Defining the residual stresses on the gear tooth

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Abstract

This paper deals with the influence of the residual stresses on the total value of loading and describes the method for measuring and calculating residual stresses and the total value of the loading on the example of a gear. In the article about the determination of the residual stresses the method of drilling a hole is introduced. On the bases of this and other experimental methods we have developed simple methods where with the help of hardness measurements the distribution of the residual stresses is calculated. The methods are suitable for the determination of the residual stresses on the tooth gears and the gear wheels. They can be also used with minimum modifications for other examples where the expected course of the residual stresses is known.

1 Introduction

The researches have shown that in the heat-treated materials the residual stresses occur and influence strongly the stress field, and this together with the normal load represents the total value of the loading. The residual stresses also influence profoundly the fracture mechanics parameters in the case of elements with the cracks.

For the accurate determination of the stress field at the gears it is important to know the amount, the direction and the application point of the force \( F_b \) [1,2]. The amount of the force \( F_b \) resulting from the operating conditions, the type of gearing arrangement, the precision of manufacture etc. can be determined fairly accurately on the basis of coefficients determined experimentally. Also the direction and the point of application represent no problems. The problems occur when the thermal treatment is taken into account in the calculations. Most gears are made of the thermally treated steels. The thermal treatment - as shown in the Figure 1 - has a significant effect particularly on the tooth root [3]. The residual
stresses from the thermal treatment are one of the important secondary influences.

![Diagram of stresses on differently heat-treated gears]

- a.) The normalized tooth of the gears
- b.) The tempered tooth through the cross-section
- c.) The carburized or quenched and tempered tooth
- d.) Plastically deformed and inductively tempered tooth

Figure 1: The distribution of stresses on the differently heat-treated gears

This is particularly important when the defects in the tooth root are expected. The Figure 2 shows that the residual stresses have a significant effect on the entire load value, which in the case of gears with the crack particularly influences the values of the parameters of the fracture mechanics and, consequently, the service life.

2 The residual stresses

The most interesting residual stresses on the gears are in the direction of the axis x and particularly in the tooth root in the section $S_F$ where the stresses are the highest due to the external loadings (Figure 2). As already stated [1, 4], for the precise analysis it is necessary to take into account the total value of loadings (the external loadings and the internal - residual stresses).

During the cooling of the steels, the surface transformation and later in the core, the transformation occurs due to the increased quantity of carbon on the surface, which lowers the temperature of starting the martensite formation. The residual stresses occur due to the lag of this transformation and the thermal contraction difference between the surface and the core. The residual stresses are normally considered to be of the secondary importance. They can be divided into two groups:

- The residual stresses with checked dimensions, which occur for various reasons, of which the main is the limited rigidity of the machine element and/or the structure with the surroundings preventing its propagation.
The residual stresses with the checked deformation that are caused by the stresses in the unlimited structure, and are the result of the temperature gradient, changes in the heat expansion, the increase in the internal stresses due to the metallurgic changes and the residual deformations, etc.

In general, the loading due to the residual stresses is a combination of both groups.

3 The experimental determinations of residual stresses

Any analytical and/or numerical determination of the residual stresses requires a comparison with the results of the experiment, if we want to estimate realistically the value of a certain analytical numerical or combined method. Each comparison of the results requires that the new knowledge and findings be taken into account in the models. In practice, several methods of measuring the residual stresses are known. One of the most popular method is the method of drilling a small hole, which is a semi-destructive method. The method of drilling a hole is also used in our case. For easier calculating we have developed a suitable algorithm complementing the program RESMET [4]. Thus we obtain the development of the stress field along the depth of drilling of the hole.

3.1 The models for determining residual stresses

If we don’t have the adequate measuring equipment we can also use the simple but enough accurate substitute the quasi-experimental models. Two models for
the computation of the residual stresses and/or their inclusion in the determining stress field by FEM [4,7] will be presented. They differ in complexity and thus also in accuracy [8].

THE MODEL 1 is based on the following assumptions:

From the known value of the surface hardening we can calculate the maximal residual stresses:

\[ \sigma_{z_{max}} = \alpha \gamma + \beta \left[ \text{N/mm}^2 \right] \]  \hspace{1cm} (1)

where

\[ \alpha = 0.105 \left[ 0.88 \exp(0.00892H_2) - 0.0406 \exp(0.0125H_2) \right] \]  \hspace{1cm} (2)

and

\[ \beta = -1.76 \exp(0.00892H_2) + 0.0406 \exp(0.0125H_2) \]  \hspace{1cm} (3)

\( H_2 \) – max. hardness according to the Figure 2 and/or 3 which approx. value we can calculate from the surface hardening:

\[ H_2 = 1.1 H_1 \]  \hspace{1cm} (4)

In our model we use the constant distribution of residual stresses for the complete area of the compressive residual stresses \( d_{eff} \):

\[ \sigma_z = 0.64 \sigma_{z_{max}} \]  \hspace{1cm} (5)

and our approximation for the depth of the compressive residual stresses:

\[ d_{eff} \approx 0.15 \text{ m} \]  \hspace{1cm} (6)

The results are shown in the Figure 11. The model loses its accuracy especially on the edges of this zone. The advantage of this model is its exceptional simplicity and easy applicability, but its results are intended only for the information. It is obvious that this model can be used only for the rough estimation of the conditions of the fracture mechanics concerning the gears.

