Matching nature with ‘Complex Geometry’ – an architectural history

J. Tomlow
Hochschule Zittau/Görlitz (FH), Germany

Abstract

In human history research in geometry takes a central place between other scientific developments. A major content was clearly defined already around 300 B.C. in Euclid’s book “Elements”. This paper gives an overview of human handling of Complex Geometry, which reaches beyond the simple forms like square/cube and circle/sphere. Complex Geometry includes the cone and its sections, ruled surfaces and polyhedra. Cone sections and polyhedra are part of Euclid’s knowledge, but geometricians and architects had a hard job to depict these forms well and thus to understand them, let alone to use them. People got interested in Complex Geometry, in order to copy “perfect” natural shapes like crystals. More important was, that simple geometry turned out to be not sufficient for establishing innovative design. By its hermetic structure, Complex Geometry has the power to generate surprising shapes along a logical path. A historic survey proves that only in cooperation with artists of superb handicraft (drawing, modeling, carving) could Complex Geometry be investigated fruitfully. It was not until the early 19th C. that ruled surfaces were defined and illustrated in a sufficiently clear way (Monge, Leroy).
Keywords: polyhedra, cone sections, ruled surfaces, descriptive geometry, model.

1 Polyhedra [1,2,3,4]

An interesting aspect of the phenomenon “polyhedra” is its generalization by Paolo Uccello (ca. 1397-1475). Ucello, being an early Renaissance painter, struggled with the correct depiction of perspective, and he experimented with detail shapes, spread over his paintings, which could be drawn in perspective by common rules. These shapes included the "mazzocchio" a ring-like hat. This

Design and Nature II, M. W. Collins & C. A. Brebbia (Editors)

torus shape is based on identical trapezes, which were structured by a poligon of 16 or 32 sides, the ring section being a hexagon or an octagon. His method included vast preparatory drawings alike web-like drawings of today's Computer Aided Design (CAD). His method of depicting objects included complex vase shapes and stellated objects [2]. See also the 'polygonic' art of Ben Jakober [10].

The theology professor Luca Pacioli, from Borgo San Sepolcro (Fra Luca di Borgo, ca. 1445-1517) was involved in a research project for the Duke of Urbino, Guido Ubaldo, and Bishop Valletari, seeking to define correctly the mathematical shapes of polyhedra [1,2], which were thought to be of symbolic significance [3]. His scientific publications included the first Italian translation of Euclid's "Elements". A new aspect in the search for representation is the devising of different kinds of models of polyhedra. Pacioli's method of working essentially develops new forms by truncation (for instance cuboctahedron) or addition (stellated polyhedra). His presentation of results is unique for his time. He found no one else than Leonardo da Vinci (1452-1519) as illustrator of the complex polyhedron models in perspective for this project [1,2,3]. Leonardo da Vinci's illustrations for Pacioli's "De Divina Proportione" show polyhedra as lattice structures. Drawing the edges of a polyhedron with more than one line allows the artist to discriminate which edges are in front and which are behind, which is a great help in visualizing structures in space. Leonardo's skill as a draughtsman shines especially in his drawings of machines and complex building designs and there is clearly a connection between his illustrations for Pacioli of latticed polyhedra and his linear stereometric images of architectural structures. The monk Pacioli's interest in polyhedra is further documented in a double portrait painting by Jacopo de' Barbari (1495), showing him with a young nobleman in the role of a pupil. On a desk are objects relevant to geometry and drawing and a solid wooden dodecahedron. Of particular interest is the model of a semi-regular polyhedron - a rhombicuboctahedron as Kepler named this shape made of eighteen squares and eight triangles [2,4]. This polyhedron of glass polygons, probably blown in one piece, is half filled with water. A thread crossing the glass bowl in its centre is fixed to its bottom and suspends it in an unstable equilibrium from the ceiling. This water-filled glass polyhedron can be interpreted as a measuring device, making use of physical laws.

