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Analysis and design of DC/DC converters using symbolic approach

S. Manetti, M.C. Piccirilli
Department of Electronic Engineering, University of Florence, Via S. Marta, 3-50139 Florence, Italy

ABSTRACT

An MS-DOS personal computer program for the analysis and design of DC/DC converters is presented. The program, named SPAC (Symbolic Program for Analysis of DC/DC Converters), is based on symbolic analysis techniques and is able to calculate automatically, starting from a graphical schematic entry, the circuit s-domain network functions in a completely symbolic form. By means of suitable algorithms, it is possible to perform their numerical inverse Laplace transform. Then the graphic representation of both time and frequency responses can be obtained.

INTRODUCTION

The symbolic analysis is a process which permits to obtain network functions of electric circuits as expressions depending on symbolic parameters. It is used for linear, time-invarying, analog or time discrete circuits, because only these kinds of networks allow a representation in transformed domains. The symbolic approach can be also applied to nonlinear time-invarying circuits by exploiting suitable linearization techniques, but generally it is not used for time-varying networks.

In this paper we propose the application of symbolic techniques to the analysis of a particular kind of circuits, which can be considered as linear, time-varying networks under suitable hypotheses. We are referring to DC/DC converters, whose periodical switching operation can be characterized by a linear, time-varying model, if the solid state devices are assumed to behave like ideal switches.

One of the most used analysis methods for these circuits is the state-space averaging technique [1]. It provides a powerful tool to analyze several classes of switching converters, because it uses appoximations which permit to represent them by means of linear systems. But this technique has the drawback of the

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decreasing accuracy as the frequencies of interest approach one half of the fundamental switching frequency of the converter [2].

The approach we propose overcomes this problem and presents advantageous characteristics due to its symbolic nature. By using suitable algorithms which permit to apply the Laplace transform to linear, time-varying circuits, we have developed a program, named SPAC (Symbolic Program for Analysis of DC/DC Converters). It performs the symbolic analysis of DC/DC converters and yields, as outputs, the graphic representations of both time domain and frequency domain responses. The s-domain symbolic network function availability gives remarkable advantages particularly in the design phase, because it makes very fast the choice of component values for the wanted behavior. In fact this procedure generally requires the repetition of a high number of simulations performed on the same circuit with the variation of component values and for this kind of application the symbolic approach is very useful, because it permits to make only once the circuit analysis and to repeat only the numerical evaluation.

In the following sections we describe the main features of the approach used for the realization of the program SPAC and, by means of an application example, we illustrate its functional characteristics.

TIME DOMAIN ANALYSIS OF DC/DC CONVERTERS

In the hypothesis of ideal behavior of the solid state devices, DC/DC converters are circuits that ciclically switch among some topological configurations constituted by linear reactive and resistive components, eventually connected to the DC generator. For such a kind of network each switched configuration can be considered as a linear circuit, but the periodical structural changes due to the different switches make the whole circuit time-varying. Then, in general, it is not possible to apply transformed methods for the analysis, because these techniques requires the fundamental hypothesis of linearity and time-invariance. Nevertheless, as we will show in the following, by particularizing each switch by means of a suitable parameter, a straightforward analysis tool such as the Laplace transform can be applied for determining the transient and steady-state response of DC/DC converters [3,4].

An efficient s-domain analysis method is the modified nodal analysis (MNA) [5]. With this approach it is possible to take into account the ideal switch by means of a symbolic parameter T, which can assume two discrete values: T=1 if the switch is closed, T=0 if it is open [5]. A generic circuit response V(s) can be obtained by solving the MNA system. If there are switches in the considered circuit, the response depends on switch parameters T_i and it is of the following kind:

$$V(s,T_i) = G(s,T_i)E(s) + \sum_{j=1}^{N_C} H_j(s,T_i)v_{Cj} + \sum_{j=1}^{N_L} K_j(s,T_i)i_{Lj} \qquad (1)$$

where: E(s) is the Laplace transform of the input voltage (a step in our case); v_{Cj} is the initial voltage of the j-th capacitor; i_{Lj} is the initial current of the j-th inductor;

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N_C is the total number of capacitors in the circuit; N_L is the total number of inductors in the circuit; $G(s,T_i)$, $H_j(s,T_i)$, $K_j(s,T_i)$ are the network functions corresponding to the input E(s) and to the initial conditions of capacitors and inductors.