THE MODEL 2 – the required input data are hardnesses \( H_0, H_2 \) and \( H_3 \), distances \( d_2, d_{eff} \) and the quantity of retained austenite \( \gamma_0 \) on the surface. The meaning of the individual symbols and the measurement results are shown in Figure 3 [3]. All these input data are measured values. The hardnesses were measured on a Zwick measuring instrument of Vickers microhardnesses under 10 N loading. The value of the residual austenite \( \gamma_0 \) was measured by the deltameter.
It can be assumed that the residual stresses of heat-treated gears depend on the difference of the volume expansion of structures in the core and on the surface after heat treatment. It can also be assumed that the structure of hardened layer consists of martensite and retained austenite. The specific volumes can be presented as the functions of carbon content in percent. The residual stresses of heat-treated gears are computed then from the volume expansion distribution as a problem of initial deformations. From the measured input data the hardness is computed with the following equation:

\[ H = H_3 + (H_1 - H_3) \exp \left[ Z \left( d - d_2 \right)^2 \right] \]  \hspace{1cm} (7)

where the coefficient is as follows

\[ Z = \begin{cases} \left( - \frac{1}{d_2^2} \right) \ln \left( \frac{H_1 - H_3}{H_2 - H_3} \right) & \text{if } d \leq d_2 \\ - \frac{1}{(d' - d_2)^2} \ln \left( \frac{550 - H_3}{H_2 - H_3} \right) & \text{if } d > d_2 \end{cases} \]  \hspace{1cm} (8a) and (8b)

The equations were verified by the measurement results of different authors [7,8]. By applying these equations, the carbon content can be calculated:

\[ C = 0.13078 \exp \left( 0.312 \cdot 10^3 \; H^{1.87} \right) \]  \hspace{1cm} (9)

The average specific volume of the martensite structure with % of retained austenite \( \gamma \) can be written as follows:
In the case, where the retained austenite content is calculated (see Figure 4) depending on its content on the surface of geometry, the formula is following:

\[ \gamma = \gamma_0 \exp\left(-2.37 \left( \frac{d}{1.5 d_{eff}} \right) \right) \]  

(10b)

The influence of the phase transformations of the residual stresses of the heat treated gears is calculated from the distribution of the volume expansion according to the equation 10 as a problem of initial deformations:

\[ \delta = \left[ \frac{\nu}{\nu_o} \right] - 1 \]  

(11)

Figure 4: The residual stresses as a function of the retained austenite

The expansion is transformed into a form of the thermal expansion and is used as the initial deformation for the usual calculations with the FEM. The input data are the temperatures in the nodes of the individual elements.

3.2 The algorithm for the determination of residual stresses

We translate the volume expansion into the form of the thermal expansion. For each time the interval \( dt \) is obtained, the distribution of temperature \( T \) as a
solution of the equation of the heat, complemented with the influence of the phase deformations is used.

\[ \frac{\partial T}{\partial t} = A_r \frac{\partial^2 T}{\partial x^2} \]  

(12)

In the above equation, \( A_r \) is the constant of the thermal diffusion calculated with the mean temperature in the gear depth at the beginning of the time interval. The boundary condition for the heat flow on the gear surface can be written in simplified from as follows:

\[ \pm \frac{\partial T}{\partial t} + h_r (T_s - T) = 0 \quad \text{by} \quad x = \pm \frac{1}{2} S_f \]  

(13)

In this equation, it is assumed that \( h_r \), i.e. the surface coefficient of the heat transmission is constant between the two infinitesimal time intervals \( dt \) for the constant surface temperature \( T_s \) and the constant temperature of the cooling medium \( T_c \) during the quenching and the tempering. For solving this problem we used hypothesis of generalized plane deformation in \( x \) and \( y \) direction:

\[ d\varepsilon_{tot} = d\varepsilon_{el} + d\varepsilon_{pl} + \lambda dT = \text{const.} \]  

(14)

That temperature distribution gives the data about the stress distribution on the critical gear cross section can calculated. Equilibrium equation:

\[ \int_{-s_F/2}^{s_F/2} \sigma_{tot} \, dx = 0 \]  

(15)

is obtained as a function of the total increase in the deformation \( d\varepsilon_{tot} \) for each point in the depth in case of each time interval and from increases of the total stresses according to:

\[ d\sigma_{tot} = d\sigma_{el} + d\sigma_{pl} + d\sigma_{gb} \]  

(16)

The residual stresses due to the heat treatment and/or stresses due to the total value of the loadings are calculated according to the procedure described above.

A special algorithm was developed for calculating the residual stresses. The program RESMET, based on the algorithm can serve as a pre-processor for different FE programs or it can be used in simple cases as an independent program [3,4].
4 The comparison of results

The results of the calculations and the experiment for the gear are given in the Figure 5.

The results show that in the accordance with the experimental results the calculated ones are very good. For the measured area 0,0 - 2,0 mm the deviation is within the scatter. The precision of the experimental method is approx 15 %. This precision was reached on the numerical model.

Figure 5 shows a comparison of distribution of residual stresses during the experimental, analytical/numerical and FEM method. It can be seen that by applying different methods, we have reached a small scattering of the results and this we have proven by the applied methods. The derived simplifications are good. The influence of the residual stresses on the enter loading of the gears in the tooth root is shown in the Figures 1 and 2. On the basis of the results we can calculate the stress intensity factors along the cracks, occurring in the tooth root, and the remain service life of the used gear.

5 Conclusion

The researches have shown that in addition to the primary loading it is necessary to take into account also the secondary loading for the heat-treated gears. The most important are the residual stresses.

The experimental methods of determining the residual stresses are usually expensive and time consuming and require special measuring equipment. Therefore, we propose to use semi-numerical method, wherever possible. In this paper we propose a combined model of measuring the hardness and on the basis of this calculating the residual stresses and the total stress distribution at the critical section of the gear tooth. On the basis of this model it is possible to
determine the service life and/or to optimize the components - gears and assemblies. Of course the model is not perfect but it assures good estimation of the problems for the plane deformation state of the narrow gears.

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