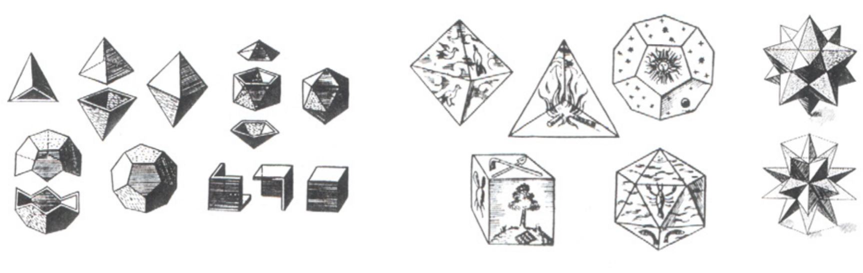

Figure 1: The five platonic polyhedra, their symbolism air, fire, universe, earth, water and stellated examples as depicted by Johannes Kepler in "Harmonices mundi" (1619).

Design and Nature II, M. W. Collins & C. A. Brebbia (Editors)

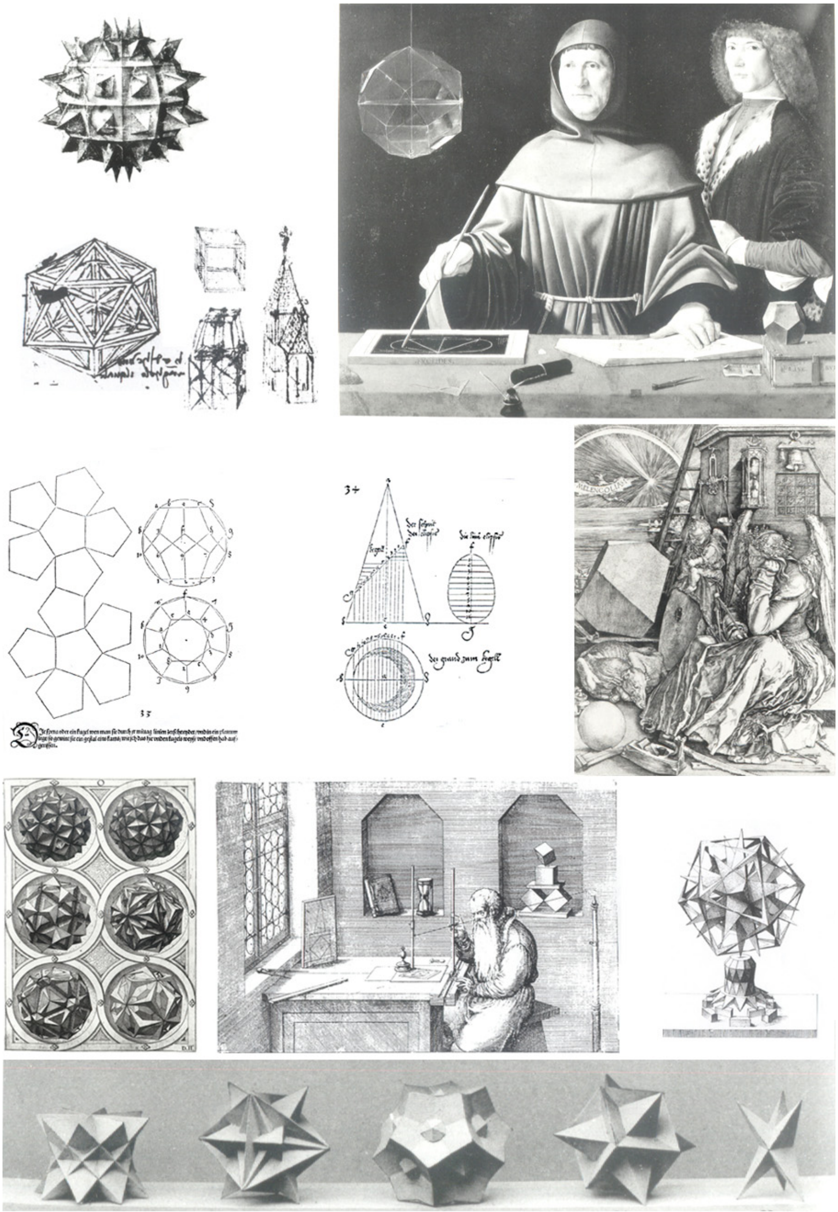

Figure 2: Uccello's stellated polyhedron; portrait of Pacioli with glass, water filled polyhedron; Leonardo's sketches of crystalline shapes (codex atlanticus); Dürer's Dodecaedron, conical section ellipse and the etching "Melancholia"; Jamnitzer's installation to draw perspectives and some brilliant results; some paper models by Brückner.

Because the suspension is vertical and the water level horizontal, any horizontal section of the polyhedron can be generated by varying the amount of water and the result can be compared with drawings. An interpretation of the glass object as symbolic for alchimistic activity has been published by Alberto Pérez-Gómez [3] which I commented on in [4].

