Applications of the finite vortex model

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Abstract

This paper focusses on the salient features and applications of the “Finite Vortex Model” (FVM) [3]. This model simulates the coupled dynamics of an oscillating airfoil (AF) in still or moving fluids. First concept and experimental FVM-validation dates back to 1963 [2]; it is based on observations in nature. After recent extensions and validation [3], [5], [12], it proved to be an effective tool for many applications in nature and engineering. After presenting the basic mechanisms, the formation of the “finite edge vortex” (FEV) and its dynamic interaction with the AF, first the FVM is applied to nature: flapping flight, swimming. Particular attention is drawn to the active attachment phase; subsequently detaching periodic FEVs are forming a typical “ladder wake”. Then sample FVM-analyses are given of the jet characteristics of a plunging AF [1]. Typical D- and T-vortex patterns do not coincide with actual force generation. General criteria are given for the transition from drag to thrust forces; they capture all relevant parameters: frequency, amplitude, freestream- and induced velocity. Next the broad field of engineering applications is addressed. All cases are of interest, where unsteady viscid flows are involved and where NS-simulations are prohibitive. Discussion starts with the steady lift generation around a fixed aircraft wing in flight: detailed NS-results prove the periodic generation of microvortices at the trailing edge (T.E.) [17]. Then several methods are presented [14], how to generate “active” vortices to improve the transport of fluids/cascade flow [8], [16], [18], cooling of hot components [16], [17] and mixing [14]. The main results are summarized and conclusions concerning the FVM-applications and future work are given.

Keywords: unsteady flow, oscillating airfoil, vortex model, analysis of forces, powers, etc., fluid-structure interaction, applications in engineering and nature.

1 Introduction and FVM

Rising attention is paid to unsteady fluid dynamics and solutions from nature. Powerful tools like 3D-NS solvers are currently used [11] but in many situations
these methods are prohibitive and risky; approximate solutions are sufficient. There are only few simpler models available for unsteady simulations; the only concept for a viscous flow is the FVM [2], [3]. This model replaces the KUTTA-condition at the T.E. by a general “finite normal velocity condition” \( v_{PRN}(t) \neq 0 \), Fig.1. The FVM is based on the following sequence for a 2D \( s(t), \phi(t) \) AF-oscillation (Fig. 1B.):

![Finite Vortex Model (FVM)](image)

Figure 1: Application of the Finite Vortex Model (FVM), A.: Analysis of the pitching flap experiment C6 [2]; “Up –stroke with counteracting D- and T- vortices (f=0.210, T=0.0490 s) B.: The FVM [3] with the finite edge vortex FEV and 2D-kinematics \( s(t), \phi(t) \), “UP”-stroke.

1. The primary “edge flow mechanism” feeds the FEV until reaching a finite, stable size (radius \( a \), mass \( m_{vortex} \)); then, with \( v_{PRN}(t) \) still rising, it is accelerated.
2. Roll-up continues (suction ‘S’) until $v_{PRN}(t)$ reaches a peak. In case of imposed freestreams $v_{\infty} \neq 0$ (or $v_{in}$) the velocity $v_{PRN}$ depends also on $v_{in}$ and the AF-position $\phi(t)$ as shown in Fig. 1A. (FVM for a pitching flap experiment).

3. The “attachment phase” of the net FEV is defined by $0 \leq v_{PRN} \leq \max v_{PRN}$; during that period (ca. T/4 [4]) the primary FEV induces a counterrotating bound vortex BV around the AF with the same circulation level as the driver FEV. This BV is made up by a so called “Mantelströmung”, close to the AF-surface [8].

4. The time dependent MAGNUS-type interaction of the BV with the external, velocity $v_{MR}(t)$ generates the circulatory force component $F_2(t)$. With a dynamic “profile quality $\zeta$ the resistance component is $F_1(t) = F_2(t) / \zeta$ ; ($\Gamma = \pm 2\pi a v_{PRN}$).
5. With the FEV detaching from the AF, the force generation is terminated. The next FEV is built up after reversal of the AF motion; steps 1–4 are repeating (two FEVs per cycle T). The “footprints” of the periodically shed FEVs are forming a characteristic inclined “ladder wake” behind the AF (see Fig. 2).

