

# The mechanical self-optimisation of trees

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## Abstract

Due to the permanent competition in nature and the survival of the fittest, trees are highly optimised structures. The expenditure of energy and material for fail-safe constructions would be uneconomically high for a species and a natural failure rate has to be accepted, but the optimisation prevents early failure. Five theorems are exemplified in the paper which describe the mechanical self-optimisation mechanisms of trees.

- Minimisation of lever arms. Trees minimise stresses by reducing the length of the loaded lever arm. Length reduction can be achieved by active self-bending of stiff parts by forming reaction wood or through passive yielding of flexible parts of trees.
- Axiom of uniform stress. Adaptive growth reduces stress-concentrations on the surface. Unavoidable stresses are distributed evenly on the surface of a tree.
- Minimisation of critical shear stresses. Wood fibres are arranged along the force flow which minimises the shear stresses between the fibres.
- Adaptation of the strength of wood to mechanical stresses. The wood quality is adapted to the degree and type of loading.
- Growth stresses counteract critical loads. Residual stresses developed by the trees counteract axial compression and tangential tension caused by external loads and reduce the resulting stresses at the surface of the trees.

Some of these biological optimisation mechanisms have been transferred by the Forschungszentrum Karlsruhe into engineering design processes and are used very successfully for the optimisation of technical components.

*Keywords: axiom of uniform stress, shape optimisation, shear stress, residual stresses, wood strength.*



## 1 Introduction

Most technical products only reach a fraction of the complexity and perfection of a simple, biological structure. The tree is a typical example of this type of development excellence: It is a natural wood construction with high stability at a minimum material expenditure and produces energy from solar radiation. Achieving high stability at minimum material expenditure means optimal material usage and prevention of early failure by avoiding of weak spots. This is guaranteed by the mechanical self-optimisation of trees.

The comprehension of the optimisation mechanisms of trees often can explain “why they grow how they grow” [1] and can also be used for the optimisation of technical components.

## 2 Self-optimisation mechanisms of trees

Outer shape, internal structure, prestressing and even the mechanical behaviour of trees under loading are influenced and adapted to the typical loading conditions. The mechanical self-optimisation mechanisms of trees are exemplified by five theorems.

### 2.1 Minimisation of lever arms

Trees minimise stresses by reducing the length of the loaded lever arm. Length reduction can be achieved by active self-bending of stiff parts by forming reaction wood or through passive yielding of flexible parts of trees.

Apical dominance and negative geotropism are balanced continuously to adjust the optimum angle between branch and stem. The potential successor's urge to straighten is suppressed by the ruling leading shoot as long as it is intact. After the loss of the leading shoot the outstretched, lateral succeeding branch straightens to take the place of its predecessor (Figure 1). This branch, mostly, but not necessarily the uppermost branch, then asserts its claim to apical dominance and suppresses the remaining potential leading shoots.

Leaning trees on slopes straighten themselves through formation of reaction wood. Depending on the taper of its stem a leaning tree grows into a sabre shape or develops a meandering shape. In both cases the loaded lever arm is reduced by positioning the centre of gravity straightly above the root plate.

In addition to the active negative geotropism that shortens the loaded lever arms, there are also passive regulating mechanisms. Grass and flowers are bent and flattened by the wind which reduces the “area of the sail”. Trees or rather the fringes of their crowns, are found to apply a similar strategy [2]. They reduce the area of their sail by bending twigs and even leaves in the wind direction.

### 2.2 Axiom of uniform stress

Trees are characterized by a homogeneous stress distribution on the tree surface. They grow such that local high stresses (potential points of rupture) as well as



locally small stresses (excessive use of material) are avoided. This was demonstrated by Metzger [3] in 1893 already, based on the taper of spruce trunks. Mattheck [4] and co-workers generalised this principle and verified by several examples that the axiom of uniform stress is a universal design rule for biological structures.