Albrecht Dürer (1471-1528) and Wentzel Jamnitzer (ca. 1508-1585) have worked on polyhedra and they lived in the city of Nürnberg. In Nürnberg, during the Renaissance period, which occurred in Germany somewhat later than in Italy, some handcrafts were highly developed: there of the printer, instrument maker, silversmith, for example. Besides being a pioneer writer on Renaissance themes in the German language Dürer was an expert in graphic design [2]. His books show a beautiful lay out, making use of excellent lettering and always giving enough white around a picture. This clearness, however, in the case of his explanation of the cone sections, or the polyhedra, turns out to be rather graphic than geometrical. Famous is his illustration of the ellipse as a cone section, of which his geometrical construction shows a somewhat pointed result. In the text he proposes the German word for the foreign word ellipse as “eyerlini” because of it egg-like appearance. Other small errors can be found in his depicting of some polyhedra, in which he draws the dodecahedron front view touching the middle section of the superscribing sphere, although in proper projection there should be a small distance. On the other hand, Dürer was a far better illustrator than most of his contemporaneous colleagues, and until 1800 mistakes in illustrating regular polyhedra are usual in the handbooks.

Wentzel Jamnitzer, a silversmith and instrument maker, made a private book project in which he thematized the shape of the five regular polyhedra by adding or subtracting [2]. In this way quite a lot of phantastic crystalline bodies occurred with alternating convex and concave parts. Some interesting aspects are that he depicted them in a perfect perspective, enhancing its spacial qualities, and that he made use of models of basic shape, which he analysed with a perspective machine, similar to the one Dürrer had explained. His etchings include funeral monuments, a thematic typical of the architectural fantasies of that time.

As a physicist of Newtonian stature, Johannes Kepler (1571-1630) made a major contribution to astronomy by showing that the orbits of the planets were ellipses having the sun as one focus. But his pursuit of the idea that the distances of the planets from the sun were proportionate to the dimensions of the five regular polyhedra nested inside one another was, in retrospect at least, a stultifying mistake. Kepler contributed to the best known astronomical tabellarium of his days, that of Tycho Brahe (1546-1601) and in his book "Harmonices mundi" (Frankfurt a.M. 1619) he discussed new concave polyhedral solutions and developed a systematic latin nomenclature, which is still valid today [2].

In the preface of his book "Vielecke und Vielflache. Theorie und Geschichte" (Leipzig 1900), Max Brückner (1860-1934) explains that the interest of mathematicians in polyhedra was diminishing because many known problems were solved by then, although a compendium such as his, bringing together a historic survey and an encyclopedic classification of polyhedral examples, still

Design and Nature II, M. W. Collins & C. A. Brebbia (Editors)

seemed worthwhile to him. A very interesting parallel to Pacioli is Brückner's care for visual presentation. By means of many conventional drawings and photographs of 146 paper models, which were made by Brückner in many years, a clear overview of the possible range of polyhedral form is presented. The reader is kindly invited to study the actual models at Brückner's work place, as he wrote. The models show also polyhedral variations of hyperboloid origin [2].

2 Ruled surfaces [1,5,6,7,8]

From a geometrical point of view the cone and the cone sections are tightly connected to the ruled surfaces. The five sections which can be cut alternatively through a (double) cone object are: a point, a (double) line, an ellipse (including a circle), a parabola, a hyperbola. By changing the inclination degree of the section plane, these different shapes are generated. If one extends these two-dimensional forms by translation or rotation, three-dimensional shapes occur like ellipsoid (including sphere), (one- or two-shell) hyperboloid, paraboloid, hyperbolic paraboloid, conoid, helicoid. In some early projects Antoni Gaudí (1852-1926), on his way optimizing his design approach in a controlled way, designed a cone shape for towers in connection with a paraboloid cupola inside. The magnificent interior hall of Palau Güell in Barcelona, shows this architecturally inventive solution, with small window holes piercing the cupola all over. In 1893 Gaudí designed an ideal project for a Franciscan mission post in Tangiers, Morocco [5]. In this project reconstructions by Torii et al show proof of an abundant use of the cone and its sections in towers and circular building aisles with oblique walls. Windows and arches in parabolic and hyperbolic shape and continuous helicoid stairs finish the image of a geometry based design.

