Complexity in architecture: 
a small scale analysis

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**Abstract**

For many centuries, architecture found inspiration in Euclidean geometry and Euclidean shapes (bricks, boards), and it is no surprise that the buildings have Euclidean aspects. Nature is fractal and complex, and nature has influenced the architecture in different cultures and in different periods. Complexity is the property of a real-world system that is manifest in the inability of any one formalism being adequate to capture all its properties. The complexity is also the theory of how emergent organisation may be achieved by the interaction with components pushed far from equilibrium (by increasing matter, information, or energy) to the threshold between order and disorder (chaos). This threshold is where the system often interacts in a new non-linear way. Modern architects study the complexity and the fractal geometry to create a new kind of buildings or to understand the problems connected to the urban growth. The aim of this paper is to present an approach that studies the complexity applied in the small scale in architecture.

**Keywords**: complexity, strange attractor, non-linear architecture, fractal geometry, self-similarity, box-counting dimension.

**1 Introduction**

In arts and architecture it is usual to search the presence of mathematical and geometrical components. For example, the golden ratio, the symmetry, the tessellations, the Fibonacci’s sequence, and the Euclidean geometry [1, 2, 3, 4, 5, 6, 7, 8]. We can also observe the architecture using a different point of view, for example we can find some complex or fractal components that are present in the buildings or in the projects [8, 9, 10, 11, 12].

The complexity is the most difficult area of chaos, and it describes the complex motion and the dynamics of sensitive systems. The chaos reveals a hidden fractal order underlying all seemingly chaotic events. The complexity can occur in natural and man-made systems, as well as in meteorological systems, human beings and social structures. Complex dynamical systems may be very small or very large, and in some complex systems small and large components exist in co-operative way. The complexity can also be called the “edge of chaos”, it is connected to the fractal geometry, and it can also inspire an aesthetic sense. In fact, in the 1930’s the mathematician George Birkhoff (1884-1944) proposed a measure of beauty defined as:

$$M = \frac{O}{C}$$

whereby M stands for “aesthetic measure” (or beauty), O for order and C for complexity. This measure suggests the idea that beauty has something to do with order and complexity.

The complexity and the fractal geometry appear in architecture because it observes and reproduces the patterns present in the nature. Robert Venturi affirms, in his book entitled *Complexity and contradiction in architecture* (1992): “The recognition of complexity in architecture does not negate what Louis Kahn has called “the desire for simplicity”. But aesthetic simplicity which is a satisfaction to the mind derives, when valid and profound, from inner complexity” [9, p. 17].

Our complex analysis in architecture has been divided in two parts:

- on a small scale analysis (e.g., to determine the complex components present in the building shape);
- On a large scale analysis (e.g., to study the urban growth and the urban development, or to analyse the organisation of the landscape).

In this paper we will present a small scale analysis that we have organised in three points:

1. the “strange attractors” used, for example, by the architect Peter Anders in the interior of his loft apartment. He has designed his loft using a complex point of view. The loft’s visual lines create a paradoxical sense of both infinitude and repetition, fragmentation and unity, like the stranger attractors;
2. the complexity in the buildings (e.g., the complex surfaces and the complex textures in the works of Franck Owen Gehry and in the projects realised by Paolo Portoghesi);
3. the box-counting dimension of a design, to determine its degree of complexity (for example applied to the Mesoamerican architecture and to the modern architecture).

In our study the process qualities that emerge spontaneously are the attractor formations, the non-linear architecture, and the fractal patterns.
2 The strange attractors

To define an attractor is not simple. Tsonis gives the definition of attractors as “a limit set that collects trajectories” [13].

There are four kinds of attractors, where the first three attractors are not associated with chaos theory because they are fixed attractors. We have:

1. Point attractor, for example a pendulum swinging back and forth and eventually stopping at a point. This kind of attractor may come as a point, in which case, it gives a steady state where it is not possible other changes.

2. Periodic attractor, that reproduces processes that repeat themselves (for example, to add a mainspring to the pendulum to compensate for friction and the pendulum now has a limited cycle in its phase space).

3. Torus attractor, mathematically the torus is depicted in the shape of a large donut or bagel. The torus attractor naturally arises whenever quasi-periodic motion is encountered in a dissipative dynamical system.

