



# AC electric traction system analysis: modelling and simulation in railway operation

P. Firpo, S. Savio

*Dipartimento di Ingegneria Elettrica, Università di Genova,  
Via all'Opera Pia, 11a - 16145 Genova, Italy*

## ABSTRACT

Simulation tools are of notable help in the design of electrified transport systems, as they make it possible to analyze the overall behaviour of a system and hence to verify the correctness of the design parameters or to establish the modifications to be performed. The computation code proposed in this article allows one to carry out the analysis of the overall steady-state behaviour of a traction system; more precisely, it is an integrated simulator for A.C. electrified double-track railway lines.

## INTRODUCTION

In recent years, the technological evolution undergone by computing systems has provided designers with very fast, reliable and efficient simulation tools for verification purposes.

In the design of particularly complex systems, a manual verification generally involves considerable simplifications which are of course conservative, hence they may often lead to excessive oversizing. In such cases, the possibility of using a simulation tool with the above characteristics, and that is also simple and easy to employ, constitutes a great advantage. By evaluating the overall dynamic behaviour of a system, a system simulator allows the designer to verify the design correctness and to quantify a possible oversizing accurately.

The design of electrified transport systems particularly benefits from such a simulator, as such design requires, during the verification

## 308 Software Applications in Electrical Engineering

phase, the integrated solution of mechanical and electrical problems, instant by instant.

When an electrified transport system is rather complex, as in the case of AC autotransformer supplied systems, a manual verification of even simplified situations involves a notably high computational load. This paper describes a simulation tool for the analysis of an AC electrified double-track railway line in two different network configurations: the traditional and the autotransformer supplied one. The characteristics of such configurations are defined and then the functioning and the structure of the simulator are described; in particular, the models of the various subsystems are detailed.

### AC TRACTION SYSTEMS

In this section, AC electrified traction systems operating at the network frequency are considered, in particular, the focus is on railway transport systems.

At present, the most widely used systems can be divided into two main categories:

1. 25 (50) kV 50 Hz single-phase systems;
2. 2x25 kV 50 Hz autotransformer supplied systems.

The elements making up the former system are the catenary (or contact line), the rails, and the electrical substations (ESSs). The contact line and the rails are subdivided into electrically isolated blocks, each of which is fed by one of the two transformer groups of the electrical substations. By using the technique of the phase rotation, it is possible to reduce the effects of the unbalance introduced into the high-voltage three-phase line by the single-phase transformers feeding the sections into which the traction line is subdivided. Figure 1 shows the scheme of a line fed by a classic single-phase system.

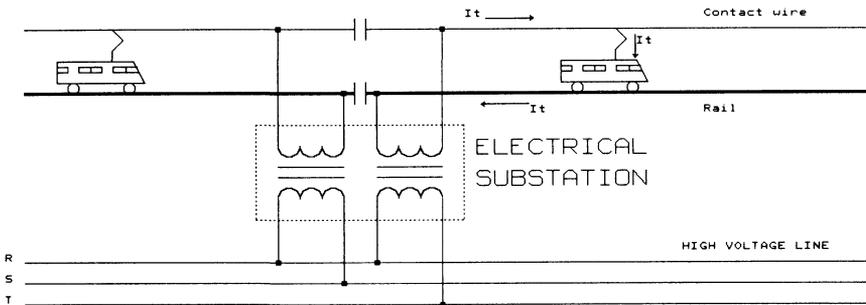


Figure 1: Principle scheme of a single-phase AC traction system

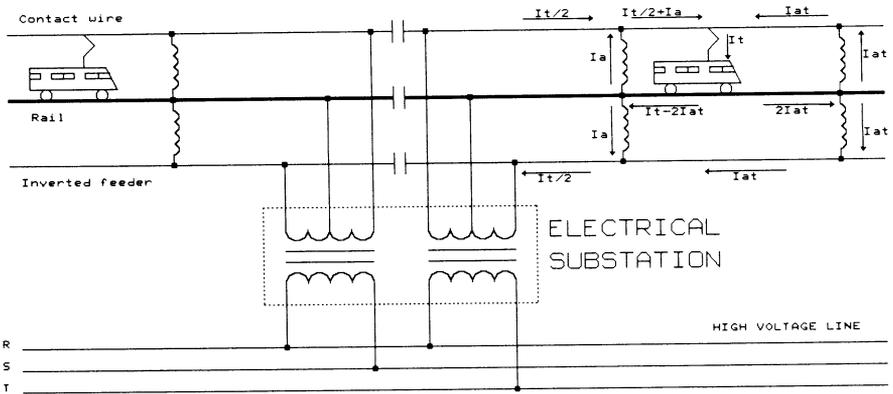


Figure 2: Principle scheme of an AC autotransformer supplied traction system

The electrical structure of the latter system is slightly more complex. In addition to the elements required by classic single-phase systems, the systems belonging to the second category use a third conductor, called “inverted feeder”, and autotransformers (ATs). Moreover, the transformer groups of an electrical substation must be equipped with a central-tap secondary winding at a rated voltage of 50 kV. In these systems, too, the contact line, the rails and the inverted feeder are subdivided into isolated blocks, each of which is fed according to the scheme in Fig. 2.

Each transformer group feeds a section: the central tap is connected to the rails, whereas the remaining terminals feed the catenary and the inverted feeder.

The autotransformers are connected along the line, at distances ranging between 10 and 15 Km; the primary is connected between the contact line and the feeder and the secondary is connected between the rails and the feeder. Each line section is always closed at its end on an autotransformer and the autotransformers are usually connected, in a redundant configuration. This makes it possible to vary the section configurations in case of failure of an electrical substation.

Figures 1 and 2 show also the distributions of line currents for two systems. For the 2x25 kV system (Fig. 2), the current configuration is valid under the assumptions of ideal ATs and of perfect symmetry between the contact line and the feeder. From a comparison of the current distribution for the two systems, one can deduce the advantages of the 2x25 kV system over the classic 25 kV one.

The connections of the ATs involves a sharp reduction in voltage drops, as shown in the graph in Fig. 3. In this diagram, the voltage

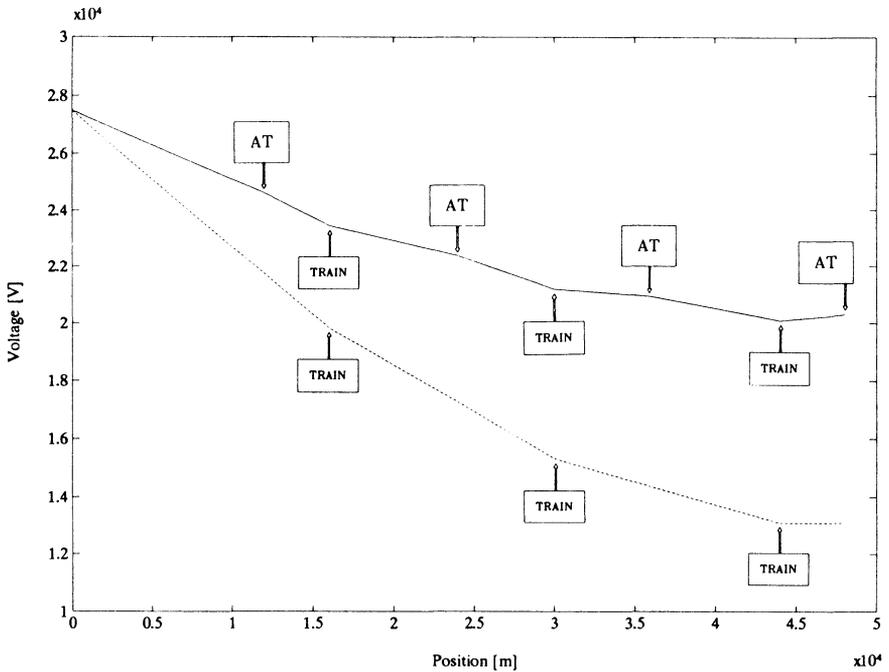


Figure 3: Pantograph voltage behaviours of an AC autotransformer supplied traction system (continuous line) and a single-phase one (dashed line); ESS located in the x-axis origin

ideal profiles along the catenary for the two systems are compared, under equal load conditions; the x-axis origin coincides with the location of the electrical substation.

For equal headways, the reduction in voltage drops makes it possible to increase the distances between the substations and then to reduce the number of substations required to ensure a given potentiality. For equal topological configurations of the substations, such reduction allows one to increase the line potentialities.

## THE 2x25 kV SYSTEM: THE ELECTRICAL MODELS OF THE SUBSYSTEMS

For simulation purposes, the solution of the electrical network for the classic 25 kV system is derived from the solution for the 2x25 kV system by imposing some constraints which will be described later on. In this section, the electrical models of the subsystems are described.

Such subsystems include the central-tap transformer groups, the substations, the autotransformers, the conductors (contact line, rails,

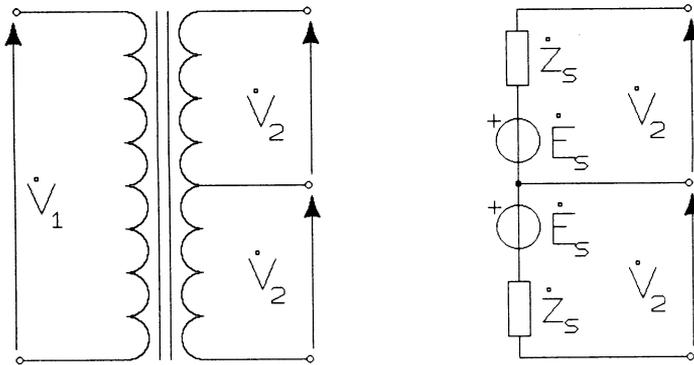


Figure 4: ESS transformer and the adopted equivalent model

and inverted feeder) and the train.

#### Substation transformer groups

Each substation consists of two central-tap transformer groups, each of which, under proper functioning conditions, fed a section, as shown in Fig. 3. The feeding scheme can be modified in case of failure of a group. In this case, the two line sections fed by the substation can be connected, thus forming a single section fed by the service group according to a T-shaped scheme. Figure 4 shows the scheme of the equivalent circuit used for this subsystem, where  $E_S$  and  $z_S$  are the no-load voltage and the leakage impedance of each secondary half-winding respectively.

#### Autotransformers

An autotransformer is modelled as two coupled inductors according to the scheme shown in Fig. 5. In the time domain, the relevant equations describing the functioning of the autotransformer are the followings:

$$v_1 = v_{2+} + v_{2-}$$

$$v_{2+} = L \frac{di_+}{dt} + M \frac{di_-}{dt} \quad v_{2-} = L \frac{di_-}{dt} + M \frac{di_+}{dt}$$

where  $L$  = self inductance of a half-winding;  
 $M$  = mutual inductance of the two half-windings;

The parameters  $L$  and  $M$  can be simply derived from the autotransformer data sheet.

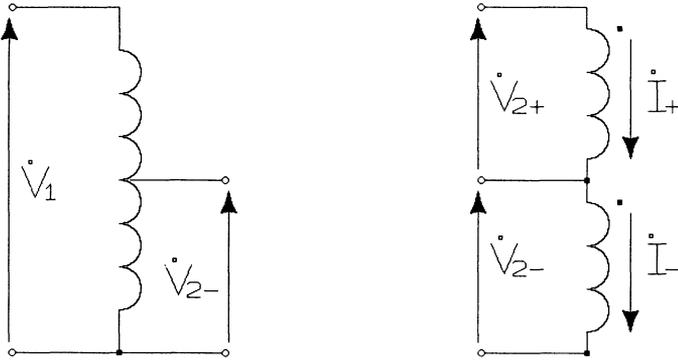


Figure 5: Line autotransformer and the adopted equivalent model

### Train

The electrical model of a train is rather a difficult problem, in that a train is a nonlinear element at least from the electrical standpoint. The known term in the equations for a train is, instant by instant, the mechanical power delivered at the axle and then the electrical power absorbed by the railway line, once known electric drive and mechanical apparatus efficiency.

This represents the product of train electrical unknowns (voltage and current) and therefore it is necessary to linearize the model in order to solve a system of linear equations for the electrical network.

In the linearized model, the train is represented as a suitable ohmic-inductive impedance, whose value can be determined through a convergence process once known train power factor. This process is summarized in the following and shown in Fig. 6. A voltage value at the train pantograph is assumed. The tentative value will be the rated system voltage during the initialization phase, that is, the voltage value at the previous step for the successive integration steps. On the basis of the tentative value, one can assume a first linear model and then solve the electric network by determining a new voltage value. If the new value differs from the tentative value by more than  $\epsilon$ , it is assumed as a new tentative value and the modelling and solution of the system are repeated until the voltage oscillation is included in a suitable tolerance range.

Obviously, the convergence process is performed for all the trains on the section under examination.