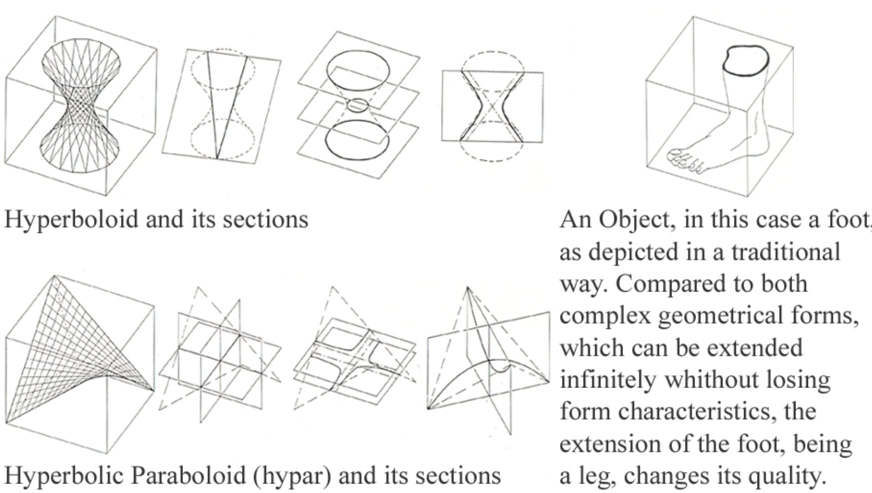

Figure 3: The abstract nature of Complex Geometry.

Design and Nature II, M. W. Collins & C. A. Brebbia (Editors)
© 2004 WIT Press, www.witpress.com, ISBN 1-85312-721-3

"What are ruled surfaces?". Viollet-le-Duc (1814-1879), a leading neogothic architect and writer explains it to a youngster in the following way. "You see this walking stick? I can throw it through the air with a turning effect, and the three-dimensional shape that thus is produced is curved, but in the mean time it shows a constant straight section!". The main two ruled surfaces are the hyperboloid ("cooling tower"), containing straight line, circle (or ellipse), and hyperbola as key sections, and the hyperbolic paraboloids ("saddle"), which owes its difficult name to the fact that besides the straight line also the hyperbola and parabola occur as key sections. As an abstract form, ruled surfaces extend virtually beyond any border, without losing its shape characteristics. Let us focus to begin with on the hyperboloid [6]. The invention of ruled surfaces, historically, can be considered "by accident". Handicraft persons, maybe women weaving a basket, discovered the stool or chair, in the shape of a hyperboloid grid.

Of considerable interest is an early use of hyperboloids by the London architect Sir Christopher Wren, who belongs to those universal artists who included inventions and scientific discoveries in their work. In the search towards the manufacture of non-spherical lenses, he proposed a combination of two hyperboloid solids, of which the first hyperboloid is giving a turning movement to the second hyperboloid, which grinds the crystal body [6]. The movement drive between both hyperboloids is transferred in a common straight line (a rule of the ruled surface). Scientists like Newton already describe the hyperboloid, but with rather poor illustrations. A full satisfactory illustration level results from the descriptive geometry method [11] by Gaspard Monge (1795) – or rather by his pupils such as Charles-Félix-Augustin Leroy – and from early 19th C. handbooks, the knowledge on ruled surface geometry will reach innovative architects.

An early and in the same time technically mature use of the hyperboloid shape is represented by the grid shell towers in steel by the Russian engineer Vladimir G. Suchov (1853-1939) since 1896. His designs included water towers and a radio transmission tower, the sabolovka tower (Moscow 1922) which was built in six hyperboloid sections, with a height of 150 m. The original design project should have had nine hyperboloid sections reaching up to 350 m, which is higher than the famous Eiffel tower, and intended to be built with much less material [1,6].

