Sacred geometry in nature and Persian architecture

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Abstract

Geometry has a ritual origin and utilisation of Sacred Geometry by man goes back many centuries. Certain specific ratios can be found in the design of lifeforms in nature

Traditional civilisations regarded architecture as a sacred means by which the heavens were manifested. Persian architecture utilised proportions comprehensively and by means of Sacred Geometry measured the proportions of heaven and reflected them in the dimensions of buildings on Earth.

In this paper, the design of a number of Persian historical buildings by the use of the science of geometry will be presented. The geometric factors upon which the design of these buildings is made, from both architectural and structural viewpoints, will be discussed and common design laws between Persian monuments and creatures in nature will be explained.
Keywords: sacred, geometry, nature, Persian, architecture, structural, golden ratio, design, aesthetics.

1 Introduction

In Persian and Arabic, the term *hindisah* (the common word for geometry) has the meaning of measuring and it is used for both the sciences of geometry and architecture. The Greek $\gamma\varepsilon\omega\mu\varepsilon\tau\rho\iota'\alpha$ (geometry) in etymological sense means *the art of measuring ground*. The Greek $\hat{\alpha}\rho\chi\iota\tau\varepsilon'\kappa\tau\omega\nu$ (architecture) literally means a master-builder or a skilled scholar of the art of building, and it is close in meaning to the Greek $\kappa o'\sigma\mu o\varsigma$ (cosmos), which means at once the world,

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order and beauty, and to $\alpha\hat{\iota}\sigma\theta\eta\sigma\iota\varsigma$ (aesthetics). Persian architects always used geometry to measure the proportions of heaven and create beauty on the earth; to put beauty in order.

2 The universe as geometrisation of Divine Unity

Plato (*circa* 427-347 B.C.) in his book *Timaeus* [1] presents the idea that the Creator created the visible world similar to a *geometric progression*:

fire/air = air/water = water/earth

He associates four of the *Platonic Solids* with the four elements, the cube with earth, the icosahedron with water, the tetrahedron with fire, the octahedron with air, and the fifth Platonic Solid the dodecahedron with *ether* or heaven or the cosmos. The *dodecahedron* with *twelve* faces that are regular *pentagons*, was used by the Creator in creating the universe. The *Golden Ratio* governs the shape of a pentagon and for Pythagoreans it symbolises the generation of the cosmos, spirit or ether.

On geometry he writes in his *Republic* [2], *"[Geometry is] ... persuaded for the sake of the knowledge of what eternally exists, and not of what comes for a moment into existence, and then perishes, ... [it] must draw the soul towards truth and give the finishing touch to the philosophic spirit."*

3 Sacred geometry in nature

In nature, systems of patterns as geometric structures of form and proportion can be found from the minutest particles to the greater cosmos. Life is interwoven with geometric forms, such as the angles of atomic bonds in the molecules, the spherical shape of the cell that itself develops with a geometric progression from one to two to four to eight cells and beyond, the helical spirals of DNA, and the lattice patterns of crystals.

Reality, as Plato stated, consisted of *Archetypal Ideas,* or pure essences, of which the visible world is only a reflection. Sacred geometry makes use of the visible forms to describe these Ideas.

3.1 The Golden Ratio

The Golden Ratio is a supra rational or transcendent ratio found in fundamental forms: plants, flowers, viruses, DNA, shells, planets and galaxies. Although the Golden Ratio is first and foremost a proportion, not a number, as a numerical quantity it is $\phi = \frac{1+\sqrt{5}}{2}$ about 1.618. The Golden Ratio is the unique ratio of two terms when the ratio of the larger term to the smaller term is in the same way as the smaller plus larger to the larger (Figure 1). It symbolises the regeneration and progression and extension from the Unity as each generation is linked to its ancestors. The Golden Ratio ϕ is the most pleasing aesthetic proportion.

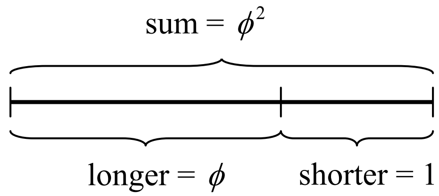

Figure 1: The Golden Ratio.