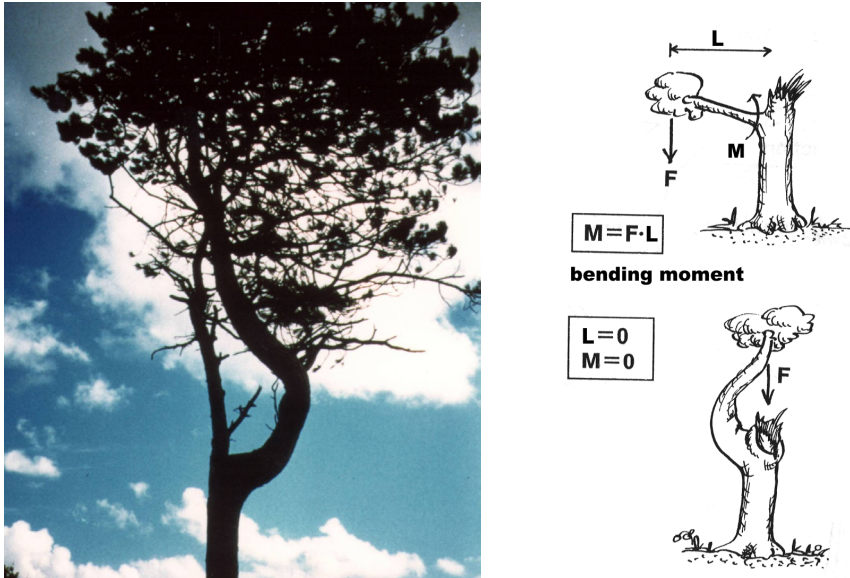


Figure 1: Minimisation of the bending load by rising up of a lateral branch after deterioration of the leading shoot.

Trees make any effort to grow into a homogeneous state of stress on their surface. The outermost annual ring always tries to adapt to the external loading by locally increased or reduced growth according to high or low stresses. For instance, the cross-section of each root adapts to the load acting on it (Figure 2). A root subjected to bending load only has an 8-shaped cross-section which is similar to that of an I-beam used for construction purposes. If for example tension and bending load are superimposed root growth takes place preferably on the upper side of the root.

### 2.3 Adaptation of the strength of wood to mechanical stresses

Wood quality is adapted to the degree and type of loading. An example for the adaptation of strength are branches.

Measurements of the axial compressive strength of deciduous tree branches with the Fractometer III [5] indicate that wood on the bottom sides of branches reacts to permanent pressure loads by developing high compressive strengths [6]. This can be seen from Figure 3, the cross section and the compressive strength



profile of the branch of an oak. Annual increment is a maximum on the top side of the branch, resulting in the oval shape of the cross section which represents an effort to match the shape to the dominant load constituted by the dead weight. The curve of compressive strength versus the branch cross section shows low compressive strengths in the wood on the top of the branch and clearly higher levels in the wood on the bottom side of the branch.