One particular invention in structural design had an enormous impact by its monument-like appearance in landscape: the cooling tower in reinforced concrete shell. Around 1914 the first cooling tower was realized by the inventive director of the Dutch state mine "Emma" in Treebeek near Heerlen, K. Iterson with G. Kuiper [6]. The first generation was 35 m high. From this invention, hardly changing the general concept, arose cooling towers of over 200 m height throughout the world. By the way, many of these cooling towers are now listed to disappear. Interesting is the way the cooling tower, but also Suchov's grid masts, have influenced modern architecture. Ivan Leonidov (1902-1959), a visionary architect of the Russian avant-garde, designed a project for the Heavy Industry Ministry (1934) in Moscow. Hyperboloid (like) shapes in this project occur in a skyscraper, a colorful decorated hall and columns [6]. Like Gaudí,

Design and Nature II, M. W. Collins & C. A. Brebbia (Editors)
 ISBN 1-85312-721-3

Leonidov was aware of the possibilities of this new geometry and a bridge is represented with a parabolic arch. A beautiful minimalist sketch of Leonidov's hand shows a stretched hyperboloid. Gaudí designed a small master piece, the Colonia Güell church, along the hanging model principle [8]. There he discovered the problem of “obliqueness” in optimized stone architecture. When structural elements tend to have a non-parallel and non-normal relation with each other, a simple geometry is not very useful. His conclusion was to define walls, vaults and pillar surfaces generally as ruled surfaces, mostly hyperboloids or hyperbolic paraboloids. (In special cases like knots of branching pillars he applied ellipsoids and paraboloids for towers.) By simply changing the corner coordinates, any shape could quickly be adapted to new static or design requirements. In a painstaking but controlled process in the Sagrada Familia church (Barcelona, design/building by Gaudí 1884-1926; still under construction) Gaudí developed a canonic kind of a new “mediterranean gothic” architecture with ruled surface, representing the shapes with gypsum models. A rational use of ruled surface shapes shows Gaudí using straight sections in neighboring hyperboloids and hyperbolic paraboloids as a common border line.

Even with the peculiar case of the hyperbolic paraboloid (hypar), one can speak of an “innocent” discovery. Gaudí used to say that every wall is crooked in some degree, what means that the upper and bottom lines are not parallel and so the plane surface is the special case of a hypar. A little known invention was made by the medieval monastic order, the Cistercians. The case here referred to is Santes Creus in Catalonia, Spain. The hypar shape is used in eightfold in a base of the octagon cross tower, matching the space between the square walls of the crossing with the octagonal shape of the tower. The simple building method was alike that of steps, each next row of ashlar stones turning slightly in the horizontal plane. Also in wooden roofs one can find crooked shapes from the middle ages. Irregular house plots, typical in old cities, caused quite a problem for the technical logical execution of wooden roofs, since either the top line of the roof was tilted or at least one of the faces became a hypar.

A decisive step to a mature use of hypars is the reinforced concrete shell structures by Félix Candela [7]. An elder pioneer of shell structures was Eduardo Torroja. The approaches of both were rather different. Whereas Torroja was a brilliant mathematician, like his father, the younger Candela was puzzled by the complicated calculation methods by Torroja, which resulted in a criss-cross layout of reinforcement parts, although the exterior shape looked beautiful simple and abstract. Candela found in the ruled surface hypar a shape which was rather flexible in its proportions, easy to use in clusters, very rich in expression, with borders of either curved or straight line.

His use of the hypar shape for shells, allowed him to develop a calculation method, in which any point on the shell could be analyzed along two straight vector axes which stayed within the hypar shape.

A well documented aspect of Le Corbusier, is the double lesson he took from Gaudí by the help of Luis Sert: ”the traditional Catalan vault method with thin tiles” and the “hypar” [9].

