case two or more stories have damage, it is necessary to raise the separation performance of the feature vector.

Reference