A mathematical model for predicting the flushing characteristics and pollution susceptibility of Centerport Harbor, New York

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Abstract

This paper describes the development and application of a simplified mathematical model for assessing the flushing characteristics and pollution susceptibility of small, well-mixed tidal basins. The model is based upon a tidal prism approach and uses a novel recursive analytical technique to predict the long-term water quality response of the embayment. Predictions from the analytical formulation are validated against experimental pollution data obtained from laboratory tests on a 1:400 scale physical model of a generic square harbour. The results from the validation tests are very encouraging and demonstrate that the analytical formulation provides a viable method of estimating the water quality response of small tidal embayments. To demonstrate the practical potential of the recursive tidal prism technique, the model has been applied to Centerport Harbor, a natural tidal embayment in the Huntington Bay Complex along the northern shore of Long Island, New York, USA.

Keywords: pollution flushing, pollution susceptibility, tidal flushing, tidal prism, water quality.

1 Introduction

The determination of the flushing characteristics of small tidal embayments and the associated water quality implications are particularly important when conducting environmental impact assessments. Excessive accumulations of nutrients, plankton or contaminants in a tidal basin will degrade the water quality and adversely impact the health of the pelagic and benthic communities.
To assess the risk of poor water quality, it is important to be able to predict the rate of exchange of sea water between the basin and the surrounding water.

In recent years, there has been a significant increase in the use of multi-dimensional hydrodynamic models for assessing the flushing characteristics and circulation patterns of tidal embayments [1-5]. Typically, these models employ two-dimensional or three-dimensional simulations of the Reynolds averaged Navier-Stokes equations and advection-diffusion equation, and can often include additional modules to account for complex physical processes such as turbulence, cohesive sediment transport and the nitrification cycle. However, multi-dimensional models are time consuming to develop and may require high spatial resolution and small time-steps for stability. Furthermore, advanced multi-dimensional simulations can sometimes be unnecessarily complicated for estimating the flushing characteristics of small, relatively simple tidal embayments. For this reason, alternative numerical techniques employing simplified mathematical formulations often provide a more practical method of assessing the dispersion of pollution from a tidal embayment.

One of the most popular methods of simplifying the complexity of the flow problem is to adopt a ‘zero-dimensional’ (i.e. spatially uniform) mathematical approach based on the concept of repeated exchange of the inter-tidal volume. Simple tidal prism models for well-mixed estuaries have been available for a considerable period of time. For example, text books such as Harleman [6], Dyer [7] and Officer [8] discuss tidal prism and estuarine box models in some detail. However, it is interesting to note that none of these reviews give a reference to the development of tidal prism theory nor do they present a comprehensive examination of the underlying theory behind the approach. Instead, most text books assume complete familiarity with the tidal prism technique. This is somewhat surprising given the usefulness of the approach. Fortunately, a number of recent articles have presented a detailed exposition on the assumptions and limitations of tidal prism models (Luketina [9]; Barber [10]; Barber and Wearing [11]).

The analytical technique described in the present paper is based upon the Simplified Tidal Embayment Assessment Model (STEAM) developed by DiLorenzo et al. [12,13] for predicting the water quality response and pollution susceptibility of small tidal basins. The mathematical model proposed by DiLorenzo and his co-workers was developed from an earlier tidal prism formulation presented by Isaji and Spaulding [14]. DiLorenzo et al. [15] subsequently improved their tidal prism approach by including a return-flow factor to account for the pollution which leaves the basin on an ebb tide but returns on the subsequent flood tide. Callaway [16], van de Kreeke [17] and Ozhan [18] have proposed similar concepts for estimating the residence of marinas. More recently, Barber and Wearing [19] have developed a novel extension of the tidal prism approach to estimate the water quality response of embayments subjected to a continuous release of pollution throughout the tidal cycle. This model has subsequently been refined by Barber and Wearing [20,21] to allow the pollution susceptibility of the embayment to be determined using an analytical formulation.
2 Mathematical model

