Estimation of fatigue crack growth retardation due to crack branching

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Abstract

Quantitative analysis is provided to estimate the reduction of fatigue crack growth rate due to overload crack branching. A recent mixed-mode fatigue crack growth model based on the dilatational component of the accumulated strain energy density near the crack tip is modified to quantify the retardation factor of crack growth rate following an overload. It is found that crack branching due to an overload results in considerable reduction of fatigue crack growth rate. The retardation factor estimated by the proposed methodology is correlated with test results for the 2090-T8E41 aluminum-lithium alloy indicating encouraging agreement.

1 Introduction

Load excursions (e.g. single tensile overloads or variable amplitude loading histories) result in crack growth retardation or arrest [1]. However, the micromechanisms governing the retardation phenomena have not enough been understood. Up to now there are several interpretations, mainly based on: (a) crack closure due to residual plastic deformation in the vicinity of crack tip [2], (b) strain hardening of the material within the overload crack-tip plastic zone [3,4], (c) plastic blunting and subsequent resharpening of the crack-tip after the loading and the unloading part of the overload [5], (d) fracture surface roughness which results in contact between the crack faces at non-zero loads and therefore in a locally reduced effective stress-intensity range [6,8] and e) crack branching or deviation of crack path due to a load excursion which results in a reduction of the driving force governing the following crack growth [9].
During the last three decades the research activities have mainly been focused on the modeling of the crack closure mechanism (case (a)) and often on the cases (b) and (d). The case (c) is mostly treated using numerical methods while for the case (e) no enough quantitative models have been obtained. Subject of present work is to quantify the fatigue crack growth retardation following single tensile overloads, due to crack branching. To achieve this target, a recent mixed-mode fatigue crack growth model [10], based on the accumulated dilatational strain energy density near the crack tip, as well as other respective models will be modified.

2 Estimation of fatigue crack growth retardation due to crack branching

Experimental observations have shown that for some alloy systems, especially for aluminum or aluminum-lithium alloys, the overload severely blunts the crack tip and produces intense shear bands and subsequent growth along one of the shear bands [11]. The change in crack growth plane (branching) due to a single overload develop a mixed-mode state (fig. 1). From information available up to now (e.g. [9]), it appears that crack branching depends on the alloy’s crystallographic system as well as the values of the overload ratio \( r_{OL} = \frac{K_{OL}}{K_{max}} \) and the baseline stress intensity factor range \( \Delta K_B \). The crack branching is more significant at higher values of \( r_{OL} \) and lower \( \Delta K_B \) and occurs only above a certain threshold \( r_{OL} \).

Figure 1: Schematic interpretation of overload retardation effect due to crack branching.
The developed mixed-mode conditions due to an overload results to significant reduction of crack growth rate or crack arrest. To quantify the crack growth retardation factor $\lambda$

$$\lambda = \frac{\frac{d\alpha}{dN}_{\text{ret}}}{\frac{d\alpha}{dN}_{\text{nom}}}$$

(1)

immediately after an overload, a recent mixed-mode fatigue crack growth model developed by the first author [10] as well as other respective models [9, 12-14] will be modified. It is necessary to give a brief description of the above model [10] in order to formulate the retardation factor $\lambda$ due to overload crack branching. The mixed-mode fatigue crack growth rate can be approximated by the equation:

$$\frac{d\alpha}{dN} = C_T \Delta T_{\text{max}}^n$$

(2)

where $C_T$ and $n_T$ are parameters depending on material and loading ratio $R = \frac{\sigma_{\text{min}}}{\sigma_{\text{max}}}$, while $\Delta T_{\text{max}}$ is the dilatational strain energy density factor range given by the equation:

$$\Delta T_{\text{max}} = \frac{1 - 2\nu}{6E} \left\{ F_1(\theta_{\text{max}}) \left[ K_{1\text{max}}^2 - K_{1\text{min}}^2 \right] + F_2(\theta_{\text{max}}) \left[ K_{2\text{max}}^2 - K_{2\text{min}}^2 \right] + F_3(\theta_{\text{max}}) \left[ K_{1\text{max}} - K_{1\text{min}} \right] \right\}$$

(3)

In this equation $\theta_{\text{max}}$ is the angle of mixed-mode crack propagation where the dilatational component of strain energy density presents a maximum (fig. 2).