### Conductors

According to the concentrated parameters equivalent model, conductors are represented as ohmic-inductive impedances, whose

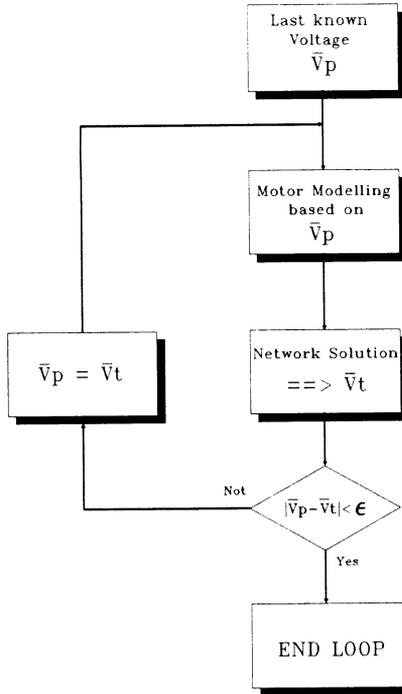


Figure 6: Convergence process block diagram

values are related, in a linear way, to the length of the line section modelled by such impedances. To simplify the model, the effects of the mutual coupling between adjacent conductors (e.g., catenary and feeder) are neglected, and only the inductance of the coil that each conductors form with the ground is considered. During simulation testing, carried on by general purpose program EMTP, this approximation has turned out to be conservative, and it leads to slight but reasonable oversizings.

## ELECTRIC NETWORK SOLUTION METHODOLOGY

Once the models of the subsystems and the equations that describe their electrical characteristics have been defined, one has to select the method for solving the electrical network related to the overall system at the integration step analyzed.

For an automatic solution of a linear electric circuit made up of passive components (like the circuit under examination), two main approaches are proposed in the literature: the node analysis and the



mesh analysis. In this paper, the method selected for the simulator is the node analysis, for which the unknowns are the potentials of all the network nodes. Such potentials are referred to the earthed secondary central points of the transformer groups in the substations. As a consequence, each unknown interacts only with the unknowns related to the previous node and to the next one, or only to one of them, if the other does not exist (as is the case with the transformer group and with the autotransformer at both ends of a section). This notably simplifies the procedure for reconstructing the system network.

The definition of the nodes and the network reconstruction must be repeated at each integration step, for the system configuration changes at every instant, while trains are running along the line.

Once known network configuration, the vector of unknowns is computed, and the admittance matrix is obtained by associating specific descriptive equations with each node. The inversion of this matrix leads to the determination of the node potentials, from which all the other quantities of interest can be derived. Figure 7 shows ESS, AT and train typical nodes.

## INTEGRATED SIMULATOR BLOCK DIAGRAM

The convergence process required to linearize the train model constitutes the main nucleus of the proposed simulator. The convergence algorithm is included in the structure shown in Fig. 8, which represents the simulation block diagram.

The program includes an off-line phase, for input reading and for data acquisition, and an on-line phase, in which the dynamic behaviour of the system is simulated. In the first phase, the input files containing the line description are read: the user must define curvatures and grades of the route, the locations of the passenger stations, the electric parameters of the network, the locations of substations, autotransformers and sectionalizing posts, the electrical characteristics of the substations and of the autotransformers and, finally, the electromechanical characteristics of the trains running on that line.

A time table programming the entry and exit of the trains is automatically prepared off line by the simulator (an example is given in Table I), on the basis of the scheduling data the user must include in an input file, together with the other parameters for simulation control, such as the tolerance of the convergence algorithm and the integration step. The time table construction represents the end of the off-line phase of the program.