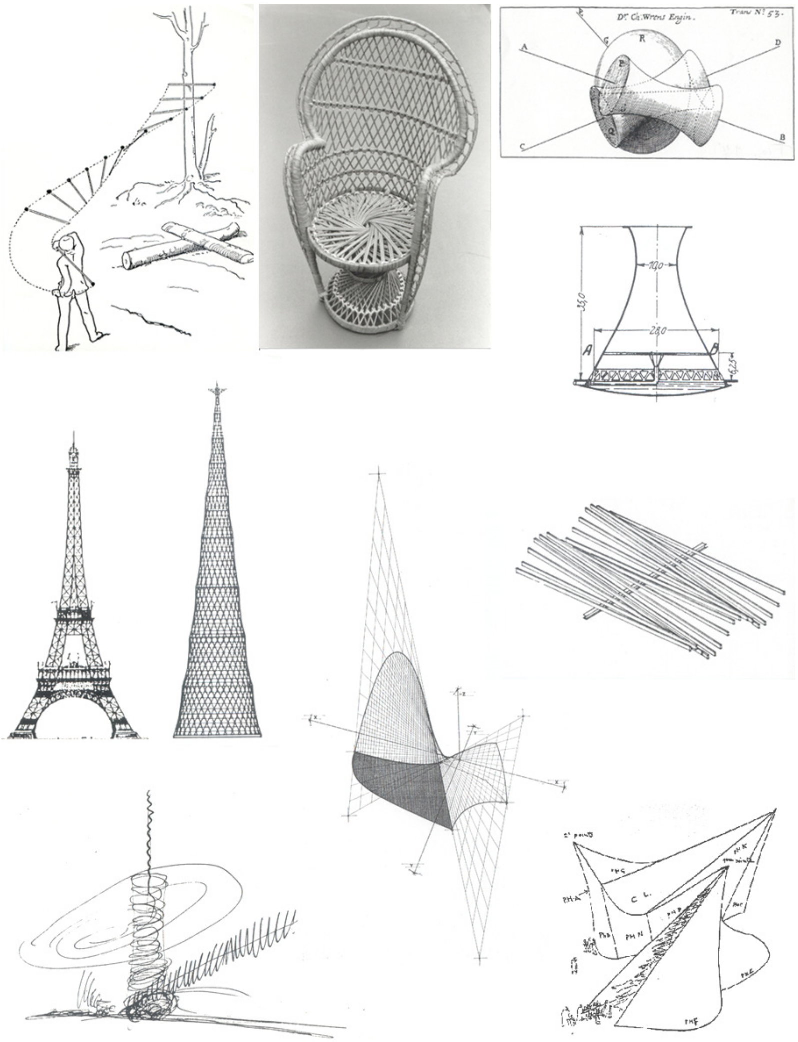

Figure 4: Viollet-le-Duc's general explanation of a ruled surface; Model of an ancient chair type based on a hyperboloid; Wren's inventive proposal using the hyperboloid shape for grinding aspheric lenses; Iterson's cooling tower for the "Emma" mine; Suchov's (not executed) design for the Moscow radio tower in hyperboloid grid segments, compared with the lower and yet heavier Eiffel tower; A Candela design of hypar shell; Gaudí's ruled surface roof system for the Sagrada Familia School; Leonidov's sketch interpreting a hyperboloid building; Le Corbusier's Philips Pavilion for the World Expo 1958 in Brussels.

Design and Nature II, M. W. Collins & C. A. Brebbia (Editors)

Both phenomena are connected since the curved shapes of ruled surfaces are rather hard to achieve with conventional brickwork. In Le Corbusier's own development both meant a change from a dogmatic "white cube" era towards an organic colorful modern architecture. Le Corbusier used the hyperboloid shape of the cooling tower, obliquely snubbed, as an assembly hall in Chandigarh (1962, India). Even more interesting may be that he made a rather free use of ruled surfaces, thus avoiding acoustical problems of a circular section. For the church Saint Pierre in Firminy-Vert (1961-1964) he projected a monumental kind of major volume. Photos of a model show his design approach. Between a square base and a tilted rounded top plane, strings are attached firmly, so that they define the shape which is similar to a snubbed cone. Interesting is the lack of respect for correct geometry which Le Corbusier displays here. Geometrically sophisticated, under the influence of LC's collaborator Yannis Xenakis, a Greek architect and music composer, is the Philips pavilion. It is a composition of hypars, pragmatically mixed with conoids, for the Brussels Expo 1958, of which the process towards realization is quite astonishing. Most firms who were asked to find a rational and economic solution were puzzled, and their proposals reached from a wooden sandwich construction, reinforced concrete, cable strings between steel borders (Xenakis). The final execution has a double net of iron cables, in between hypar elements of concrete. These hypar elements were produced on a sand bedding.

Acknowledgements

Part of my studies on the history of Complex Geometry were inspired in the mid nineties by Jean-François Gabriel, editor of "Beyond the cube", a book with the best fitting title one can think of [2]. In the preparatory studies on the history of polyhedra in the *Institut für leichte Flächentragwerke IL* of Stuttgart University, founded by Frei Otto, directed by Werner Sobek, were involved Elisabeth Trautz-Fülöp, Gabriela Heim and Brigitte Trappe. In a long-year cooperation with my senior colleagues Rainer Graefe and Jan Molema structural design methods in history were analyzed, developing an appropriate methodology, focusing on geometric aspects. All studies were part of the special research program SFB 230 "Natürliche Konstruktionen" and/or the Gaudí Research Group Delft. This has been continued, assisted by Jan Fallgatter, in my research in the Hochschule Zittau/Görlitz (FH). Special thanks to Nic Tummers, Heerlen, for our talks on Iterson's first cooling tower, which he tried to save. Of certain relevance is my private collection of rare old books. The reader is asked to excuse the brevity of the reference list, due to lack of space.