The obtained expression $V(s,T_i)$ is the same in each switching topology that the circuit goes through. By particularizing the switch parameters for each switch stage, the explicit continuous solution for v(t) is found for each configuration in closed form. Obviously, even if at the beginning the initial conditions of the reactive elements are zero, after each switching the network functions change because of the different value assumed by characteristic parameters T_i relative to each switch. Consequently the inverse Laplace transform has to be calculated again when the circuit changes from one topology to another and, then, it is necessary to take into account the initial conditions of capacitors and inductors corresponding to each switching instant. This means that it is necessary to determine also each $V_{Cj}(s,T_i)$ and $I_{Lj}(s,T_i)$ (they have the form in (1)) and evaluate their inverse transform in the switching instants. Furthermore we are considering circuits which ciclically switch among some topological configurations. Then, due to the periodicity of the behavior, the expressions of $V(s,T_i)$, $V_{Cj}(s,T_i)$ and $I_{Lj}(s,T_i)$ are the same for each cycle, the difference from one cycle to another being due to the values of the initial conditions.

If such method is implemented on a computer by means of symbolic techniques, a suitable numerical inverse Laplace transform algorithm has to be used in order to determine the time domain response. To this end it is worth pointing out that the considered circuits have a single input with the form of a step. Then it is convenient to adopt for inductors and capacitors s-domain equivalent models with generators relevant to the initial conditions (I.C.) of the kind I.C./s. In this way the equation (1) can be written as:

$$V(s,T_i) = F(s,T_i)E(s) \qquad (2)$$

where E(s) is equal to 1/s and F(s) is a global network function, in which the initial conditions appear only within the numerator coefficients. All this is possible because the denominators of $G(s,T_i)$, $H_j(s,T_i)$, and $K_j(s,T_i)$ are the same in each switching configuration. By proceeding in this way, the MNA has to yield global network functions as in (2) relevant to both the desired output and each capacitor voltage and inductor current. Only in a successive phase the moltiplicative coefficient 1/s is considered in such a way to permit to apply for the inverse Laplace transform a state variable approach, that exploits a numerical method based on the Pade' approximation [4, 5, 6]. By following this approach the time response up to a desired instant can be obtained with a high computational speed also for long time response calculations [4].

FREQUENCY DOMAIN ANALYSIS OF DC/DC CONVERTERS

In this section we show how it is possible to perform the frequency domain analysis of a DC/DC converter by exploiting the numerical values of the state

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variable and output samples relevant to the corresponding transient responses. The proposed approach is based on the knowledge of the z-domain network function of the circuit, considered as a sampled-data system. In fact, if this network function is known, by replacing z with $e^{j2\pi f/fs}$ (f_s is the sampling frequency for the sampled-data system), the frequency response can be obtained, with $0<f<f_s/2$.

First of all we determine a general form for the z-domain network function. To this end let us consider a linear time-varying state-space model for the converter [7], in the hypothesis of ideal solid state devices and one-input-one-output system. The equations describing the network are the following:

$$\begin{aligned} \dot{\bar{x}}(t) &= A_m \bar{x}(t) + b_m u(t) \\ y(t) &= c_m \bar{x}(t) + d_m u(t) \end{aligned} \qquad (3)$$

where: u(t) is the input signal; $\bar{x}(t)$ is the state vector; y(t) is the output; m is equal to 1 for the time interval T_{on} and equal to 2 for the time interval T_{off}, with the period $T_s=T_{on}+T_{off}$ (we suppose a continuous mode operation, that is two phases of operation in the period T_s).

Starting from the equation (3), it is possible to obtain a sampled-data system with the sampling instants t_k coincident with the circuit commutation instants. We obtain:

$$\begin{aligned} \bar{x}(t_{k+1}) &= A_{d,k} \bar{x}(t_k) + b_{d,k} u(t_k) \\ y(t_k) &= c_{d,k} \bar{x}(t_k) + d_{d,k} u(t_k) \end{aligned} \qquad (4)$$

$$k = 1,2,\ldots$$

where $A_{d,k}$, $b_{d,k}$, $c_{d,k}$ and $d_{d,k}$ can be easily determined [7] by applying the very well known tools of the linear system theory.

