Using electrical circuit model to simulate heat transfer for environmental applications
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Abstract

The similarity between a thermal system and an electrical system is used as an alternative method for solving thermal problems. Numerical equations of the thermal system are converted to electrical equations to form an electrical circuit simulation model. Computer programs are readily available for solving the electrical circuit problem. Numerical solutions to the heat transfer phenomenon in question can be easily obtained by converting the electrical voltage and current to heat and temperature. In this paper, the electrical circuit simulation technique is used to simulate heat transfer in a passive solar sludge drying system. This simulation technique is expected to be a valuable tool for simulating and solving other environmental heat and mass transport problems.

1 Introduction

Transport of heat from a point of higher temperature to another point of lower temperature is accomplished through conduction, convection and radiation or combinations of these mechanisms. The rate of heat flux is expressed by the product of a driving force (temperature difference) and conductance (heat conductance of the medium). Numerous numerical thermal equations can be formulated to describe the complicated heat transfer phenomena, but the degree of difficulty in solving these equations is proportional exponentially to the number of equations. Thus, solving a system of a large number of simultaneous equations is almost impossible without developing a computer program. An alternative solution is to
simulate heat transfer by the analogy of the heat flow in a thermal medium to the electrical current in an electrical circuit. The major advantage of converting the thermal system to an electrical system is that computer programs specially developed for solving electrical circuit problems are available. Once the electrical system is solved, the calculated electrical currents and voltages can be easily converted to their equivalent heat flows and temperatures.

2 Basic Thermal Equations in a thermal system

Basic thermal equations describe heat transfer in a medium and its absorption by the medium. Heat is transferred from a point of higher temperature to another point of lower temperature through conduction, convection and radiation. The rate of heat flow across a thermal medium for all three types of heat transfer is expressed by Equation (1):

\[ Q = \frac{T}{R_t} \]  

(1)

Where

- \( Q \): heat
- \( T \): temperature difference
- \( R_t \): thermal resistance

When heat is transferred in a medium, it may be absorbed or released by the medium thus causing a rise or drop of the medium temperature. This phenomenon is expressed by Equation (2):

\[ T = \left( -\frac{1}{C_t} \right) \int_{t_1}^{t_2} Q \, dt \]  

(2)

Where:

- \( C_t \): thermal capacitance of the medium
- \( t_1 \): initial time
- \( t_2 \): final time
- \( T \): temperature difference
- \( Q \): heat flow

A passive greenhouse type solar sludge drying device is used as an example to illustrate the complicated heat transfer phenomena. The system consists of 6 physical components: outside air, vinyl cover, inside air, sludge drum, heat absorbers and the sludge being dried. The main source of heat comes from solar radiation which is converted to heat at the surface of heat absorbers. Heat in the system may also be gained from or lost to the surrounding outside air across the vinyl cover. The heat generated or gained by the system is distributed among inside air, sludge drum, heat absorbers
and the sludge being dried through convection, conduction and radiation.

Although only Equations (1) and (2) are used to delineate the heat transfer phenomena occurring in the example solar sludge drying system, the transfer of heat may occur between any two of the six components through convection, conduction and radiation. Thus approximate 57 heat transfer equations are needed to delineate the heat transfer phenomena completely. Solving 57 simultaneous thermal equations is not an easy task. One has to develop a computer program that may take months before the program can be successfully tested and debugged.

Figure 1: Schematic diagram of the example passive solar sludge drying system that involves heat transfer phenomena.

3 Electrical circuit model to simulate the thermal system

Thermal flow and temperature are analogous to electrical current and voltage in an electrical circuit. Thus, heat, temperature difference and thermal resistance of the thermal system can be simulated by electrical current, electrical voltage and electrical resistor, respectively. The rate of heat flow across the thermal medium as expressed by Equation (1) can be converted to an electrical equation expressed by Equation (3):
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$$I = \frac{V}{Re}$$

(3)

Where:  
I: electrical current  
V: voltage  
Re: electrical resistance

The analogy between Equations (1) & (3) applies to conduction, convection and radiation heat transfer. Similarly, the medium that absorbs or releases heat resulting in a temperature change can be simulated by an electrical capacitor to store or release electrical current due to a change of the electrical voltage or vice versa as shown by Equation (4).

$$V = \left(\frac{1}{C_e}\right) \int_{t_1}^{t_2} I \, dt$$

(4)

Where  
Ce: electrical capacitance of the medium  
t1: initial time  
t2: final time  
V: electrical voltage  
I: electrical current

A complete electrical circuit model that simulates the thermal system of Figure 1 is shown in Figure 2. This diagram shows conductive heat transfer and heat absorption within the solid media such as cover, drum and heat absorbers shown in the blocks. Radiative heat transfers between air and other components are shown by three resistances representing the surface and the bulk radiative resistances. Physical and thermal properties of the component media such as the vinyl cover, the metal drum and the heat absorbers can be obtained from data published in the ASHRAE handbook (ASHRAE). 