Figure source

Figure 1: Kepler, J.: Weltharmonik, München 1990. Figure 2: Borsi, F.: Paolo Uccello - Florenz zwischen Gotik und Renaissance, Stuttgart/Zürich 1993; Pedretti, C.: Leonardo – Architect, London 1986; Jacopo de' Barbari. Portrait of Fra Luca Pacioli with Young Man, 1495, Museo National di Capodimonte,

Napels, The Renaissance from Brunelleschi to Michelangelo, Milan 1994; Dürer A., Underweysung der Messung, 1525, Nördlingen 1983; Bundesdruckerei Berlin; Jamnitzer, B.: Perspectiva corporum regularium, Nürnberg 1568 and a print by Jost Amman ca. 1568: Germanisches National museum Nürnberg; Brückner, M.: Vielecke und Vielflache, Leipzig 1900. Figure 3: Jos Tomlow; [6]. Figure 4: Viollet-le-Duc, Histoire d'un dessinateur, Paris (1879); Foto G. Heim, IL-Stuttgart; Wren, C.: Philosophical Transactions, London 1669; Suchov; Beton und Eisen, Heft 15, 1925; [6]; Faber, C.: Candela und seine Schalen, München 1965; Gaudí Research Group Delft; Gozak, A.: Ivan Leonidov, New York 1988; Capanna, A.: Conoids and hyperbolic Paraboloids in Le Corbusier's Philips Pavilion, Nexus Network Journal, vol. 2, no. 3, July 2000.

References

[1] Tomlow, J., Der geometrische Faktor beim Konstruieren, *Prozeß und Form 'Natürlicher Konstruktionen", Der Sonderforschungsbereich 230*, hrsg. v. K. Teichmann, J. Wilke, Berlin, pp. 75, 77-83, 1996.

[2] Tomlow, J., Polyhedra, from Pythagoras to Alexander Graham Bell, *Beyond the Cube. The Architecture of Space Frames and Polyhedra*, ed. J.-F. Gabriel, John Wiley & Sons, Inc. New York, pp. 1-34, 1997.

[3] Pérez-Gómez, Alberto: The Glass Architecture of Fra Luca Pacioli, in: *architectura - Zeitschrift für Geschichte der Baukunst* 2 /1998, pp. 156-180.

[4] Tomlow, J., Eine physikalische Interpretation des gläsernen Objekts auf dem Porträt Luca Paciolis von Jacopo de'Barbari (1495), *architectura - Zeitschrift für Geschichte der Baukunst* 2/2000, pp. 211-213, 2000.

[5] Tomlow, J., Neue Interpretation von Antoni Gaudís Entwurf der Franziskaner-Mission in Tanger, *Zur Geschichte des Konstruierens*, hrsg. von R. Graefe, Stuttgart, pp. 129-137, 1989.

[6] *V.G. Suchov (1853-1939) Die Kunst der sparsamen Konstruktion*, bearb. von R. Graefe, M. Gappoev, O. Pertschi, Stuttgart, 1990 (Moscow 1994).

[7] Tomlow, J.,"Ich zähle nur auf meine Zahlen" - Ein Interview mit Félix Candela über die historischen Hintergründe seiner Werke, *Deutsche Bauzeitung*, 126. Jahrgang, 8 / August 1992, pp. 72-81, 1992.

[8] Tomlow, J., The spirit of calculation in the architectural work of Antoni Gaudí, *Gaudí 2002 Miscelánea.* Barcelona, pp.176-199, 2002.

[9] La bóveda tabicada a la catalana y el naciemento de la „cerámica armada„ en Uruguay, in: Huerta, S.(Ed.), Las Bóvedas de Guastavino en América, Instituto Juan de Herrera, Madrid 1999/2001.

[10] *Perspectivas – Ben Jakober & Yannick Vu (1984-2002)*, Zaragoza 2002.

[11] Leroy, C.F.A., E. F. Kauffmann, *Die darstellende Geometrie*, Stuttgart 1838

Design and Nature II, M. W. Collins & C. A. Brebbia (Editors)