4. Strange attractor, it deals with the three-body problem of stability. The strange attractor shows processes that are stable, confined and yet never do the same thing twice. The strange attractor can take an infinite number of different forms. One classical strange attractor is the “Lorenz attractor” that is used for the weather forecast.

Chaos is based on the concept of strange attractor [14]. The flow of the water is governed by dynamic and chaotic laws. The various kinds of flow represent different patterns to which the flow is attracted. Figure 1 shows a painting that contains a study on the movements of the water, where there are in evidence some attractors. This painting has been realised by Leonardo da Vinci (1452-1519) that was one of many artists across the centuries inspired by the mysteries of turbulence [15, p. 66].

![Figure 1: Da Vinci’s painting dedicated to the water movements.](image-url)
We can analyse the complex components in architecture observing the small scale, for example to examine the presence of strange attractors in a plan. The architect Peter Anders, fascinated by the chaos and fractal theory, has conceived the interior of his loft apartment in the shape of a “strange attractor” (see figure 2) [16, p. 173]. The visual lines create a paradoxical sense, like the stranger attractors, for the presence of the repetition, fragmentation and unity [16].

Figure 2: Peter Anders’ loft: it contains some “strange attractor”.

3 The complexity in the buildings

A new kind of architecture was born by the complexity: non-linear architecture [10]. To research the complexity in the buildings, we can observe the complex textures (e.g., in Gehry’s works) and the complex surfaces (e.g., in the Gehry’s or Portoghesi’s projects) derived by the observation of the nature, and the natural phenomena.

Frank Owen Gehry is one of the most inventive and pioneering architects working today. He has used the complexity and the fractal geometry in his recent works. Since his fish lamps of 1983, Gehry has applied the property of the self-similarity to realise the complex textures of his buildings. The self-similarity is a fractal geometry’s property that permits to an object to repeat its shape in
different scales. This is evident observing the metal shingle, repeated in different scales, that covers the Vitra Headquarters (1989-1992), and the Guggenheim Museum, Bilbao (1992-1997). Figure 3 shows the skin of a snake, where it is clear the complexity and the self-similarity. The figure 4 illustrates a portion of the Guggenheim Museum’s skin. The analogy with the nature is fascinating.

Figure 3: The skin of the snakes is self-similar.

Figure 4: Frank O. Gehry has used the self-similarity to realise the skin of the Guggenheim Museum, Bilbao (1992-1997).

Italian architect Paolo Portoghesi, inspired by the chaotic movements connected to the gas or liquid motion, has realised the Hotel Savoia (1992-1996) in Rimini (Italy) that contains smoothed surfaces inspired by the wave motion as a metaphor of the sea (shown in figure 5). The hotel is situated on the Adriatic [17]. Figure 6 illustrates a particular of the model of Hotel Savoia.
Figure 5: The wave motion is an example of complex system.

Figure 6: Portoghesi’s Hotel Savoia (Rimini, Italy).

Portoghesi has used the fractal geometry, in particular the self-similarity, in the Casa Baldi, Rome (1959), in the Villa Papanice, Rome (1966), and in the Islamic Cultural Center and Mosque in Rome (1975) [18]. His more notable works include Casa Andreis a Sandriglia, Rieti (1964), the Istituto Tecnico Industriale, Aquila (1969), the Chiesa della Sacra Famiglia in Fratte, Salerno (1969), the City Library and Social Center, Avezzano (1970), the Royal Palace
of Amman, Jordan (1973) and the Urban Planning Scheme and International Airport for Khartoum, Sudan (1973).

Gehry and Portoghesi have developed a non-linear architecture in conscious way.

4 The box-counting dimension

The box-counting dimension is connected to the problem of determining the fractal dimension of a complex two-dimensional image. It is defined as the exponent $D_b$ in the relationship:

$$N(d) \approx \frac{1}{d^{D_b}}$$

where $N(d)$ is the number of boxes of linear size $d$, necessary to cover a data set of points distributed in a two-dimensional plane. The basis of this method is that, for objects that are Euclidean, equation (2) defines their dimension. One needs a number of boxes proportional to $1/d$ to cover a set of points lying on a smooth line, proportional to $1/d^2$ to cover a set of points evenly distributed on a plane, and so on. Applying the logarithms to the equation (2) we obtain: $N(d) \approx -D_b \log(d)$.

