Assessment of fatigue and fracture strength of transportation energy storage flywheel using finite element simulation

L. Wang, X.J. Ren & G. Shatil

Structural Integrity Group, CEMS, University of the West of England, Bristol, U.K

Abstract

The safety of an energy storage flywheel for a light rail transportation system was assessed using structural and damage tolerance analyses. The flywheel’s strength was estimated using a detailed 3-D finite element (FE) simulation incorporating contact surfaces and sub-modelling techniques. Critical areas were identified near the bolt’s holes and, although the simulated strength was considered safe under normal operation (about 3000 RPM), it was conceived that failure due to fracture under extreme conditions such as over-speed may occur. Crack growth at these critical locations near the disk's bolt's holes was considered in an elastic fracture analysis. For three crack positions the maximum stress intensity factor ($K_{\text{max}}$) was calculated by conducting separate simulations for different crack lengths. The material fracture resistance was compared to the FE results using an R-curve procedure and the critical crack lengths to cause fracture were obtained. The critical cracks were then used to predict the life of the flywheel in a fatigue crack growth rate analysis. Following the investigation a structural integrity analysis was introduced into the design process for further development.

Introduction

Flywheels store kinetic energy mechanically by means of rotation of a mass and the use of the inertia. They are an alternative to chemical cell batteries [1, 2]. Several applications, including spacecraft and aircraft power systems, already use flywheels [3, 4]. However, a number of design challenges remain before this technology can reach operational status and those considered as safe for service.
A typical flywheel system consists of a mass for storing the energy; a supporting mechanism and the rotating assembly. Centrifugal forces and subsequent stresses are developed in the flywheel assembly due to the relatively high rotational speed. In the past, several simple analytical methods have been used to assess flywheel structural integrity. For example, Stodola [5] developed an empirical model to calculate the stress of a rotating body with either uniform or non-uniform thickness. Numerical methods were also developed to analyse axisymmetric flywheels with several layers [6] and for composite materials [7]. However, most of these methods were concerned with a single component and were used for a particular industrial application.

In this work initially the structural strength of the flywheel components was assessed using a solid FE assembly model where the interaction between the components during rotation was simulated by using contact elements procedure. The FE results were compared to simple analytical models. Subsequently, a local two-dimensional FE model was used with one of the flywheel discs to simulate fracture. Stress intensity factors were calculated for different crack lengths and directions, and compared with the material $R$-curves [8]. The critical crack length to cause fast fracture was obtained and was used to predict the fatigue life of the flywheel by applying the material fatigue crack growth relation.

**Structural analysis**

The flywheel assembly included a flange, three steel rotor disks and an end plate.
as shown in Fig. 1a. The rotor discs were made of the EN8 structural steel with yield stress of about 360 MPa and ultimate stress of about 650 MPa and were evenly clamped using standard M12 bolts located around the circumference at a radial distance of 230mm from the edge. The elastic modulus and Poisson's ratio for this material were approximately 200 GPa and 0.3 respectively. A solid model of the assembly and the disks are shown in Figs. 1a and 1b respectively. Near to the bolt/hole regions the mesh was partitioned into small volumes spaced symmetrically around the hole to allow local refinement. The model was centrally constrained to allow free rotation and standard bolt tightening pressure of 100 MPa was applied at each bolt location. The interfaces between the flange and the outer disks and between each of the rotor discs were simulated by using contact elements and assuming a friction coefficient of 0.3.

Initially, FE results (deformation and stress distributions) were obtained from applying the nominal operating rotational speed of 3000 RPM. Higher speeds of 4500 and 6000 RPM were used later to study the effect of increase in rotating speeds on the deformation and stresses of the flywheel structure.

**Nominal rotation speed results**

Simulation results using the nominal rotation speed are summarised in Table 1. During rotation, both the flange and the flywheel discs extend radially outward due to the centrifugal forces. A maximum radial displacement, of about 0.23mm, occurred at the rim of the flywheel rotors (Table 1). Due to the different size, and hence different stiffness of the flange and the rotor, they deformed differently during the rotation. For example, near the bolt's radial location the flange deforms 0.1mm radially and it results in relative displacement of about 0.12 mm. This radial deformation difference leads to shear load on the bolts in the interface between the parts and slight bolt deformation.