A full derivation of the water quality model has previously been presented by Barber and Wearing [20]. The mixing and dilution processes within the tidal basin are markedly different during the flood and ebb cycles and therefore the analysis is partitioned into separate intervals depending upon the direction of the tide. To obtain a tractable set of equations that can be solved analytically, the basin is assumed to experience repetitive sinusoidal tides of constant amplitude. During the ebb tide, the pollutant concentration, \( C \), (an analogue for the water quality), and the volume of water, \( V \), inside the basin satisfy the following equations:

\[
\frac{d}{dt}(CV) = C \frac{dV}{dt} + V \frac{dC}{dt} = Q C + k \quad \text{and} \quad \frac{dV}{dt} = Q
\]

where \( Q \) is the discharge through the entrance (\( Q > 0 \) on the flood tide, \( Q < 0 \) on the ebb tide) and \( k \) is the internal pollution loading rate (measured as a mass per unit time). Combining these expressions leads to the following differential equation governing the water quality response on the ebb tide:

\[
\frac{dC}{dt} = \frac{k}{V} .
\]

Integration of eqn. (2) over the \( n^{th} \) ebb tidal cycle gives

\[
C_e(n) = C_f(n-1) + \frac{k \pi}{\omega \sqrt{V^2_m - V^2_t}}
\]

where \( C_e(n) \) is the pollutant concentration at the end of the \( n^{th} \) ebb tide, \( C_f(n-1) \) is the concentration at the end of the previous flood cycle, \( V_m \) is the mean volume of the tidal basin, \( V_t \) is the amplitude of the oscillatory component of the tidal volume, and \( \omega \) is the tidal angular velocity given by \( \omega = 2\pi / T \) where \( T \) is the period of the tide.

Similarly during the flood tide, the pollutant concentration and volume of water inside the embayment satisfy the following differential equation:

\[
\frac{d}{dt}(CV) = k .
\]

Integration of this equation over the \( n^{th} \) flood tide, yields

\[
C_f(n) = \left( \frac{V_m - V_t}{V_m + V_t} \right) \left( C_e(n) + \frac{k \pi}{\omega \sqrt{V^2_m - V^2_t}} \right)
\]

while substitution of \( C_e(n) \) from eqn. (3) enables the water quality response to be written as a recurrence relationship:

\[
C_f(n) = \left( \frac{V_m - V_t}{V_m + V_t} \right) \left( C_f(n-1) + \frac{k \pi}{\omega \sqrt{V^2_m - V^2_t}} + \frac{k \pi}{\omega (V_m - V_t)} \right) .
\]
Repeated application of eqn. (6) for \( n = 1, 2, 3, \ldots \) allows the pollutant concentration after \( n \) flood tides to be written in terms of the initial concentration, \( C_0 \):

\[
C_{f(n)} = C_0 \left( \frac{V_m - V_t}{V_m + V_t} \right)^n + \left( \frac{k \pi}{\omega \sqrt{V_{m}^2 - V_t^2}} \right) \left( \frac{k \pi}{\omega (V_m - V_t)} \right) \sum_{i=1}^{n} \left( \frac{V_m - V_t}{V_m + V_t} \right)^i
\]

while substitution of \( C_{f(n-1)} \) into eqn. (3) yields the pollutant concentration after \( n \) ebb tides:

\[
C_{e(n)} = C_0 \left( \frac{V_m - V_t}{V_m + V_t} \right)^{n-1} + \left( \frac{k \pi}{\omega \sqrt{V_{m}^2 - V_t^2}} \right) + \left( \frac{k \pi}{\omega (V_m - V_t)} \right) \sum_{i=1}^{n-1} \left( \frac{V_m - V_t}{V_m + V_t} \right)^i
\]

The predicted water quality response has to be adjusted to account for pollution which leaves the embayment on an ebb tide and returns on the subsequent flood tide: the so-called ‘return-flow’ effect. In the present model, a convenient (although approximate) method of accounting for the return-flow is to reduce the amplitude of the oscillatory tidal prism. Thus, the effective amplitude of the oscillatory tidal volume can be expressed as:

\[
V_t' = (1 - b) V_t
\]

where \( b \) is the pollution return parameter between 0 and 1. The pollution return factor can sometimes be evaluated theoretically (Sanford et al. [22]) although in most cases the value has to be found by calibrating the analytical model against observed pollution data.