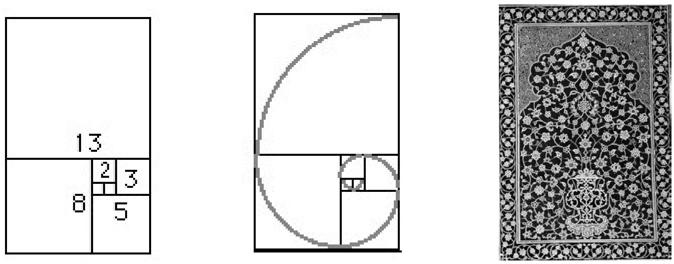

Figure 2: The Golden Mean spiral, and arabesque as the cosmic spiral, the journey towards the Unity through multiplicity, Chahar-Bagh madrassa, Isfahan, 1706-14 A.D.

The Golden ratio ϕ is the ratio of adjacent terms of the Fibonacci Series evaluated at infinity. The Fibonacci Series can be found in the ratio of the number of spiral arms in daisies, in the chronology of rabbit populations, in the sequence of leaf patterns twisted around a branch, and many places in nature.

3.2 Spirals and arabesque patterns

The spiral created by a recursive nest of Golden Rectangles (rectangles with relative side lengths of 1 and ϕ, or successive terms in the Fibonacci Series) can be found in a myriad of places in nature; in a snake coil, in an elephant trunk, and in the cochlea of the inner ear. In Persian architecture, arabesque patterns are based on ascending spirals with succession of form elements indicating the idea of infinity and multiplicity, as the creation of the universe (Figure 2).

4 Sacred geometry in Persian architecture

Geometry plays a fundamental role in design of Persian architectural monuments. From the viewpoint of *exterior* functioning, the use of geometry as *art* for creation of shapes, patterns and proportions reminds the *Great Architecture of the World* and recalls the Archetypes. From the viewpoint of *interior* functioning, geometry as *science* for selection of structural dimensions such as height, length and width of the building and its structural elements

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governs the *structural behaviour* of the building, the *behaviour* that follows the geometry. The right geometry makes the building behave correctly.

4.1 The mathematics of two-dimensional geometric patters

In Persian architecture geometric patterns are as aspects of the multiplicity of the Unity. Geometric patterns as spatial concepts are used to fill surfaces. It could be shown that as a mathematical fact solving the equation

$$\frac{360^\circ}{((n-2)/n)180^\circ} = 2 + \frac{4}{n-2} \qquad (1)$$

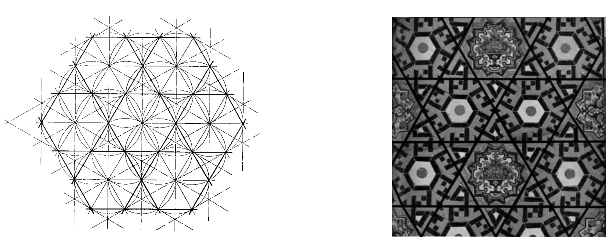
where n is the number of sides of each regular polygon, for a whole number for n greater than 2 there are only three regular polygons, known as the regular equipartitions, that may be used to fill a surface area exactly where the vertices sum up to 360 degrees: the triangle, the square, and the hexagon [3, 4].

There cannot be less than three polygons nor more than six around a vertex, thus the equation

$$(\sum_{i=1}^{m} (n_i - 2)/n_i)180^\circ = 360^\circ \qquad (2)$$

where m=3, 4, 5, or 6 and n_i is the order of regular polygon at face m, yields *seventeenth* possible solutions in whole numbers. The combinations of these three regular polygons form eight *semi-regular* equipartitions in which the vertices are similar on all occasions and fourteen *demi-regular* equipartitions in which the vertices vary.

Geometric patterns have been used widely in Persian architecture. Figure 3(a) shows a semi-regular pattern, a combination of triangles and hexagons. In Figure 3(b) the same pattern is used in a tile lattice pattern in the Jami mosque in Yazd (fourteenth century A.D.), Central Iran.



(a) (b)

Figure 3: Semi-regular tile lattice pattern, Jami mosque, Yazd, fourteenth century A.D.

4.2 Mechanical features of Persian architectural patterns

In Figure 4 tile, door and window decorations based on mathematical patterns are shown. In design of wooden doors and windows, geometrical patterns enable

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small pieces of wood to be used economically and allow the whole combination to conform to climatic changes of temperature and humidity. There is no need for string, glue or screws to hold the wooden pieces together.

(a) (b) (c)

Figure 4: Decoration based on mathematical patterns: (a) tile, Jami mosque, Isfahan, fifteenth century A.D.; (b) door, Imam-Zadeh Ismail Shrine, Isfahan, fifteenth and seventeenth centuries A.D.; (c) window, Chahar-Bagh madrassa, Isfahan, 1706-14 A.D.