4 Computer circuit analysis program

A microcomputer-based program has been developed (Chao & Yu) for solving the electrical circuit currents and voltages of the simulation model. The calculated thermal parameters are converted to their equivalent electrical parameters using a Time Factor (TF), Capacitance Factor (CF) and Resistance Factor (RF). Outputs of the electrical voltage and current are converted to thermal temperature and heat, respectively using a voltage factor (VF) and current factor (IF). Values of these conversion factors can be arbitrarily selected but are subject to the following relationships: $VF = IF \times RF$ and $TF = CF \times RF$. The current factor (IF), capacitance factor (CF) and resistance factor (RF) are assigned with appropriate numerical values so
that the voltage factor (VF) and time factor (TF) are 1.0 and 2.0, respectively. Thus, the electrical voltage (V) can be directly converted into

Figure 2: Electrical Circuit Simulation model of the sludge drying system.

The computer simulation program input consists of data of the electrical simulation circuit in a convenient format as shown in the following section:

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$ Transient Response of Four Time-Constant Amplifier
TR ANALYSIS
C data obtained on November 11, 1981
C Feb. 20, 1993
B1 N(1,0), R=8.376E1
E1 P(0.5),
  *0.7,0.0,1.7,2.3,3.0,4.4,5.5,5.5,6.2,6.9,7.3,8.1,8.5,8.5,8.5,8.5,8.5,
  *8.8,8.5,9.2,7.7,6.2,5.5,4.8,2.7,2.3,2.0,1.7,1.3,0.7,0.0,0.0,0.0,0.0,0.0
B2 N(2,0), R=2.365E1
-----------(Programs lines are deleted)----------
B14 N(13,0), C=2.595E-5
```
Table 1: Listing of example computer electrical circuit simulation program.

The input starts with a statement to indicate the type of analysis, e.g. TR ANALYSIS. Its main body consists of descriptions of all electrical elements included in the simulation circuit as shown in Figure 2. Each line starts with an identification letter followed by a number. Letters "B" represents "branches", "E" indicates voltage input (for temperature input to the thermal system) and "I" shows current input (for heat input to the system). Following the identification, the two end nodes of the branch are specified. Type and value of the element contained in the branch are indicated. Letter "R" represents resistor while "C" represents capacitor. For example, B15 N(12,13), R=2.293E5 indicates Branch #15 which is between node 12 and 13, and contains a resistor with a value of 2.29x10^6 Ohms. Near the end of this program control commands for carrying out calculations and printing the results are included. In this program, the simulation is performed with a time increment of 0.05 hour (DT=0.05) but the results are printed every 1.0 hour (OUT INT = 1.0) for a total of 20 hours (FINISH = 20). The outputs are plotted for node 1 (PLOT, NV(1)) and node 18 (PLOT, NV(18)) only in this program. However, results of other nodes can be easily added by including statements such as "PLOT, NV(2)" for outputting the voltage or temperature for node #2. The current (or heat) of any node can also be plotted versus time by specifying "PLOT, NI(5)" for node #5.

5 Results and Conclusion

After executing the computer program, tabulated results and line drawings are produced and saved in an ASCII file. With the electrical circuit simulation model, time-dependent temperature variations of any component medium can be easily obtained. The model outputs of voltage and current are converted directly to temperature and heat. Examples of the simulated results are shown in Figures 3 and 4.
Figure 3: Simulated inside air temperature to fit the observed field data.

Figure 4: Simulated temperature of (A) inside air, (B) heat absorber facing the sun, (C) heat absorber in the shade and (D) sludge drum.
The simulation technique can be applied to other environmental systems as well. These systems may involve actual heat transfer such as an anaerobic digestion tank, or kinetic phenomena that can be represented by equations similar to those used for heat transfer. In both cases, if the system is complicated and numerical solutions are difficult to solve, the electrical circuit simulation is a valuable tool to simulate these systems and solve the mathematic problems.

References