Figure 3: Generation of fluid forces by oscillating airfoils (AF): A.: FVM-analysis of an AF pitching in air with 25.0 Hz (FEV-size a(f) derived in [5] from validations [2], [9]) B.: Vortex wakes from plunging NACA 0012 in water (R=100.mm); Lai and Platzer [1]; D,N,T.; Pattern transition “drag-neutral-thrust”; FVM-analysis of cases I) II) III) to V) (Table 1).
Apparently the mass/energy/momentum of the moving FEV does simulate the interaction with both, the AF-structure and the external velocity \( v_{MR} \) very well; its dimensionless size \((2a/R)\) turns out to be a key-parameter for the overall dynamics. The example in Fig. 1A. shows details of an FVM-analysis for a pitching flap test in air [2]: One can see that actually two counteracting edge vortices are “competing”- the passive D-vortex (drag-type /lee side) and the active T-vortex (thrust-type/luff side). The formation of the FEV is delayed to when D cancels T \((v_{PRN} = 0)\). Similarly the detachment of the net FEV is delayed also by 18 ms; force interaction terminates. Mean force /power required are \( H_m = + 0.0748 \) N / \( P_m = 1.51 \) W. The velocity ratio \( f \) is the general characterizing parameter. Here it becomes \( f = 0.2100 \) (-) (being indicative for a thrust force, i.e. \( C_m > 0 \)), similarly \( V_A = 0.107 \) (-). The induced velocity \( v_{idz} \) is 1.426 m/s.

The following equations are used:

\[
f = \left( \frac{v_m}{v_{max}} \right) = \left( \frac{v_\infty}{v_{max}} \right) + f_a = \left( \frac{v_{idz}}{2} \right) / v_{max} \quad \text{with} \quad Str = \left( \frac{\omega R \phi_0}{v_m} \right) / f = 1 / Str
\]

For pure, harmonic pitching \( \phi(t) = \phi_0 \sin(\omega t) \) within \( \pm \phi_0 \):

\[
v_{max} = \left( \frac{\omega R \phi_0}{v_{max}} \right) \quad \text{and} \quad \left( \frac{v_{idz}}{2} \right) = \left( \frac{2a/R}{2} \right) R \omega / (16 \sin \phi_0)
\]

In general the computer solves four coupled systems of nonlinear equations:

- Equilibrium of forces and moments (from D’Alembert-Lagrange)
- Arbitrary, prescribed AF-motion with resulting kinematic relations
- Compatibility relations for the \( v_{PRN} \) (slip-free contact between AF and FEV)
- Physical relations between velocities, circulations and fluid forces \( F_{1,2} \)

Main results are time dependent forces/moments/circulations/power requirements.

2 FVM application in nature

It is well known, that pitching fins/flapping wings in nature do generate considerable forces from an efficient formation/control of vortices, leaving wakes of minimum energy content. Nature very effectively combines pitching, plunging, sculling and rotation of wings [10], or fins. The FVM-application to flapping flight in nature is depicted in Fig. 2 (birds, bats, insects; similarly fish): The attachment phase is shown in an exploded T.E.-view with the driver FEV, forces \( F_{1,2} \) induced by the BV around the wing. After detachment - as a “footprint”- the FEVs assemble to a typical inclined “ladder wake”, recently confirmed by DPIV-experiments [13]. Fig. 3 shows the generation of forces by AFs oscillating in fluids: Large amplitude \((20^\circ, 40^\circ)\) high frequency \( (25 Hz) \) pitching FVM-results present the variation of \( C_m(f) \), where a typical vortex radius function \( 2a/R \) \( (f) \) is taken from [4]; thrust /drag force regimes are quantified. Plunging experiments in water [1] are also shown and analyzed by FVM (table below). Clearly the D-/N-/T-patterns from tests [1] are not coinciding with drag or thrust-force prediction by FVM. This was found by others, too [7], [9]. Apparently here it needs higher \( v_{PRN} \): 1.5-3.0 instead of 0.1-0.2 m/s. Thrust
is predicted for the data in [1] from \( f \leq 0.25 - 0.38 \) on \( \text{VA} \leq 0.095 - 0.14 \), and \( 2a/R \geq 0.52 - 0.56 \), also found in [5],[12]. The power required in cases III) to V) is 0.450, 71.88 and 580.7 W.

Table 1.

<table>
<thead>
<tr>
<th>CASE (water)</th>
<th>( \nu_\infty ) m/s</th>
<th>( \omega/2\pi ) Hz</th>
<th>( s_0 ) mm</th>
<th>PLATZER (K/h)</th>
<th>( f ) -</th>
<th>VA -</th>
<th>a mm</th>
<th>H_m N</th>
<th>( v_{edx} ) m/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>I)</td>
<td>0.2</td>
<td>5.</td>
<td>2.50</td>
<td>0.393</td>
<td>25.50</td>
<td>0.6375</td>
<td>23.95</td>
<td>-18.25</td>
<td>3.606</td>
</tr>
<tr>
<td>II)</td>
<td>0.2</td>
<td>5.</td>
<td>5.00</td>
<td>0.785</td>
<td>8.041</td>
<td>0.4021</td>
<td>26.00</td>
<td>-10.75</td>
<td>2.127</td>
</tr>
<tr>
<td>III)</td>
<td>0.2</td>
<td>5.</td>
<td>7.50</td>
<td>1.178</td>
<td>3.991</td>
<td>0.2993</td>
<td>26.55</td>
<td>-6.934</td>
<td>1.481</td>
</tr>
<tr>
<td>IV)</td>
<td>0.2</td>
<td>10.</td>
<td>36.4</td>
<td>11.44</td>
<td>0.2443</td>
<td>0.0889</td>
<td>27.95</td>
<td>+12.09</td>
<td>0.7176</td>
</tr>
<tr>
<td>V)</td>
<td>0.6</td>
<td>20.</td>
<td>36.4</td>
<td>7.624</td>
<td>0.2881</td>
<td>0.105</td>
<td>27.95</td>
<td>+39.91</td>
<td>1.435</td>
</tr>
</tbody>
</table>