**Table 1: Stress levels and the deformation of the flywheel at a rotation speed of 3000 RPM.**

<table>
<thead>
<tr>
<th>flange</th>
<th>flywheel discs</th>
<th>bolt</th>
</tr>
</thead>
<tbody>
<tr>
<td>peak level</td>
<td>location</td>
<td>peak level</td>
</tr>
<tr>
<td>Max. principal stress (MPa)</td>
<td>217-246</td>
<td>bolt-hole</td>
</tr>
<tr>
<td>Max. shear stress (MPa)</td>
<td>50</td>
<td>bolt-hole</td>
</tr>
<tr>
<td>Max. extension (mm)</td>
<td>0.1</td>
<td>edge</td>
</tr>
</tbody>
</table>

Peak stress levels of the flange, the flywheel discs and the bolts are also listed in Table 1. Two critical regions with higher stresses have been identified. One region is where the maximum principal stress is near the bolt holes with maximum level of stress of 266 MPa, which suggested that the bolt hole region
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is the critical location for initiation of fatigue cracks. In addition, a second region
near the centre hole of the flange was found to be under relatively high stress of
about 200 MPa (not shown). As mentioned above, due to the relative
deformation of the flywheel discs in respect to the flange, the flange and the
flywheel discs were also subjected to an in-plane shear stress. However, the
maximum shear stress level of about 50 MPa, near the bolt’s hole, was low in
comparison to the direct stress. The shear stress developed in the bolts was about
40 MPa, considered to be safe for the bolt operation.

Simulated hoop stresses at the edge of the flange and the flywheel’s rotor
was compared to analytical calculations. According to Stodola [5], for a solid
disc with uniform thickness the hoop stress level could be estimated as follows:

\[ \sigma_t = \frac{(1-\nu)\sigma_u}{4} \]  \hspace{1cm} (1)

where \( \sigma_u \) denotes the tangential stress which occurs at the free rotating rim:

\[ \sigma_u = \rho \omega^2 R^2 \]  \hspace{1cm} (2)

\( \rho \) is the density of the wheel, \( \omega \) is the angular velocity of the rotating wheel and
\( R \) is the radius of the wheel. Table 2 lists the simulated and analytic hoop
stresses at the edge of the flange and the flywheel rotor. It is clearly shown that
the simulation results are in good agreement with Eqs. (1,2) results at about 3%
difference.

Table 2: Comparison of the simulated and the calculated hoop stress at the rim of
the flange and the flywheel rotor (\( \omega = 3000 \) RPM).

<table>
<thead>
<tr>
<th></th>
<th>Flange edge (( \sigma_t ))</th>
<th>flywheel rotor edge (( \sigma_t ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simulated result</td>
<td>34.9</td>
<td>67.3</td>
</tr>
<tr>
<td>Calculated result</td>
<td>34.1</td>
<td>65.5</td>
</tr>
<tr>
<td>Percentage difference</td>
<td>+2.3%</td>
<td>+2.7%</td>
</tr>
</tbody>
</table>

**Increased in rotational speed results**

Commonly, stress levels of rotating discs are related to the rotating speeds using
a square power low:

\[ \sigma_{\text{max}} \propto \omega^2 \]  \hspace{1cm} (3)

In Fig 2 the stress results from the FE simulations show a similar trend between
the local stress and the rotation \( \omega \). Fig 2 also shows that in general the local
stress distribution follows Eqn. 2. This analysis indicates that if, for some reason,
the nominal rotation increased by over 10-20% (over-speed) the material near the
hot spots may yield and may result in sudden fracture.
Figure 2: Effect of rotation speeds on the stress level of the flywheel.

Damage tolerance analysis

Figure 3: the FE fine mesh near the hole area, with three possible crack growth directions.
A separate, 2D plane-stress FE model of the flywheel disc with the same dimensions as the previous 3D FE model was used for the fracture analysis, applying the nominal rotation speed of 3000 RPM. Since the area near the bolt holes have shown the highest stresses, a refined mesh was used near one of the holes assuming that cracks initiate from that hole. Three possible different crack growth scenarios were considered as shown in Fig. 3.; a crack grows from the hole outwards along the radial direction; a crack propagates inwards along the radial direction, and a crack grows along the circumference direction. Multiple damage sites analysis was not considered.

A semi-numerical procedure was adopted for the fracture calculation. Different crack lengths were introduced in each of the crack orientation and a stress intensity factor was evaluated using the FE results from each crack length and for each orientation, as follows:

\[
K = \sigma_{y_{FEM}} \sqrt{\pi a_{FEM}} \quad (\theta = 0)
\]

(4)

Figure 4: Using resistance curves to determine the critical initial crack length.