4.3 Platonic Ideas in Persian architectural patterns

Recall that the pentagon, containing the Golden Ratio, is the shape for the faces of the Platonic Solid the dodecahedron, symbolising the cosmos or ether. The pentagon is in mutual relation with the pentagram and the spiral all representing the generation of the cosmos, universal love and *rebirth*. Such concepts can be explained through Persian Islamic architectural patterns. Figure 5 demonstrates a combination of geometric patterns and calligraphy using tile in a wall in the Jami mosque in Yazd. It shows the arrangement of ten peripheral pentagons, with a pentagram (five-pointed star) inside, arranged symmetrically around a star decagon, with arms related in a Golden Mean proportion to the side of the pentagon. The sacred name Muhammad, the Cosmic or Divine Man, is rotated around a five-pointed star. The number five, represented by the five-pointed star (pentagram) standing on two *legs*, symbolises the Perfect Man. The name Muhammad calls for the pentagram (man) to be *reborn* as a *Whole Person.* The number ten, represented by the decagon outside and the decagon (ten-pointed) star inside, symbolises the return to the Unity. The whole pattern indicates that creation is a continual inversing exchange between eternal Divine Man and humanity. The Divine Man incarnates continuously, and the Divinity reflects Itself in matter so as to become perceptible. Man is not a mere constituent part of the universe, but the original goal and the final stage of creation.

4.4 Geometrical analysis of historical buildings

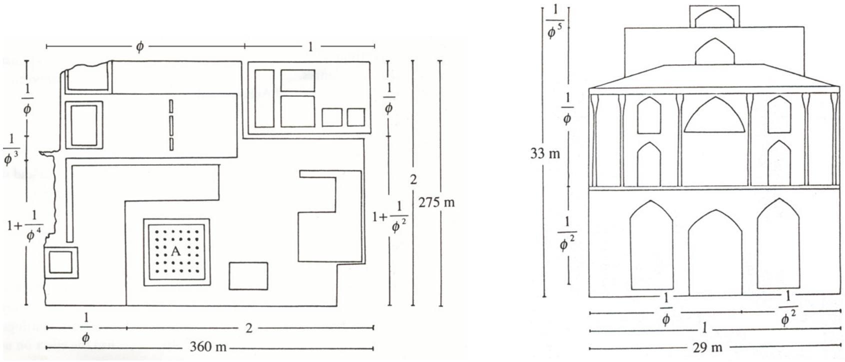
Geometrical analysis of many Persian historical buildings has proven that a complete knowledge of proportions, in particular the Golden Ratio, was widely used in Persian architecture and it was the basis of Persian aesthetics.

Figure 5: Combination of geometric patterns and calligraphy indicating the Cosmic Man, Jami mosque, Yazd, fourteenth century A.D.

In many Persian buildings the plan and elevation were set out in a framework of squares and equilateral triangles, whose intersections gave all the important fixed points, such as the width and height of doors, the width, length and height of galleries, the position of inscriptions, etc.

For example, geometrical analysis shows that a complete knowledge of the Golden Ratio is applied in the plan of Persepolis (518-330 B.C.) as shown in Figure 6(a).

Aesthetically, the Ali Qapu building (1597-1668 A.D.), in Isfahan, shows the application of the Golden Ratio in architecture. If the width of the building is considered as unity, important points such as the corners of the entrance to the building and the heights of different levels produce ratios of the Golden Ratio (Figure 6(b)).



(One) (b)

Figure 6: The use of the Golden Ratio: (a) plan of Persepolis, Shiraz, 518-330 B.C.; (b) elevation of Ali Qapu, Isfahan, 1597-1688 A.D.

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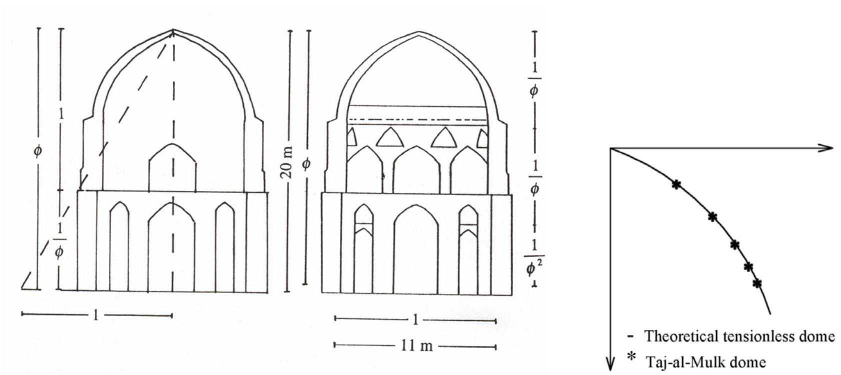

(a) (b) (c)

Figure 7: The Taj-al-Mulk dome and the Golden Ratio, Isfahan, 1088 A.D.: (a) Schroeder's [5] geometrical analysis; (b) Hejazi's [6] suggestion; (c) comparison of the meridional shape of theoretically perfect dome and the Taj-al-Mulk dome [7].