### 3 FVM application in engineering

It is interesting not only to look at nature’s evolution but also to recall human history of aerodynamics: In the early days (1910-30) unsteady aerodynamics was addressed but within the Prandtl, v.Karman-school. The big success of aeronautics shifted attention towards steady flow, conformal mapping, lifting line theory (1930-50).

Rebirth of work on unsteady and vortex flow did focus on wake analyses (1960-80).

Later (1970-2000) many researchers “moved ahead”, towards the leading edge, concentrating on the slowly spinning L.E.V. (dynamic stall vortex); since about 4 years, also due to new experimental techniques, more light is shed on the strong, fast spinning T.E. vortex [6] (FEV). This development is also linked to the rising possibility to use large 3D-NS-solvers. The steady lift analysis around a fixed aircraft wing is based on the successful Kutta-condition (smooth detached, inviscid flow at T.E.) and on Prandtl’s lifting line theory. Detailed NS-results, however, prove periodic, high-frequency microvortices (ca.10 KHz) due to viscous flow around the T.E. Fig. 4 gives mean power requirements \( P_m \) and thrust \( H_m \) from FMV-analyses in water (no imposed freestream flow) with/without circulatory forces \( F_{1,2} \); it underlines the importance of the aerodynamic MAGNUS-effect to the pure inertial action.

Fig. 5 presents several methods how to generate “active”, optimum vortices (\( \omega, \varphi_0 \) etc.) for the following broad FVM-applications in engineering [8], [11], [14], [15], [17], [18]. Improvements are visible in: 1) Transport of fluids, pulsed jet pumping (mechanical & process engineering, fluidics) 2) Cascade flow, blade flutter, noise & cavitation control (power generation, pumps & hydroturbines) 3) Propulsion, lift & thrust control (aeronautical & ship engineering, bionics) 4) Cooling of hot components (power generation gas turbines, jet engines) 5) Mixing of fluids, combustion, emissions (power generation, process engineering, biotechnology, biofluiddynamics).
Figure 4: Mean power requirement $P_m$ and thrust $H_m$ from FVM-results with and without circulatory forces: 

- **A.** $P_m$ for a pitching AF in water over a wide range of frequencies and amplitudes (full vortex interaction: “inertia + circulatory” = I+C).
- **B.** Significance of adding the circulatory forces $F_{1,2}$ to the dynamic equations (MAGNUS-effect).

### Table: Significance of forces $F_{1,2}$

<table>
<thead>
<tr>
<th>Sample results for 24 Hz:</th>
<th>Inertial+ circulatory 1) $F_{1,2} \neq 0$ (I+C):</th>
<th>Inertial only 2) $F_{1,2} = 0$ (I):</th>
<th>Factor $(I+C) / (I)$:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amplitude $\phi_o$:</td>
<td>$30^\circ$</td>
<td>$10^\circ$</td>
<td>$30^\circ$</td>
</tr>
<tr>
<td>Thrust $H_m$ N</td>
<td>174.3</td>
<td>475.6</td>
<td>89.52</td>
</tr>
<tr>
<td>Power $P_m$ W</td>
<td>1917</td>
<td>20.56</td>
<td>1173</td>
</tr>
</tbody>
</table>

1) From above FVM-numerical solutions (I+C)  
2) From explicit integration results, equ. (1) and (2) below (see 5 Nomenclature):  

Mean thrust $H_m$ over period $T$ is:

\[(1) \quad H_m = \left(\frac{1}{\pi}\right) m_{vortex} a \phi_o \omega^2 \text{ with } m_{vortex} = \pi a^2 B \rho\]

Mean overall power $P_m$ over period $T$ is:

\[(2) \quad P_m = \left(\frac{1}{2\pi}\right) \left[1 + \left(\frac{1}{4}\right) (2a/R)^2\right] m_{vortex} R^2 \phi_o \omega^3\]
Figure 5: Efficient means to generate FEVs for applications in engineering: Optimum frequency, amplitude Cases (a) to (c): With mechanical drives DR and “Gap ventilation” GP Case (d): No moving parts; pulsing flow generating periodic ring vortices (circular pipe), see [14].