However, due to stress singularity at the crack tip this approach contains a large error. Instead, an apparent intensity factor (Kapp) was calculated at several increments away from the tip and the stress intensity factor value at the tip was extrapolated. The resulting modified stress intensity factor for each crack orientation was used to obtain the local crack geometry factor (\(\beta\)) by using the local (normal to the crack position) stresses (local hoop or radial stresses) obtained from simulation without cracks as follow:
\[ \beta = \frac{K}{\sigma_{ref} \sqrt{\pi a}} \]  

where \( a \) is the crack length in the FE model, and \( \sigma_{ref} \) is the FE local stress.

To determine the length of the initial crack in the flywheel disks that will cause fast fracture (\( a_F \)) the material resistance curve obtained from a separate test programme [8] was used together with the simulated driving force curves by employing the tangency method [9]. This is shown in Fig. 4 for the three crack directions investigated where \( a_F \) was found to be 3.2mm long for the radial inward direction, 4.5mm for the radial outward direction and 6.5mm for the hoop direction.

Finally, fatigue life of the flywheel up to an unsustained cracking was estimated using the Paris-Erdogan relation:

\[ \frac{da}{dN} = C(\Delta K)^m \]  

where \( C \) and \( m \) were taken as \( 10^{-11} \text{ mm/cycle} \) and 3, respectively, and \( \Delta K \) is in \( \text{MPa m}^{1/2} \). The estimated life in terms of on/off rotation cycles Vs crack length is shown in Fig. 5 for the three crack directions.

Figure 5: Fatigue crack growth vs. cycles at different directions.
Figure 6: (a) Calculated $K_{\text{max}}$ using numerical simulation, (b) Calculated geometry factor, $\beta$, using numerical simulation.

Discussion

The maximum hoop and radial stress distributions obtained from the 2D plane-stress analysis for the flywheel without crack were similar to those obtained from the 3-D model. The stresses increased towards the flywheel centre and The 2D
analysis peak stress around the hole area was about 258 MPa (at the crack tip), which was similar to that obtained from the 3D model using the contact analysis.

The simulated $K_{max}$ for the flywheel with different crack lengths, at different locations is shown in Fig. 6a. The $K_{max}$ values for the crack growing radially inwards, outwards and in the hoop directions are denoted as $a_{rad}$, $-a_{rad}$ and $a_{hoop}$, respectively. The $K_{max}$ values for the crack growing inward were the highest and the $K_{max}$ values for the crack growing in the hoop direction were the lowest. This is because the bolt hole area is under higher hoop stress in comparison to radial stress and the hoop stress is acting perpendicularly to the radial crack propagation direction. The $K_{max}$ values for the outward direction are lower in comparison to the inward direction due to the increase in hoop stress towards the disk centre.

The calculated geometry factors, $\beta$, (using Eqn. 4) for the three different crack growth directions are shown in Fig. 6b. The applied reference hoop and radial stresses are taken as the remote hoop and radial stress for the bolt hole area, which were 114 MPa and 83 MPa, respectively. The $x/R$ ratio (Fig. 6b abscissa) is the crack length over the radius of the hole.

The simulated sets of data points in terms of crack length and stress intensity factors ($a$, $K_{max}$) were used to calculate the crack growth rate $dN/da$, by applying Eqn. 6, and integrating the number of cycles for a particular crack length. The crack length, $a$, versus cycles, $N$ shown in Fig. 5, indicate that cracks initiated radially would propagate at a similar rate independent of location inward or outward. Cracks initiate near the hoop direction would propagate at a much slower rate and may not be as critical.

Concluding comments

A procedure using commercial FE code is developed to assess the structural integrity of a flywheel assembly contain a bolted structure. High stresses were found near the bolt’s holes between the flange and the flywheel discs.

The stresses were within safe limit when the flywheel was running at or below the nominal speed of 3000 RPM. However, when the rotation speed increased by more than 15% the component was over stressed. The stress level increased with the rotating speed, approximately following a relation of $\sigma_{max} \propto \omega^2$.

The maximum allowed flaw size in the flywheel rotor was estimated to be about 3mm using the nominal rotation of 3000 RPM which, if allow to propagate will result in failure after about 60000 operating cycles.

References

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