The on-line phase consists of a main loop, which is repeated at



# Software Applications in Electrical Engineering 315

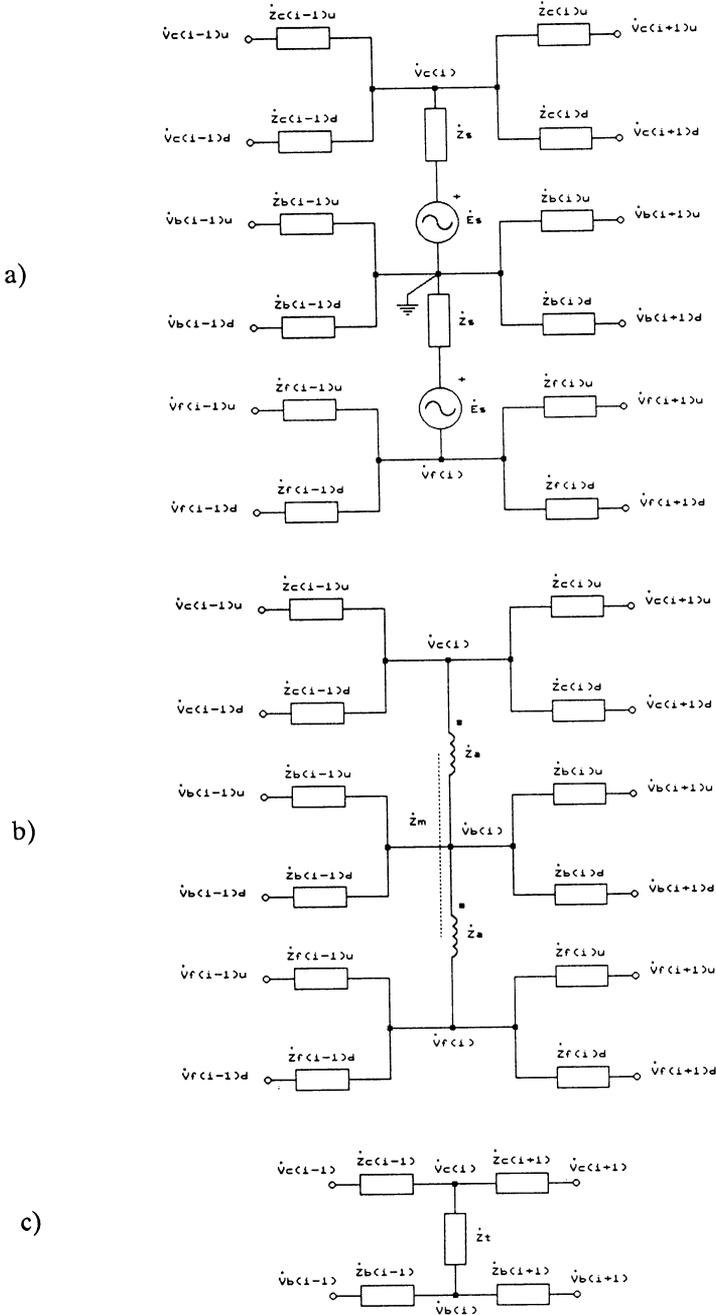


Figure 7: Electrical substation (a), autotransformer (b) and train (c) nodes

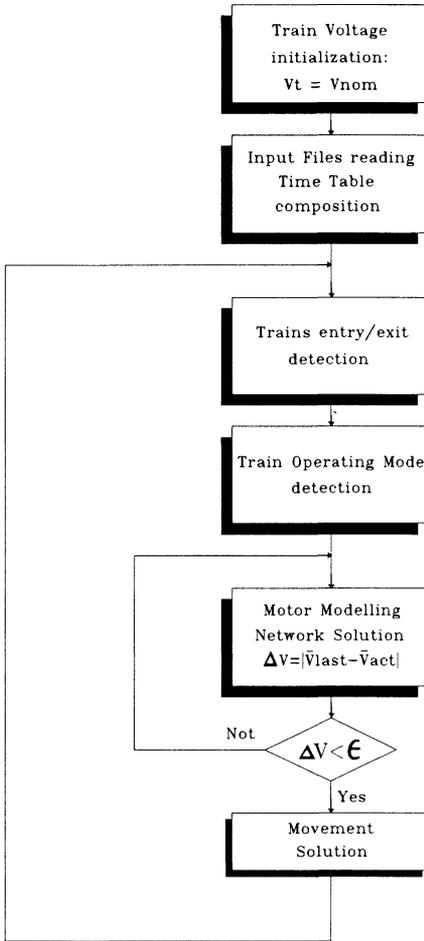


Figure 8: Simulator block diagram

Table I

Train number	Entry headway	Entry line	Entry position [m]	Exit position [m]	Entry speed [km/h]	Vehicle consist	Reverse time up/down [s]	Reverse time down/up [s]	Vehicle model
1	1	uproad	50	149560	0	1	3600	3600	1
3	2	uproad	50	149560	0	1	3600	3600	1
5	3	uproad	50	149560	0	1	3600	3600	1
7	4	uproad	50	149560	0	1	3600	3600	1
9	5	uproad	50	149560	0	1	3600	3600	1
2	1	downroad	40	149550	0	1	3600	3600	1
4	2	downroad	40	149550	0	1	3600	3600	1
6	3	downroad	40	149550	0	1	3600	3600	1
8	4	downroad	40	149550	0	1	3600	3600	1
10	5	downroad	40	149550	0	1	3600	3600	1



each integration step.