In the hypothesis $u(t_k)=u(t_{k+1})=\text{const.}$, a recursive application of the previous equations over an entire period T_s yields the expression of the state variables at even (odd) switching time points as functions of the state variables evaluated in the previous even (odd) switching time points. Then the system can be written in the following way:

$$\begin{aligned} \bar{x}(t_{2(k+1)}) &= A\bar{x}(t_{2k}) + bu(t_{2k}) \\ y(t_{2k}) &= c\bar{x}(t_{2k}) + du(t_{2k}) \end{aligned} \qquad (5)$$

$$\begin{aligned} k &= 1,2,\ldots \\ A &= A_{d,2k+1} A_{d,2k} \\ b &= A_{d,2k+1} b_{d,2k} + b_{d,2k+1} \\ c &= c_{d,2k} \\ d &= d_{d,2k} \end{aligned}$$

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where also the output variable is sampled at the time points t_{2k}. Finally, labelling t_{2k} with the discrete index n=2k (t_{2k}=nT_s), the following system is obtained:

$$\bar{x}(n+1) = A\bar{x}(n) + bu(n)$$
$$y(n) = c\bar{x}(n) + du(n) \qquad (6)$$
$$n = 1,2,\ldots$$

It is worth pointing out that the obtained result can be applied also to discontinuous mode operation. Furthermore, from the system in the equation (6), it is possible to obtain the discrete network function W(z) as:

$$W(z) = c(zI - A)^{-1}b + d \qquad (7)$$

In order to determine A, b, c and d for the evaluation of W(z) it is not necessary to know the space-state system. In fact, if a suitable number of samples of the state variables and output are known, the system in the equation (6) becomes a system where A, b, c and d are unknown quantities. In general terms, if N is the dimension of the state vector, a state vector sample number equal to (N+2)N and a output sample number equal to N+1 are necessary in order to determine the matrix A, the vectors b and c and the scalar d from the linear system in the equation (6).

SPAC PROGRAM

On the base of the approaches described in the previous sections, a program for the analysis and design of DC/DC converters, named SPAC (Symbolic Program for Analysis of DC/DC Converters), has been developed. It is a symbolic interactive workbench, constituted by two main working environments. The first one is the "drawing environment", which is used to give the program the circuit scheme. This environment performs also the symbolic analysis of the converter and calculates the necessary s-domain network functions in completely symbolic form with respect to both component values and switch parameters. It exploites the MNA for the building of the solving system and uses a particular algorithm developed by the authors [8] for the complete symbolic solution of the linear system. The necessary s-domain network functions are constituded by the desired output network function and all the circuit capacitor voltage and inductor current network functions: the form of these functions is like that in the expression (2). In fig.1 the screen representing the Boost converter and the desired output symbolic network function is shown as an example. In the screen the allowed components are also displayed.

The second environment is the "evaluation environment", which is used to specify the numerical value of components, switching times, period duration and so on. This environment performs the numerical inverse Laplace transform as reported in [4] and plots the graphic representation of the transient and steady-state responses. Once the time domain response has been evaluated, by exploiting the samples of both the state variable and output transient responses, SPAC can give the z-domain network function of the sampled-data model of the circuit. In this phase the program evaluates the matrix A, the vectors c and b and the scalar d of the equation (6) as reported in the previous section and gives the coefficients of W(z) by using the Souriau-Frame algorithm [9]. Finally, by replacing z with $e^{j2\pi f/fs}$, the frequency response amplitude and phase can be plotted.

Referring again to the Boost converter in fig.1, the time response and the frequency response amplitude are shown in fig.2 as an example (numerical values of components and times are like those specified on the bottom of the screens). It is also possible to obtain parametric graphics for all the allowed responses.

It is important to outline that the frequency domain behavior can be obtained only from the z-domain network function, because the s-domain network functions depend on switch parameters T_i. Nevertheless the availability in symbolic form of the s-domain network functions constitutes the fundamental characteristic of the program. In fact the numerical values of components, switching times, period duration, etc. must be assigned only before the computation of the numerical inverse Laplace transform. This feature can be very useful in design phase for selecting the values of components and times in order to obtain the requested behavior, because, if component values are changed, there is no need to run all the analysis again, as it happen in a completely numerical approach. On the other hand the s-domain network function availability allows to use very fast algorithms for the evaluation of both time domain and frequency domain responses. Then the numerical evaluation phase requires very little computational times and it is the only part that has to be repeated if component values are changed. It is true that the initial symbolic phase requires computational times enough high, but the s-domain network function evaluation is performed once for a given converter.