4 Results and conclusions

1) Unsteady aerodynamics play an increasing role in understanding nature and improving engineering design. A large potential for FVM-applications is visible.
2) For approximate analyses of oscillating AF the FVM is confirmed experimentally and by NS-analyses. It bridges large solvers and classical aerodynamics.
3) The coupled fluid-structure responses from FVM are governed by $\omega$, $\varphi_0$, $v_{idz}$, $v_\infty$; new parameters are introduced /criteria are quantified for drag-thrust transition.
4) More experimental work is needed to generalize/extend the relations between 2a/R and f, φ₀, f∞, edge radius and type of AF-kinematics for optimisation.

5 Nomenclature

\[ \begin{align*}
\text{a}, \text{B}, \text{R} & \quad (\text{m}) \quad \text{Radius of stable, finite vortex; span, chord of airfoil} \\
\text{C}_m & \quad (-) \quad \text{Coefficient of mean horizontal force } \text{H}_m \text{ (reference velocity } v_m) \\
\text{f} = f_{\text{inf}} + f_{\text{a}} & \quad (-) \quad \text{Velocity ratio } f = (v_m / v_{\text{max}}) \text{ (newly introduced)} \\
\text{F}_2(t), \text{F}_1(t) & \quad (\text{N}) \quad \text{Circulatory forces, MAGNUS: } \text{F}_2(t) \text{; drag: } \text{F}_1(t) = \text{F}_2(t) / \zeta \\
\text{h} = \text{s}_o / \text{R} & \quad (-) \quad \text{Nondimensional amplitude of oscillation} \\
\text{H}, \text{H}_i, \text{H}_z & \quad (\text{N}) \quad \text{Horizontal forces } \text{H}(t); \text{ (total, inertial, circulatory)} \\
\text{H}_m & \quad (\text{N}) \quad \text{Mean force on airfoil (over T): Thrust } (>0), \text{ Drag } (<0) \\
\text{K} = \omega R / v_m & \quad (-) \quad \text{Reduced frequency (newly introduced)} \\
\text{m}_{\text{vortex}} & \quad (\text{kg}) \quad \text{Mass of finite cylindrical vortex (radius a)} \\
\text{m} & \quad (\text{kg} / \text{s}) \quad \text{Fluid mass flow} \\
\text{P}, \text{P}_1, \text{P}_2, \text{P}_B & \quad (\text{W}) \quad \text{Powers } \text{P}(t); \text{ (total, inertial, circulatory, rotational)} \\
\text{P}_m & \quad (\text{W}) \quad \text{Mean total power required (over } \text{T}) \\
\text{s}(t), \text{s}_o & \quad (\text{m}) \quad \text{Plunging displacement of } \text{2D-airfoil motion} \\
\text{v}_{\text{PRN}}(t) & \quad (\text{m} / \text{s}) \quad \text{Net normal component of flow velocity around } \text{T.E.} \\
\text{v}_{\text{idz}} & \quad (\text{m} / \text{s}) \quad \text{Mean flow velocity induced by the AF-motion (over } \text{T}) \\
\text{v}_\text{M}(t) & \quad (\text{m} / \text{s}) \quad \text{Pitching flap normal velocity of } 1/4 \text{ chord point } \text{M (Fig. 1)} \\
\text{v}_{\text{MR}}(t) & \quad (\text{m} / \text{s}) \quad \text{Instantaneous flow velocity around AF, "middle section"} \\
\text{v}_m & \quad (\text{m} / \text{s}) \quad \text{Mean flow-thru-velocity across pitching flap (over } \text{T}) \\
\text{v}_i & \quad (\text{m} / \text{s}) \quad \text{Incoming flow velocity (generally } v_\infty) \\
\text{T} & \quad (\text{sec}) \quad \text{Oscillation period } 2\pi / \omega = (1/ \text{frequency}) \\
\text{VA} = (f h) & \quad (-) \quad \text{Nondimensional product } \text{“flow-thru-velocity x amplitude”} \\
\alpha(t) & \quad (\text{rad}) \quad \text{Instantaneous angle of flow attack around airfoil} \\
\Gamma(t) & \quad (\text{m}^2 / \text{s}) \quad \text{Instant. bound circulation } \pm 2\pi a \text{ v}_{\text{PRN}} \text{ around AF (Fig. 1)} \\
\zeta(\alpha) & \quad (-) \quad \text{Dynamic profile quality } = C_2 / C_1 = C_L / C_D \text{ of the AF} \\
\omega & \quad (1 / \text{s}) \quad \text{Circular frequency of AF oscillation} \\
\phi(t), \phi_0 & \quad (\text{rad}) \quad \text{Pitching angle of AF, amplitude of pitching AF} \\
\phi(t) & \quad (\text{rad}) \quad \text{Rotational angle of finite vortex (Fig. 1)} \\
\end{align*} \]

References