![Figure 2: Geometrical parameters of mixed-mode crack propagation.](image)

The functions $F_1, F_2, F_3$ can be obtained by the equations:

$$F_1 = f_1^2 + f_3^2 + 2f_1f_3$$

(4)

$$F_2 = f_2^2 + f_4^2 + 2f_2f_4$$

(5)

$$F_3 = f_1f_2 + f_2f_3 + f_3f_4 + f_1f_4$$

(6)

where $f_1, f_2, f_3, f_4$ are the following dimensionless functions:
\[ f_1 = \cos \frac{\theta}{2} \left( 1 - \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right) \]  

\[ f_2 = \cos^2 \frac{\theta}{2} \left( 2 + \cos \frac{\theta}{2} \cos \frac{3\theta}{2} \right) \]  

\[ f_3 = \cos \frac{\theta}{2} \left( 1 + \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right) \]  

\[ f_4 = \cos \frac{\theta}{2} \sin \frac{\theta}{2} \cos \frac{3\theta}{2} \]  

The parameters \( C_T \) and \( n_T \) can be obtained by the equations

\[ C_T = C \left[ \frac{6E}{1 - 2\nu} \frac{(1 - R)}{16(1 + R)} \right]^{\frac{n}{2}} \]  

\[ n_T = \frac{n}{2} \]  

where \( E \) is the Young modulus, \( \nu \) is the Poisson ratio, \( R \) is the fatigue loading ratio \( R = \sigma_{\text{min}}/\sigma_{\text{max}} \) and \( C, n \) are the parameters of the Paris type equation.

\[ \frac{d\alpha}{dN} = C\Delta K^4 \]  

derived by mode-I test data. With the aid of equations (3), (11), (12) and taking into account that

\[ K_{i_{\text{min}}} = R K_{i_{\text{max}}} \]  

\( i = 1,2 \)  

equation (2) can be written:

\[ \frac{da}{dN} = C \left( \frac{1 - R}{4} \right)^n \left( F_1 K_{1_{\text{max}}}^2 + F_2 K_{2_{\text{max}}}^2 + F_3 K_{1_{\text{max}}} K_{2_{\text{max}}} \right)^{n/2} \]  

Experimental observations have shown that the geometric parameters of the crack branch (fig. 3) take the values:

\[ \beta = 45^\circ \]  

\[ \vartheta = 53^\circ \]  

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Figure 3: Geometry of the overload fatigue crack branching.
For these geometric values of the crack branch, Suresh [9] has suggested that the mode-I and mode-II stress intensity factors $K_{I_{\text{max}}}$, $K_{II_{\text{max}}}$ of the mixed-mode state can be expressed versus the mode-I stress intensity factor $K_{I_{\text{max}}}$ of the main crack:

$$K_{I_{\text{max}}} = 0.8K_{I_{\text{max}}},$$  \hspace{1cm} (18)

$$K_{II_{\text{max}}} = 0.3K_{I_{\text{max}}},$$  \hspace{1cm} (19)

Using these values, equation (15) can be written:

$$\frac{da}{dN} = C \left( \frac{1 - R}{4} \right) \left( 0.61F_1 + 0.09F_2 + 0.24F_3 \right)^{n/2} K_{I_{\text{max}}}^n $$  \hspace{1cm} (20)

Considering that:

$$\Delta K_I = K_{I_{\text{max}}} - K_{I_{\text{max}}}$$  \hspace{1cm} (21)

or

$$\Delta K_I = K_{I_{\text{max}}} - RK_{I_{\text{max}}}$$  \hspace{1cm} (22)

then:

$$K_{I_{\text{max}}} = \frac{\Delta K_I}{1 - R}$$  \hspace{1cm} (23)

With the aid of eqn (23), eqn (20) can be written:

$$\frac{da}{dN} = \lambda_P \left[ C\Delta K_I^n \right]$$  \hspace{1cm} (24)

where:

$$\lambda_P = \left( 0.038125F_1 + 0.005625F_2 + 0.015F_3 \right)^{n/2}$$  \hspace{1cm} (25)

The parameter $\lambda_P$ is named here retardation factor because it expresses the ratio of the retarded true crack growth rate over the nominal mode-I crack growth rate. