The phi code in nature, architecture and engineering

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Abstract

This paper examines how the golden ratio, Φ (Phi), appears systematically coded (the phi code) in nature, architecture and engineering. The phi code helps to explain the growth patterns in nature, the aesthetics of design in architecture and the behavior of structural elements used in engineering. The phi code defines the morphogenesis (development of forms) in nature as well as the aesthetics of forms in architecture and its structural parts.

Keywords: golden ratio, golden mean, golden section, divine proportion, geometric proportion, biomimetics, design and nature, architecture.

1 Introduction

The golden ratio, phi, is coded symbolically manifesting itself in geometric patterns and forms and in mathematical expressions defining the behavior of natural and physical phenomena. As discussed in this paper, certain geometries describe growth patterns in nature, whereas other geometric shapes are employed in art and architecture in order to establish a sense of harmony and proportion in design. In engineering, the phi code appears in mathematical expressions related to structural mechanics particularly in the area of stress analysis of structural elements. The phi code identifies aspects of nature, architecture and engineering in which the golden ratio plays a determinant role.

The phi code establishes rules of behavior. In nature it is growth behavior. In architecture it is aesthetic behavior and in engineering it is structural behavior. In nature, the golden ratio defines patterns of growth in the form of spiral shapes and pentagonal geometries. In architecture, some styles and forms are created by employing different systems of proportion, most of which following golden ratio
proportions in the form of geometric shapes [1]. In engineering, the golden ratio
also plays a significant role in the area of stress analysis of basic structural
elements (columns, beams, domes) as predicted by elastic theory. In the context
of this paper, it is not feasible to discuss all aspects of the phi code in detail. This
would entail a lengthy digression into the history of “phi”. In consequence, this
article discusses various themes of the phi code, as they are relevant to nature,
architecture, and engineering (structural mechanics). The present work examines
in a condensed form, how the phi code is embedded in the growth pattern of
shells and pinecones. It also discusses how the golden ratio was employed by the
Greeks in the Parthenon. And finally, the paper describes how this ratio defines
the structural behavior of simple beams as characterized by the principal stresses.
But first, let us discuss some of the properties of the golden ratio to better
understand the theme of the phi code.

2 The golden ratio Φ (phi)

The golden ratio is a proportion found in nature. The Ancient Greeks applied it
to the design of their temples [2]. In graphical terms, it can be described as “the
whole is to its larger part, what the larger part is to the smaller”. It is also
known as the golden mean, the golden section, or the divine proportion. As
shown in Figure 1, it is a way to divide a line in such a manner as to create an
ideal relationship between the parts.

\[
\frac{AB}{CB} = \frac{CB}{AC} \quad \text{or,} \\
\frac{\Phi x}{x} = \frac{x}{x(\Phi - 1)}, \quad \text{that is:} \\
\Phi = \frac{1 + \sqrt{5}}{2} = 1.61803 \\
\Phi^2 - \Phi - 1 = 0 \quad \text{(solving for } \Phi \text{ we obtain } \Phi = \frac{1 + \sqrt{5}}{2} = 1.61803) \\
\]

The qualities of the golden ratio Φ (phi) were central to the numerological
philosophy of Plato and Pythagoras. The mathematician Filius Bonacci
(Fibonacci) wrote a treatise on the number series related to the golden ratio.
0,1,1,2,3,5,8,13,21,34,55,89,144,233,377,610,987,1597. If we take the ratio of
two successive numbers in Fibonacci’s series, we find: \(1/1=1\); \(2/1=2\); \(3/2=1.5\); \(5/3=1.666\); \(8/5=1.6\); \(13/8=1.625\); \(21/13=1.61638\); \(34/21=1.61904\); \(55/34=1.6176\); etc. The ratio converges to a value very close to the golden ratio (\(\Phi\)) just as for the’’ line’’ above! The Greeks studied phi closely through their mathematics and used it in their architecture. The Parthenon at Athens is a classic example of the use of the golden ratio, as employed geometrically in a rectangular shape. This shape, known as the golden rectangle, is shown in Figure 2.

The golden rectangle is produced mathematically when the side DC of a square ABCD is bisected at X and an arc with radius XB is swung on to DC at F. The resultant rectangles AEFD and BEFC are both golden rectangles whose sides conform to the golden ratio, thus: \(DF/AD = BC/CF = 1.618\).

Golden rectangles can be approximated by using Fibonacci series to construct “Fibonacci rectangles”. Fibonacci rectangles are those built to the proportions of consecutive terms in the Fibonacci series: \(\text{Fib} (n) = 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, \ldots\) \(\text{Fib} (n+1)\). Because of the nature of the series, any \(\text{Fib} (n+1): \text{Fib} (n)\) rectangle can be divided into all the previous Fibonacci rectangles. For example, Figure 3 shows a 34:21 rectangle (the numbers inside the squares show the length of the sides) within which is contained the 21:13, 13:8, 8:5, 5:3, 3:2, 2:1 and 1:1 rectangles. Thus, each of the rectangles is divided into squares of length of side conforming to the Fibonacci series. As \(n\) tends to infinity, the proportion of the rectangle will tend to phi. This is a ‘very good’ approximation of the golden rectangle discussed previously.