### 3 Long-term water quality response

The long-term water quality response of the embayment can be determined by considering the values of \( C_{f(n)} \) and \( C_{e(n)} \) as the number of tidal cycles, \( n \), tends to infinity. By definition:

\[
\left( \frac{V_m - V_t}{V_m + V_t} \right) < 1
\]

and therefore the influence of the initial concentration, \( C_0 \), becomes less significant as the number of tidal cycles increases. Using geometric series, it can be shown that

\[
\lim_{n \to \infty} \sum_{i=1}^{n} \left( \frac{V_m - V_t}{V_m + V_t} \right)^i = \frac{\left( V_m - V_t \right)}{2V_t}
\]

and therefore the limit-cycle concentrations at the end of the flood and ebb tides are given by
\[
\lim_{n \to \infty} C_f(n) = \left( \frac{k \pi}{\omega \sqrt{V_m^2 - V_t^2}} + \frac{k \pi}{2V_t} \left( \frac{V_m - V_t}{\omega (V_m - V_t)} \right) \right)
\] (12)

and

\[
\lim_{n \to \infty} C_{e(n)} = \left( \frac{k \pi}{\omega \sqrt{V_m^2 - V_t^2}} + \frac{k \pi}{2V_t} \left( \frac{V_m - V_t}{\omega (V_m - V_t)} \right) \right) + \frac{k \pi}{\omega \sqrt{V_m^2 - V_t^2}}. \] (13)

### 4 Pollution susceptibility

The concept of *pollution susceptibility* (PS) of a tidal embayment was first proposed by Weyl [23] and can be defined as the spatially and temporally averaged concentration of a dissolved conservative pollutant which results from a unit rate of discharge. For example, a pollution susceptibility of 10 ppb cyc/kg indicates that a steady discharge of 1 kg of conservative pollutant per tidal cycle would result in an average basin contamination of 10 ppb (10^-2 ppm). The concept can therefore be used to identify tidal basins at risk from potential water quality problems.

Applying the Simplified Tidal Embayment Assessment Model (STEAM) to a hydraulically-efficient embayment, DiLorenzo *et al.* [13] derived an expression for the pollution susceptibility of a tidal basin in terms of the volume of the tidal prism, \( V_t \), the pollution return-flow factor, \( b \), and the freshwater inflow rate, \( Q_f \). They demonstrated that the pollution susceptibility could be written as:

\[
PS = \frac{10^6}{2(1-b)V_t + Q_f} \text{ ppb cyc/kg} \] (14)

where the freshwater inflow rate is expressed in units of cubic metres per tidal cycle.

To determine the pollution susceptibility from the present analytical model, the contaminant loading rate is assumed to be \( k = 1/T \text{ kg/s} \) (corresponding to a loading rate of 1 kg of pollutant per tidal cycle). Since the pollutant concentrations presented in Sections 2 and 3 are measured in kg/m^3, a conversion factor of \( 10^6 \text{ ppb m}^3/\text{kg} \) (based upon an assumed water density of 1000 kg/m^3) must be applied to maintain the desired units for pollution susceptibility. Eqs. (7) and (8) can thus be recast as pollution susceptibilities at the end of the \( n^{th} \) flood and \( n^{th} \) ebb tide:

\[
PS_{f(n)} = C_0 \left( \frac{V_m - V_t}{V_m + V_t} \right)^n + \left( \frac{10^6}{2\sqrt{V_m^2 - V_t^2}} + \frac{10^6}{2(V_m - V_t)} \right) \sum_{i=1}^{n} \left( \frac{V_m - V_t}{V_m + V_t} \right)^i
\] (15)

and
\[ PS_{e(n)} = C_0 \left( \frac{V_m - V_t}{V_m + V_t} \right)^{n-1} + \frac{10^6}{2\sqrt{V_m^2 - V_t^2}} + \left( \frac{10^6}{2\sqrt{V_m^2 - V_t^2} + \frac{10^6}{2(V_m - V_t)}} \right) \sum_{i=1}^{n-1} \left( \frac{V_m - V_t}{V_m + V_t} \right)^i \] 

where the initial concentration, \( C_0 \), is now measured in parts-per-billion (ppb). The long-term (or limit-cycle) pollution susceptibilities (measured in ppb cyc/kg) can similarly be written as

\[
\lim_{n \to \infty} PS_{f(n)} = \left( \frac{10^6}{2\sqrt{V_m^2 - V_t^2}} + \frac{10^6}{2(V_m - V_t)} \right) \left( \frac{V_m - V_t}{2V_t} \right)
\]

and

\[
\lim_{n \to \infty} PS_{e(n)} = \left( \frac{10^6}{2\sqrt{V_m^2 - V_t^2}} + \frac{10^6}{2(V_m - V_t)} \right) \left( \frac{V_m - V_t}{2V_t} \right) + \frac{10^6}{2\sqrt{V_m^2 - V_t^2}}.
\]