The first operation to be performed, at each time step, lies in checking if any train has to entry or, being at the end of its route, has to leave the line. Of course, this operation is performed for both tracks and, if a train, for instance, has left the line, the list of trains on the line is updated. If a reverse is required, the program places the trains to run on the opposite track after the reverse time.

At the end of this phase, one knows how many trains are running on the line; then one can start managing the traffic.

On the basis of the position and of the speed of each train, and according to the characteristics of the line (speed limits, station locations), a specific type of operation is chosen for that train in terms of traction, braking, coasting, and stop times in the stations. Then the electrical part of the simulator is considered, and the convergence process described in one of the previous sections is started.

The inversion of the admittance matrix leads to the determination of all node potentials, from which one can derive the pantograph voltages of all trains, the voltages of the substations and of the autotransformers, and the currents in each branch.

At the output of the convergence process, the consistency of the electrical parameters with the mechanical ones is ensured. Therefore, the equation of motion can be solved, which updates the cinematic variables (position and speed) of each train. At this point, the main loop ends, the next integration step begins, and so on, until the end of the simulation.

The simulation of 25 kV lines, can be performed very simply by defining a line with no autotransformers, and by considering, for the transformer groups of the substations, only the part of secondary through which the current flows.

Results are stored in the simulator's output files. The program produces ASCII files that contain all simulation results. A suitable post-processor, developed by the authors, is able to provide integral results on a headway for substations and autotransformers.

This post-processor also produces files of a format compatible with MS-DOS graphic programs, which may be used to print the graphs related to the temporal load diagrams of the substations, to the pantograph voltage and current profiles, and to the train running curves.

The code has been written in FORTRAN 77 language and implemented on the Data General Aviiion 3000 workstation, which is a machine based on the RISC 88000 processor.



## CONCLUSIONS

The complexity of an electrified transport system, such as the 2x25 kV 50 Hz autotransformer supplied one, requires the use of automatic computation codes able to simulate the real behaviour of the system, thus avoiding onerous oversizing during its design.

The simulation program proposed in this paper is a useful verification tool which needs anyway a basic design realized by classic methods. It is well known that software design tools cannot replace the designer but can support his work by enabling him to verify the correctness of his choices or by suggesting appropriate modifications.

The availability of fast and efficient computing machines has made it possible to develop an integrated code simple to use, requiring short execution times, and easily utilizable in real applications. Such characteristics are basic to an efficient tool for system verification.

## REFERENCES

1. Duncan Glover, J., Kusko, A., Peeran, Syed M. "Train Voltage Analysis for AC Railroad Electrification", *IEEE Trans. on I.A.*, Vol. IA-20, No.4, July-August 1984.
2. Mellit, B., Allan, J., Shao, Z.Y., Johnston, W.B., Goodman, C.J. "Computer-based methods for induced-voltage calculation in AC railways", *IEE Proc.*, Vol. 137, Pt. B, No. 1, January 1990.
3. Roussel, H. "Power Supply for the Atlantic TGV High Speed Line", *IEE Int. Conf. on Main Line Railway Electrification*, No. 312, London, 1989.
4. Kneschke, Tristan E. "Simple Method for Determination of Substation Spacing for AC and DC Electrification Systems", *IEEE Trans. on I.A.*, Vol. IA-22, No. 4, July-August 1986.
5. Tierney, J.R., Turner, R.J. "Improvement of the booster transformer/return conductor method of suppressing 50 Hz interference from a.c. electrified railway systems", *IEE Proc.*, Vol. 128, Pt. B, No. 1, January 1991.
6. Mellit, B., Johnston, W.B., Allan, J., Denley, M. "Whole system compatibility measurements of the autotransformer supplied 50 kV railway in New Zeland", *IEE Int. Conf. on Main Railway Electrification*, No. 312, London 1989.
7. Firpo, P., Savio, S., Sciutto, G. "SIAV: un codice di calcolo automatico per l'analisi elettrica del sistema Alta Velocità 2x25 kV 50 Hz", *Convegno "Sviluppo e prospettive dei trasporti elettrificati: ricerca ed innovazione"*, Vol. 1, Genova, November 25-27, 1992.