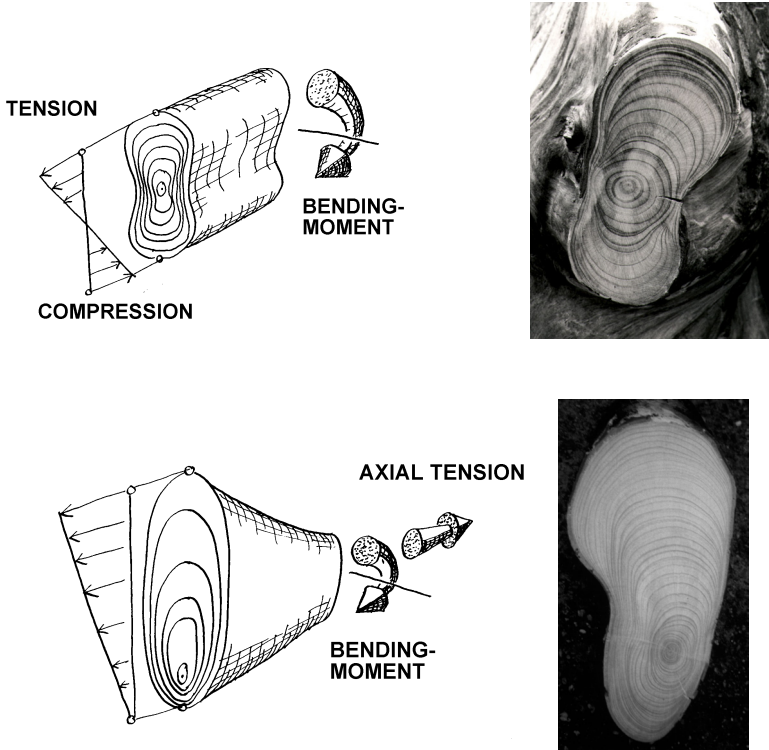


Figure 2: The approximate symmetrical 8-shape indicates a pure bending load, the combination of tension and bending load causes root growth to take place preferably on the upper side.

#### 2.4 Minimisation of critical shear stresses

It is common to most biological load carrying structures that they are not made of homogeneous, isotropic material, but of fibrous structures. In the case of the tree, these are microscopically small fibrils. They form the fibrils of the cell walls which generate the macroscopic wood fibres and eventually join in fibre bundles.



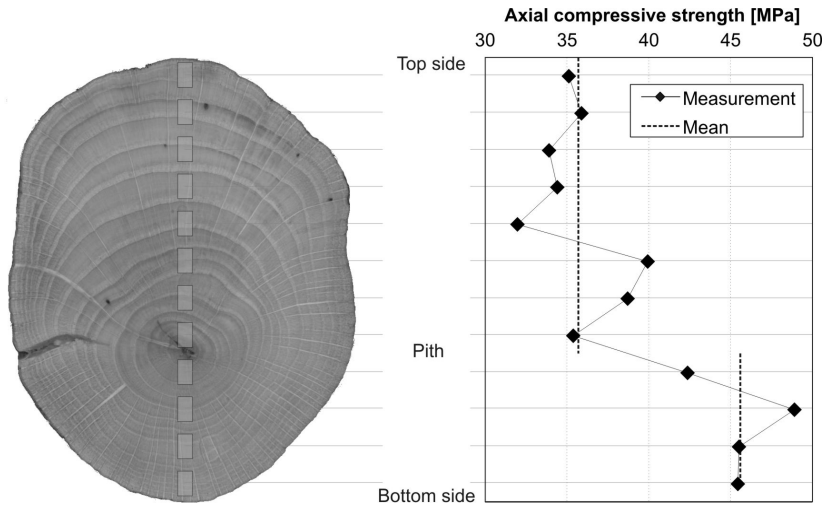


Figure 3: Cross section of a branch (*Quercus* spp.) and its compressive strength profile. The bright rectangles in the cross section indicate the original positions of the compression samples taken with an increment borer.

Trees and other biological structures, e.g. bones, try to arrange their fibrous pattern along force flow. This means that the stiffest material directions of an orthotropic material is locally aligned along the direction of the maximum principle stresses [7]. The modulus of elasticity of the wood-fibres in grain direction is much higher than the modulus of elasticity perpendicular to the grain.

Having a look at a died off branch or a decayed trunk, even the patterns of the wood fibres can be studied. In smooth curves, the fibres are passed around branch connections (Figure 4). These almond- or spindle-shaped contours divert the axial force smoothly and minimise the shear stresses between the fibres. Figure 4 also shows that uniaxial fibre arrangements around structural disturbances cause much higher shear stresses between the fibres than spindle shaped.

Only the orientation of the outer fibres is controlled mechanically and follows the force flow unless hindered by genetic programming. The fibres in the interior of the tree are no longer aligned with the force flow. This can cause failure.

When analysing a cross-section e.g. of a sweet chestnut tree under the microscope, it can be seen how the wood rays are arranged around the vascular cells. Even the fibrils which make up the cell walls of the wood cells exhibit this optimised form. Thus, self-iteration in nature is shown in a rather impressive manner. Direction of the fibres along the force flow can be found in all dimensions of the tree.