The Golden Ratio has been masterly used in the design of the Taj-al-Mulk dome dated 1088 A.D., in Jami mosque in Isfahan. Schroeder [5] thoroughly explains the aesthetic and geometrical features of the monument. He shows the sophisticated application of the Golden Ratio, in such a way that the lesser part is below, in the dimensions of the dome and the chamber below. His geometrical analysis proves that the architect of the building has taken a pentagon, which is generated between the sides of a grand equilateral triangle the apex of which is the peak of the dome, as a symbol for the ratio (Figure 7(a)). Hejazi [6] independently shows that the rule of the Golden Ratio, in such a way that the lesser part is above, can be applied to the structure. He suggests that the dimension of the Golden Rectangle in which the vertical cross-section of the dome is lying could have been alternatively used as a module by the architect (Figure 7(b)).

4.5 Relation between geometry and structural features

In the field of Persian historical buildings, it is meaningless to consider structural phenomena such as strength, stiffness and stability as the main and determinant design criteria. From the viewpoint of a traditional architect, although being fully aware of forces, resulting stresses and structural failures, the calculation of stress is of secondary importance. It is the function of structural elements that follows the overall form of building, as form itself has no meaning without the right function.

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As a fundamental principle in traditional art of building, *the functioning and stability of a building follow its geometry; a perfect geometry guarantees the stability*. This principle can be traced in many historical Persian buildings.

4.5.1 Optimum design of wooden structures

It has been shown that structural design of the wooden structure of the Ali Qapu building has been relatively optimum and structurally a masterpiece according to the modern codes [6]. The optimum design is very much related to the Golden Ratio used in the dimensions of the whole building (Figure 6(b)).

4.5.2 The shape of momentless tensionless masonry domes

Recalling the Taj-al-Mulk dome with its striking geometry, Farshad [7] shows that for weight loading the dimensions of the Taj-al-Mulk dome exactly match the mathematical formulae for the shape of the meridional curve and thickness variation of masonry domes without tensile stresses and bending forces. The equilibrium equations for symmetrically loaded shells of revolution are:

$$\frac{d}{d\varphi}(rN_\varphi) - rN_\theta \cos\varphi = -p_\varphi r r_1 \tag{3}$$

$$\frac{N_\varphi}{r_1} + \frac{N_\theta}{r_2} = p_r \tag{4}$$

where φ and θ are meridional and circumferential angles, N_φ and N_θ are meridional and hoop forces, r is the radius of the circle normal to the axis of revolution, r_1 and r_2 are meridional and circumferential radii, and p_φ and p_r are the components of external load per unit area in the φ and r directions, respectively (Figure 8(a)). For weight load, that is p (per unit area), N_φ is always compressive, then by eliminating N_θ, that is $N_\theta = 0$, in regions where it may become tensile, the formulae for the variation of meridional thickness h and radius r of the shell will respectively be:

$$h = h_0 r^{1\backslash\nu} \tag{5}$$

$$r^2 = -\frac{2A}{ph}\ln\frac{1+\sin\varphi}{\cos\varphi} + B \tag{6}$$

where h_0 and ν are reference thickness and Poisson's ratio, respectively, and A and B are constant values that can be obtained according to boundary conditions. The meridional shapes of theoretically perfect dome and the Taj-al-Mulk dome are plotted in Figure 7(c). The agreement between the two shapes is striking.

Hejazi's [6] structural analysis of the dome, using the finite element method, shows that the resultant stresses due to the system of bending forces are negligible compared with the system of membrane forces not only for weight load but also for wind and temperature and more significantly for the dynamic effects of earthquakes.

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Finite element analysis of the Taj-al-Mulk dome proves that if different shapes of cross-section, or different variation of meridional thickness, were used for the dome shell the magnitude of stresses and forces induced in the dome would increase and the design would not be perfect any more.