5 Experimental validation

Barber and Wearing [20,21] and Wearing [24] have previously described the laboratory validation tests for assessing the accuracy of the analytical water quality model. The experiments considered a series of idealised flat-bottomed rectangular embayments with vertical side-walls. For compatibility with experimental data reported by Falconer and Yu [25], the plan-form area of the basin was varied from 432 m x 432 m (18.7 hectares) to 864 m x 432 m (37.3 hectares). The laboratory model was constructed according to Froude number similarity at a horizontal scale of 1:400 and a vertical scale of 1:40. Pollution levels in the tidal basin were measured using sodium fluorescein dye and a luminescence spectrometer. The embayment was subjected to a known discharge of sodium fluorescein flowing from a constant head Marriotte vessel to simulate a continuous release of pollution. Water quality sampling was carried out at high tide until the pollution levels within the basin reached limit-cycle conditions.

Inspection of the analytical formulation demonstrates that the water quality response depends crucially on the value of \( \alpha = (V_m-V_t)/(V_m+V_t) \), the ratio of the volume of water in the embayment at low tide to the volume of water at high tide. Prototype semi-diurnal tidal ranges of 2, 4 and 6 m were considered and the maximum water depth was assumed to be 8 m, giving values of \( \alpha \) of 0.75, 0.50 and 0.25, respectively. The water quality predictions were computed using a pollution return parameter of \( b=0.135 \) determined from an initial calibration test. Figs. 1 and 2 present the results of the pollution discharge tests and show good correlation between the analytical predictions and the observed pollution susceptibilities. The stray experimental data clearly demonstrate the difficulties in measuring the fluorescein dye concentrations even under carefully controlled laboratory conditions. Nevertheless, the close correlation between the
experimental data and the analytical predictions demonstrates that the proposed mathematical formulation offers a viable and accurate method of determining the water quality response in well-mixed tidal embayments. Fig. 1 also demonstrates that the tidal range has a significant effect on the length of time required to achieve the long-term water quality response. For example, the test involving the 2 m prototype tide took in excess of 15 tidal cycles to reach steady-state, while the test involving the 6 m tide reached steady-state after only 4 tidal cycles.

Figure 1: Experimental validation of proposed analytical pollution susceptibility model (432 m x 432 m square embayment).

Figure 2: Experimental validation of proposed analytical pollution susceptibility model (864 m x 432 m square embayment).
6 Application to Centerport Harbor

To demonstrate a practical application of the proposed mathematical model, the technique has been used to estimate the water quality response of Centerport Harbor in the Huntington Bay Complex – a series of interconnected tidal embayments along the northern shore of Long Island, New York, USA (Figs. 3 and 4). Centerport Harbor is a natural tidal basin which serves as both a recreational and commercial port as well as a productive shellfish habitat. The inevitable conflicts between the economic and environmental aspirations of the local community create problems when managing the aquatic ecosystem.

Figure 3: Location map of the Huntington Bay Complex, Long Island, USA.

Figure 4: Digital elevation model of the Huntington Bay Complex, Long Island.
The Huntington Bay Complex is characterised by wide peninsulas (or necks) of land separated by long, finger-like natural harbours. The land making up the necks is underlain by a complicated mixture of glacial deposits laid down twenty thousand years ago during the last ice age. These deposits include outwash, till, loess as well as older Cretaceous sediments. The distinctive coastal morphology around Huntington Bay is typical of a micro-tidal estuary and includes numerous spits and bars together with the Asharoken tombolo.

Centerport Harbor is connected to Huntington Bay by a narrow inlet channel extending some 500 m (1,640 ft) into the interior of the basin. The overall dimensions of the harbour are approximately 1270 m (4,165 ft) long, 250 m (820 ft) wide with a mean-low-water basin depth of 0.6 m (2ft). The mean tidal range is 2.2 m (7.2 ft) while the spring tidal range is 2.6 m (8.4 ft). There are no significant freshwater streams entering the harbour, but a limited amount of freshwater enters the embayment in the form of direct surface runoff and groundwater seepage. The spring tide conditions in the harbour can therefore be characterised by the following parameters: average surface area of basin, \( A = 317,500 \text{ m}^2 \), mean volume of water, \( V_m = 539,750 \text{ m}^3 \) and volume of oscillatory component of tidal volume, \( V_t = 412,750 \text{ m}^3 \).