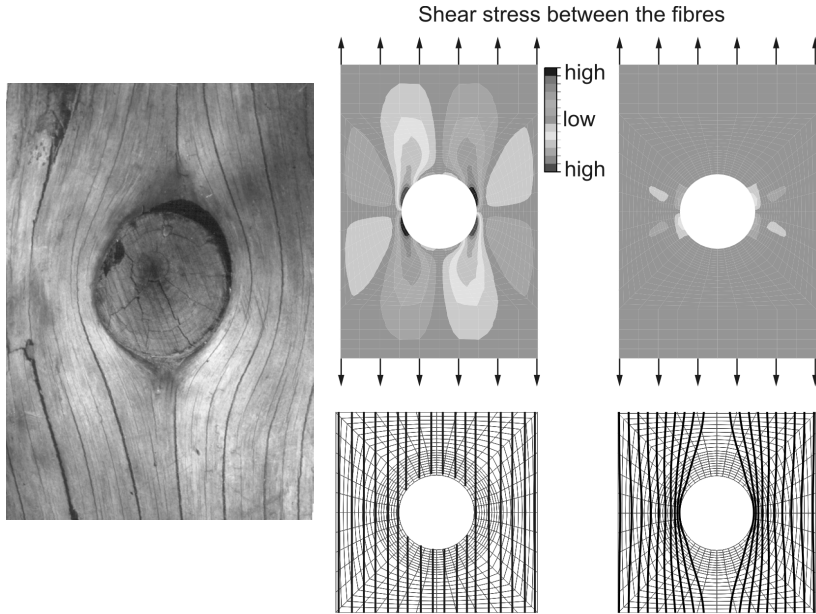


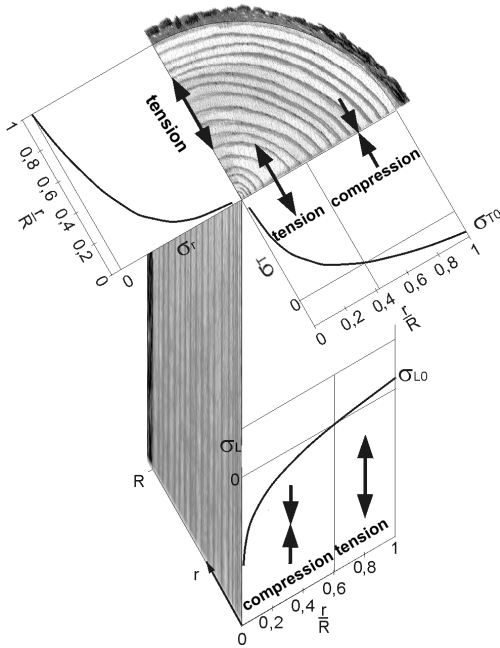
Figure 4: The wood fibres are laid in the shape of a spindle around a branch. This optimised form minimises shear stresses between the fibres as it can be seen in the Finite Element calculations. Uniaxial fibre arrangements around a hole cause much higher shear stresses between the fibres than the optimised spindle shaped.

## 2.5 Growth stresses

Growth stresses were investigated and described first by Jacobs [8]. Kubler examined the growth stress phenomenon and established a simple theory [9].

The high risk of fibre buckling reveals that the compressive strength of wood is only about a quarter to a half as high as its tensile strength. Trees could resist much higher bending moments if they have support in resisting the hazardous compressive stresses on the compression side of bending, which is the lee side for wind loading. This assistance is given by the axial growth stresses that act in them. Figure 5 shows the distribution of growth stresses and how the marginal areas of the tree cross section are longitudinally prestressed by tension while the interior is compressed.

The longitudinally tensile prestressing of the marginal areas minimises the hazardous compressive stresses on the lee side, but increases the less critical tensile stresses on the windward side (Figure 6). The resulting stress distribution, adapted to the strength of wood, would lead to failure due to tension and failure due to compression at the same time in the ideal case.



$$\sigma_L = \sigma_{L0} \left(1 + 2 \ln \frac{r}{R}\right)$$

$$\sigma_T = \sigma_{T0} \left(1 + \ln \frac{r}{R}\right)$$

$$\sigma_R = \sigma_{T0} \ln \frac{r}{R}$$

$\sigma_L$  = longitudinal growth stress in grain direction

$\sigma_T$  = tangential growth stress

$\sigma_R$  = radial growth stress

$\sigma_{L0}, \sigma_{T0}$  = stress at the surface of the trunk

Figure 5: Distribution of longitudinal, radial and tangential growth stresses.