Also of interest is the Fibonacci spiral, shown in Figure 4 constructed using the diagonal arcs (arcs of demi-semi-circles) of radius \(\text{Fib} (n)\) for all natural \(n\) up to a given point (depending on the required size/complexity of the spiral). The diagram shows a spiral constructed for all \(n\) up to \(n=21\), following Figure 3. Let us now turn our attention to the Fibonacci spirals and see how they exemplify the appearance of the phi code in nature.
Fibonacci spirals occur widely in nature as for instance, the growth pattern of shells and pinecones shown in Figure 5 (a) and (b) respectively. In each growth phase characterized by a spiral, the new spiral is very close to the proportion of the golden ratio (a golden rectangle to a square) larger than the previous one. The contour spiral shape of shells, illustrated in Figure 5 (a), reveal a cumulative pattern of growth. The growth patterns of shells are logarithmic spirals which are close to golden ratio proportions, as illustrated in Figure 4. The growth patterns of the nautilus and other shells are never in the exact golden ratio proportions. The nautilus shell is a logarithmic spiral. Such a shape arises because a growing animal has the same proportions as it grows and the spiral fits the requirement to protect this shape as it gets larger.
The spiral growth patterns of the pinecone each grow along two intersecting spirals, which move in opposite directions. Upon examining the pinecone seed spirals, shown in Figure 5 (b), eight (8) of the spirals move in a clockwise direction and thirteen (13) in a counterclockwise direction, closely approximating golden ratio proportions.

That is $\frac{13}{8} = 1.625$. The numbers 8 and 13 as found in the pinecone spiral are adjacent pairs in the mathematical sequence of the Fibonacci series as shown previously. The ratio of adjacent numbers in the sequence progressively approaches the golden ratio of $1:1.618$. The phi code in nature is also found in the proportions of the human body. As an example, consider the ratio of a person’s total height to the distance from the navel to the foot. These measurements were made in a number of people and the ratio turned out to be
very close to the golden ratio! [3]. After all, “Man is the measure of all things,” according to Protagoras, the Greek philosopher of the fifth century B.C.

4 Phi code in architecture

The Parthenon is an interesting example of a mathematical approach to art. Once its ruined triangular pediment is restored, the ancient temple fits almost precisely into a golden rectangle, Figure 6. The height (in meters) of the building follows golden ratio proportions [4].

That is: \[
\frac{AD}{AB} = \frac{AB}{BD} \approx \frac{19.87}{12.28} = \frac{12.28}{7.59} \approx 1.618 = \Phi.
\]

In addition, as shown in Figure 7, the facade of the Parthenon was designed around the proportions of two large and four small golden ratio, or \( \sqrt{5} \), rectangles (refer to Figures 7a, 7b), placed above four squares [4]. These proportions are in agreement with the golden rectangle, shown in Figure 2.

![Figure 7: The Parthenon and the golden ratio.](image-url)
5 Phi code in engineering

Let us consider the beam shown in Figure 8 (a). A stress analysis of the beam results in the plane stress condition shown in Figure 8 (b).

The Normal Stresses $\sigma_x$, $\sigma_y$, and Shear Stresses $\tau_{xy}$, $\tau_{yx}$ are acting as shown in the figure for a given x-y coordinate system. The Principal Stresses $\sigma_1, \sigma_2$ (critical in stress analysis) are also illustrated in Figure 8 (b).

![Simple beam](image1)

(a) Simple beam

![Principal stresses](image2)

(b) Plane stress condition

Figure 8: Stress analysis of a simple beam.

The Principal Stresses $\sigma_1$ and $\sigma_2$ are given by:

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

(2)
And the Shear Stresses by:

\[ \tau_{\text{max/min}} = \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \]  

(3)

Normally, beam behavior entails flexural and shearing actions in which \( \sigma_y \) is negligible when compared with the other stresses acting on the beam. That is \( \sigma_y = 0 \) for all practical purposes. In addition, for the particular condition in which \( \sigma_x = \tau_{xy} \) (and \( \sigma_y = 0 \) as indicated earlier) we obtain the following relationships for the principal stresses:

\[ \sigma_{1,2} = \sigma_x \[ 1 \pm \sqrt{5} \] \text{; and} \]

\[ \sigma_1 = \sigma_x \[ 1.61803 \] = \sigma_x [ \Phi \] \text{ tension} \quad (4a) \]

\[ \sigma_2 = \sigma_x \[ -0.61803 \] = \sigma_x \[ -1/\Phi \] \text{ compression} \quad (4b) \]

The Maximum Shear Stress is given by:

\[ \tau_{\text{max}} = \sigma_x \[ \sqrt{5} \] , \text{ Where } \sqrt{5} = 1 + \Phi^2 \]  

(5)

Indeed, the golden ratio \( \Phi \) definitely is a defining parameter in the stress analysis of beams. See also reference [5] for a discussion on “the golden ratio and the stress analysis of domes”.

6 Conclusion

We have seen how the phi code identifies growing patterns in nature in the form of Fibonacci spiral shapes. How the phi code identifies architectural designs in geometrical golden ratio proportions.

And finally, it was shown how the phi code also identifies aspects of engineering in the stress analysis of simple beams. In all of these areas, the golden ratio, phi, plays a significant and determinant role. The geometric forms and mathematical expressions already discussed (Fibonacci spirals, \( \Phi \)-rectangles and principal stresses) represent, on the whole, rules of behavior associated with the specific category (nature, architecture, engineering) under consideration. As discussed in this paper, in nature it is growth behavior, in architecture it is aesthetic behavior and in engineering it is structural behavior.

In conclusion, “the golden ratio” appears to be the principal invariant in patterns of growth described by nature as well as among the various systems of proportion employed in architecture. It is also a significant parameter
encountered in the area of stress analysis when considering the principal stresses acting on a beam as predicted by elastic theory. The phi code, in the form of the golden ratio, is clearly a pervasive theme in nature, architecture, and structural mechanics. The golden ratio may very well be a definitive characteristic of “design” in nature, architecture, and engineering (structural mechanics) as well as an important element of aesthetic expression in all of these areas. Our fascination with the golden ratio and all it represents will continue to be of interest for scientific speculation and artistic expression. We need only to search our environment and hope to discover these Φ-geometries and Φ-relationships that are encoded in that universe in order to decipher the phi code.

References