DiLorenzo et al. [15] reported the results of a field experiment carried out in Centerport Harbor to address some of the environmental concerns about the water quality. Tidal elevations were continuously sampled in the embayment, while an instantaneous dye-release experiment was performed by injecting 1.8 kg (4 lbs) of rhodamine WT dye into the upstream tributary at high water on a spring tide. The resultant spatial distributions of the rhodamine dye were sampled until the concentration approached the limit of detection (approximately two tidal cycles after the initial release). The field survey revealed that the semi-diurnal tides in Centerport Harbor are almost identical (in both magnitude and phase) to the tides in the adjacent coastal bay, demonstrating that the embayment may be classified as a ‘hydraulically efficient’ system by virtue of its unattenuated tidal response. This was quite an important finding since DiLorenzo et al. [13] had previously shown that hydraulically efficient tidal basins are generally less susceptible to water quality problems in comparison to ‘hydraulically constricted systems’ which have an attenuated tidal response.

The results of the rhodamine dye experiment can be used to estimate the pollution-return parameter for Centerport Harbor. The return-flow factor, \( b \), was found by DiLorenzo et al. [15] to be approximately 0.1, indicating relatively efficient mixing between the effluent plume and the surrounding coastal water just outside the embayment. Knowledge of the return-flow parameter then enables the pollution susceptibility of the harbour to be determined. Applying eqn. (14) to Centerport Harbor and assuming that the freshwater inflow is negligible yields a pollution susceptibility of 1.35 ppb cyc/kg. In other words, a toxic discharge of 1 kg/cycle of a conservative substance (for example, lead) would increase the pollutant concentration in Centerport Harbor by about 1.4 parts-per-billion (ppb).

The present mathematical formulation has the advantage over more traditional tidal prism models of being able to estimate the average basin...
concentration at the end of both the flood and ebb tides. Applying eqns. (17) and (18) to Centerport Harbor and assuming a 10% reduction in the oscillatory tidal volume to account for the return-flow effect yields pollution susceptibilities of 0.96 ppb cyc/kg at the end of the flood tide (high water) and 2.24 ppb cyc/kg at the end of the ebb tide (low water). While the pollution susceptibilities could be averaged to obtain a temporal mean value of 1.6 ppb cyc/kg, it is generally more appropriate to express the water quality response of the embayment in terms of both these values. The mean pollution susceptibility agrees reasonably well with DiLorenzo et al.’s [15] estimate for Centerport Harbor. However, the present analytical model suggests that the water quality in the embayment deteriorates considerably during the ebb tide, with the pollution susceptibility increasing well in excess of the value predicted by DiLorenzo et al. [15]. The ebb tide pollution susceptibility eqn. (18) provides a more stringent estimate of the likely long-term maximum pollution level and would therefore highlight any potential water quality problems at an earlier threshold than existing tidal prism models.

7 Conclusions

This paper has described a novel analytical method for estimating the flushing characteristics and pollution susceptibility of well-mixed tidal embayments. The mathematical model employs a tidal prism approach and uses an analytical technique to predict the water quality response. Predictions from the mathematical formulation are first compared with experimental pollution data from a series of laboratory tests on square and rectangular harbours. The results are very encouraging and show that the tidal prism approach has considerable promise for predicting the water quality response of well-mixed tidal basins. The proposed model is able to provide accurate estimates of pollution susceptibility over a wide range of tidal regimes and offers a computationally inexpensive alternative to conventional multi-dimensional pollution transport simulations.

The mathematical model has also been applied to Centerport Harbor, a natural tidal embayment in the Huntington Bay Complex along the northern shore of Long Island, New York, USA. The water quality predictions show that conventional tidal prism models considerably underestimate the pollution susceptibility of the basin and fail to account for the deterioration in water quality during the ebb tide. By analysing the flood and ebb tidal cycles separately, the present tidal prism formulation is able to account for the different mixing and dilution processes on the incoming and outgoing tides.

Further investigations are required to test whether the proposed mathematical techniques are appropriate for complex embayments which have less internal mixing. Complex tidal basins are likely to experience large internal gradients in pollutant concentration, with areas remote from the entrance exhibiting slower flushing rates than predicted by the present analytical model. An extension of the tidal prism approach could be feasible using two or more separate ‘prisms’ to represent the behaviour of a system of inter-connecting tidal basins. However, the problem of estimating the individual return-flow factors may be unacceptably empirical. Nevertheless, the present model has been shown to provide accurate
estimates of the flushing characteristics for simple basin geometries and can produce water quality predictions quickly with the minimum of computational effort.

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References