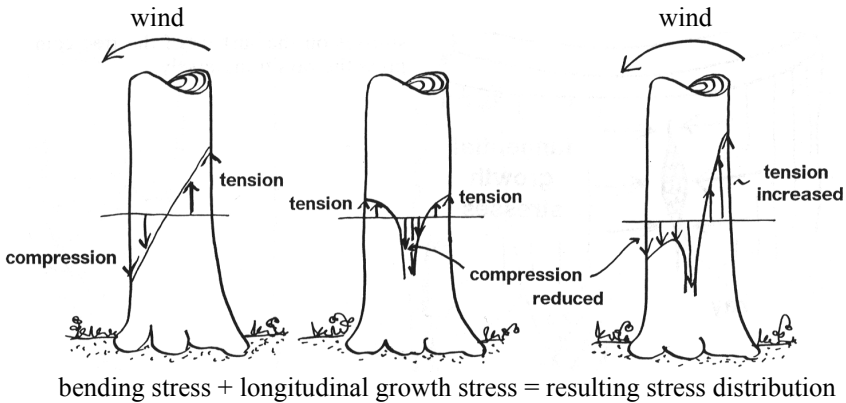


Figure 6: Longitudinal tensile growth stresses on the surface of trees minimise the critical compressive stresses induced by wind-bending load [10].

The wood fibres are deflected around the wood rays and are endangered by local buckling and separating of fibres due to tangential tensile stresses. In that case the wood rays become crack starters. The tangential compression prestressing at the surface minimises the risk of radial cracks at the surface due

to local fibre buckling and due to transversal tension on the convex side of bended sabre-shaped trees. Dietrich [11] found that the compressive stresses in the tangential direction are largest where the external loading or curvature of the trees or the branches cause the largest tangential tensile stresses.

### 3 Biologic optimisation mechanisms and engineering design

Some of the biological optimisation mechanisms explained above have been transferred by the Forschungszentrum Karlsruhe into computer algorithms (Figure 7) and are used very successfully for the optimisation of technical components [12].

In most cases local stress concentrations on the surface due to notches or changes in geometry are responsible for fatigue failure. Therefore, a prevention of such stress raising effects is of great importance in nature as well as in engineering design. The effort of trees to grow into a homogeneous state of stress on their surface (axiom of uniform stress) was shown in section 2.2. The outermost annual ring always tries to adapt to the external loading by locally increased or reduced growth according to high or low stresses. The Computer Aided Optimization method, CAO, simulates this effect by a Finite Element calculation with a fictitious thermal expansion dependent on previously calculated stresses whereat high stresses lead to a great thermal expansion and vice versa. By this, the stress peaks will be reduced and finally a homogeneous state of stresses on the surface is achieved. The CAO method is used for the enlargement of the lifetime of a lot of technical components in the automotive industry as well as in orthopaedics. Now a new analytical method [13] for shape optimisation which works without of the use of FE calculations is in progress.

The Soft Kill Option (SKO) is also based on the axiom of uniform stress. In contrast to the CAO method this method removes underloaded material not only at the surface but also inside of a component. The SKO-method copies bone mineralisation. Unlike trees, bones are able to remove underloaded material and save weight. This strategy of nature makes sense, especially when taking into account that the tree does not move during its often several hundred years of life. An animal often survives by moving its lightweight design as quickly as possible to catch a prey or not to become a prey. Bone mineralisation means that areas which are subjected to higher loads are hardened by adaptive mineralisation, whereas less heavily loaded sections are softened and finally discarded or 'killed'. The main procedure needs again a standard Finite Element program. The stress distribution is calculated for an initial design area, the limit dimensions and bearing and loading conditions of which correspond to those of the component in service. In accordance with the calculated stresses, the Young's modulus is increased in areas subjected to higher loads and is reduced in more lightly loaded areas. The load carrying sections are reinforced while the more lightly loaded sections are softened and increasingly relieved. Some iterations suffice to separate sharply the passive, idle areas from the active, useful ones. All areas serving no useful purpose are finally eliminated. Result of the SKO method is a lightweight design proposal.