The box-counting dimension can be produced using this iterative procedure:

- superimpose a grid of square boxes over the image (the grid size as given as $s_1$);
- count the number of boxes that contain some of the image ($N(s_1)$);
- repeat this procedure, changing ($s_1$), to smaller grid size ($s_2$);
- count the resulting number of boxes that contain the image ($N(s_2)$);
- repeat this procedures changing $s$ to smaller and smaller grid sizes.

The box-counting dimension is defined by:

$$D_b = \frac{\log(\frac{N(s_2)}{N(s_1)}) - \log(\frac{N(s_1)}{N(s_2)})}{\log\left(\frac{1}{s_2}\right) - \log\left(\frac{1}{s_1}\right)}$$

where $1/s$ is the number of boxes across the bottom of the grid.

We can apply the box-counting dimension in architecture, too. It is calculated by counting the number of boxes that contain lines from the drawing inside them. Next figure 7 illustrates the box count for the elevation of a Frank Llyod Wright’s building (Robie House, 1909) [19, p. 122]. Table 1 contains the number of boxes counted, the number of boxes across the bottom of the grid, and the grid size. The box-counting dimension of Robie House, calculated using (3), is a value between 1.441 and 1.485.

To determine the degree of the complexity in the Mesoamerican arts and architecture, Burkle-Elizondo et al. have collected more than a hundred of
images of Mesoamerican artistic and architectural works by reviewing literature on archeology [20, 21, 22]. All these images have been digitized using a Printer-Copier-Scanner (Hewlett Packard®, Model LaserJet 1100A) and saved in bitmap (*.bmp) format on a Personal Computer (Hewlett Packard®, Model Pavilion 6651). Thereafter, these images were analyzed with the program Benoit®, version 1.3 in order to calculate Box ($D_b$), Information ($D_i$), and Mass dimensions ($D_M$), and their respective standard errors and intercepts on log-log plots. It was taken under consideration that the information dimension differs from the box dimension, because its boxes contains more points. For all the cases the fractal dimension values were high from a $D_b = 1.803 \pm 0.023$ for the left and superior side of the “Vase of seven gods”, to a $D_M = 2.492 \pm 0.195$ for the left side of the “Door to underworld of the Temple 11, platform” at Copán.

![Figure 7: Box-counting method applied to Wright’s Robie House.](image)

<table>
<thead>
<tr>
<th>Box count</th>
<th>Grid size</th>
<th>Grid dimension</th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td>8</td>
<td>24 feet</td>
</tr>
<tr>
<td>50</td>
<td>16</td>
<td>12 feet</td>
</tr>
<tr>
<td>140</td>
<td>32</td>
<td>6 feet</td>
</tr>
<tr>
<td>380</td>
<td>64</td>
<td>3 feet</td>
</tr>
</tbody>
</table>

The degree of complexity found in the Mesoamerican buildings can be explained if we remember the basis of Mesoamerican Cosmo vision, and the idea how the Universe works, that have influenced this architecture [22].

5 Conclusions

The complexity paradigm has developed two different traditions: one in architecture, another in science. In this work we have described only an approach where the complexity has been analysed using three different points of view:
1. to search the “strange attractors” in a plan,
2. to find the complex components in the buildings,
3. to determine the degree of complexity in a building (using the box-counting dimension).

The examples described in the points 1 and 2 introduce the concept of non-linear architecture. Non-linear architecture has influenced Peter Eisenman’s Aroff Center in Cincinnati, Frank Gehry’s Guggenheim Museum in Bilbao and Daniel Libeskind’s Jewish Extension to the Berlin Museum. All three buildings were partly generated by non-linear methods.

The complexity in architecture is also connected to the concept of contradiction, as Robert Venturi affirms: “I like complexity and contradiction in architecture. I do not like the incoherence or arbitrariness of incompetent architecture not the precious intricacies of picturesqueness or impressionism. Instead, I speak of a complex and contradictory architecture based on the richness and ambiguity of modern experience, including that experience which is inherent in art. Everywhere, except in architecture, complexity and contradiction have been acknowledged, from Gödel’s proof of ultimate inconsistency in mathematics to T.S. Eliot’s analysis of “difficult” poetry and Joseph Albers’ definition of the paradoxical quality of painting” [9, p. 16].

The complexity paradigm in architecture, based on the science of complexity, has reached maturity, and it will influence the architecture of this new millennium.

References