It is worth pointing out that the frequency domain behavior of DC/DC converters is completely defined if are known: the input-to-output frequency response $M_v(j\omega)=V_o(j\omega)/V_i(j\omega)$, the open-loop input impedance $Z_i(j\omega)=V_i(j\omega)/I_i(j\omega)$, the open-loop output impedance $Z_o(j\omega)=V_o(j\omega)/I_o(j\omega)$, the control-to-output frequency response $T_p(j\omega)= V_o(j\omega)/V_c(j\omega)$. At present SPAC is able to calculate $M_v(j\omega)$, $Z_i(j\omega)$ and $Z_o(j\omega)$, but not $T_p(j\omega)$, that is fundamental for the design of the complete system including also the control circuit. Furthermore only converters with continuous mode operation can be considered. In the near future it is in our intention to extend the potentialities of SPAC program introducing the possibility of considering discontinuous mode operation converters and evaluating $T_p(j\omega)$ frequency response.

CONCLUSIONS

An MS-DOS personal computer program for the analysis and design of DC/DC converters has been presented. The program, named SPAC, is able to produce the graphic representation of transient and steady-state responses, the z-domain network function of the sampled-data model of the converter and the relative graphic representation of the frequency response amplitude and phase.

An important characteristic consists in the symbolic nature of the program. In fact the s-domain symbolic network function availability permits not only to avoid the repetition of the converter analysis every time the numerical values are changed, but also to use very fast algorithms for the evaluation of all the allowed responses.

Finally SPAC is very easy to use, requiring only the knowledge of few rules for circuit drawing, and has very satisfactory computational times.

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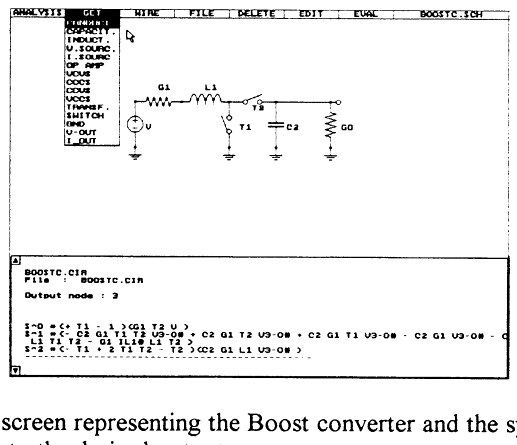

Figure 1: SPAC screen representing the Boost converter and the symbolic network function relevant to the desired output.

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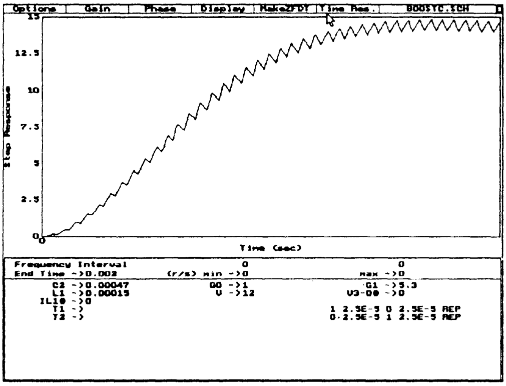
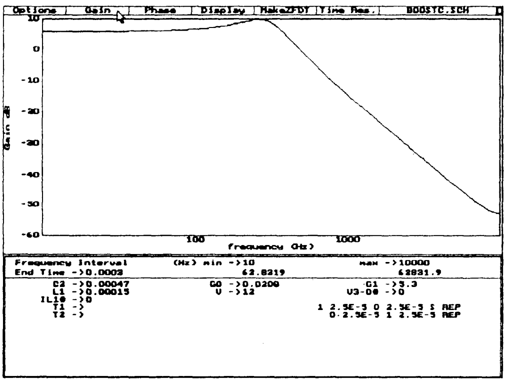


a)



b)

Figure 2: Time response (a) and frequency response amplitude (b) of the Boost converter.