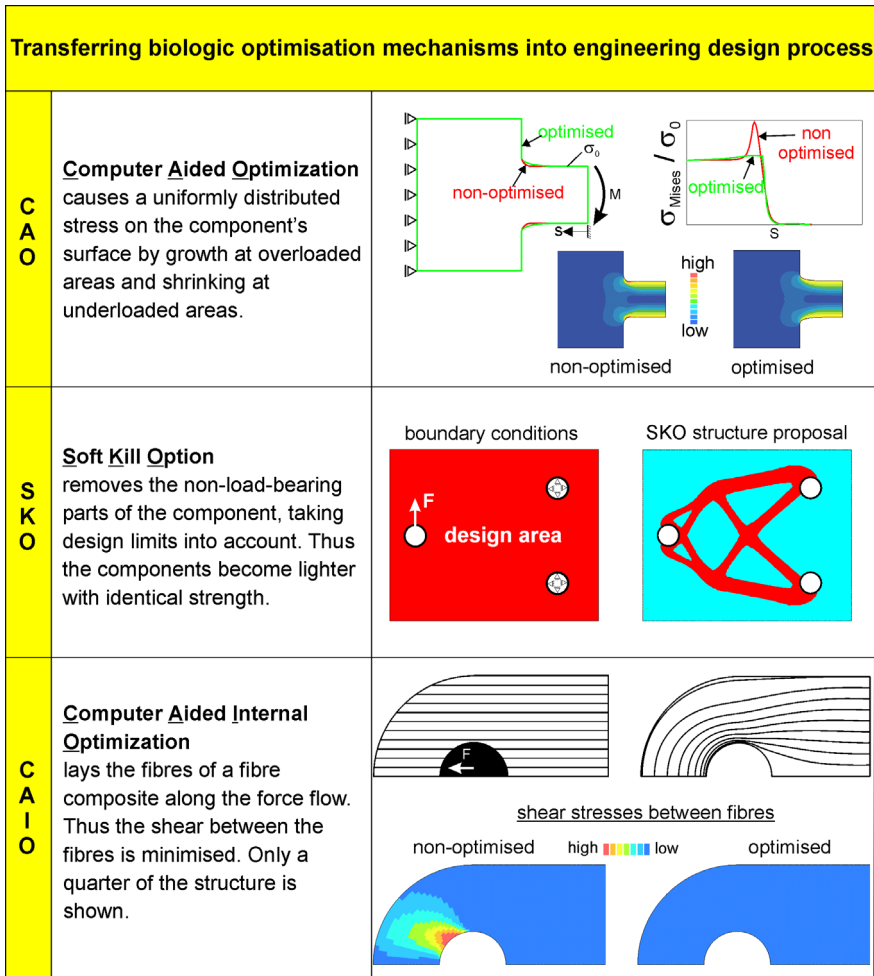


Figure 7: Biological based computer methods for the optimisation of technical components.

Finally the Computer Aided Internal Optimization (CAIO-method) optimises the performance of composite materials by aligning the fibre distribution with the force flow in order to avoid shear stresses in between the fibres, again mimicking the structure of trees (see section 2.4). Avoiding shear stresses in technical fibre components can increase the maximum load capacity. First, an orthotropic material of any orientation is assigned to the FE model to be optimised. After an FE analysis, the optimised material directions are calculated by means of the CAIO routine. From the results of the stress analysis the new orientation of the material axes along the principle stress trajectories is computed. The results of the CAIO calculation serve as a basis for further stress analysis with the material axes being oriented along the force flow.

While CAO and SKO are used very successfully for the optimisation of technical components, technical feasibility constricts the application of CAIO because an accurate realising of the complex spatial fibre arrangement mostly is too expensive or even not possible.

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