4.5.3 Relation between the Golden Ratio and crack patterns in circular shapes

Persian architects have always disliked the use of circular shape in constructing load bearing arches, vaults and domes, because hinging cracks occurred at a certain meridional angle that caused the failure mechanism and collapse of the structure. In cases that they used this shape, they changed the radius of the shape well before the meridional angle $51^\circ 50'$ in order to avoid tensile forces. Solving the equilibrium equation for a spherical dome under weight load shows that this is the angle where the sign of hoop force N_θ changes from negative (compressive) to positive (tensile), that is $N_\theta = 0$. Masonry materials have no tensile strength and therefore cracks occur at this angle (Figure 8(b)). For a hemispherical dome of radius *r* subjected to its self weight *p* (per unit area), it can be shown that:

$$N_\theta = pr\frac{1-\cos\varphi-\cos^2\varphi}{1+\cos\varphi} \tag{7}$$

The sign of the hoop force changes at a value of φ given by:

$$1-\cos\varphi-\cos^2\varphi = 0 \tag{8}$$

This yields:

$$\cos\varphi = \frac{-1+\sqrt{5}}{2} = \frac{1}{\phi} \tag{9}$$

or

$$\varphi = 51^\circ 50' \tag{10}$$

It is interesting to evaluate the meridional force N_φ at this angle to show that it is inversely proportional to the Golden Ratio:

$$N_\varphi = -pr\frac{1}{1+\cos\varphi} = -pr(\frac{1}{\phi}) \tag{11}$$

This angle and meridional force could be called the *Golden Angle* and the *Golden Force*, respectively. Therefore, the meridional angle dominated by the Golden Ratio is the critical location for the stability of a spherical dome, or a circular arch or vault.

Similar to the cases of the Ali Qapu building and the Taj-al-Mulk dome it can be concluded that structural stability has direct relation with geometrical characteristics of a building. Again the Golden Ratio (geometric proportions) rules the stability (mechanics).

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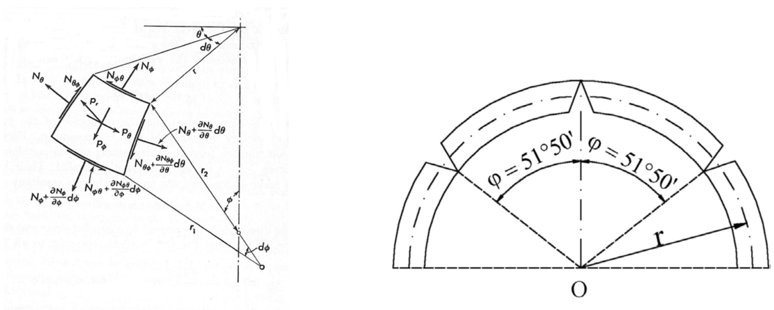

(a) (b)

Figure 8: (a) Shell element; (b) crack pattern dominated by the Golden Ratio in a circular arch under weight load.

5 Conclusions

An intrinsic character of Persian traditional architecture is the practice of Sacred Geometry, both in scientific and artistic dimensions. Sacred Geometry is the powerful tool by which Persian architecture has been able to create right proportions in order to reflect Divine Beauty, which could only be reflected through patterns that are exactly constructed upon *right* proportions. Many proportions used by Persian architecture to build up a traditional style of architecture that indicates the methods of right design and correct engineering can be found in many natural life-forms. In many traditional structures geometry rules the stability. Persian architecture is the *Sacred Geometrisation* of *Divine Beauty.*

References

[1] Plato, *Timaeus* (trs. Thomas Taylor), Wizard's Bookshelf: Minneapolis, 1975.
[2] Plato, *Republic* (trs. Robin Waterfield), Oxford University Press: Oxford, 1993.
[3] Kritchlow, K., *Order in Space*, Thames and Hudson: London, 1969 (Viking, New York, 1970).
[4] Kritchlow, K., *Islamic Patterns*, Thames and Hudson: London, 1976.
[5] Pope, A. U., and Ackerman, P. (ed.), *A Survey of Persian Art: from Prehistoric Times to the Present*, 6 Vols, Oxford University Press: London and New York, 1938, (and 3rd edn 1965).
[6] Hejazi, M., M., *Historical Buildings of Iran: their Architecture and Structure*, Computational Mechanics Publications: Southampton, 1997.
[7] Farshad, M., On the Shape of Momentless Tensionless Masonry Domes. *Building and Environment*, **12**, pp. 81-85, 1977.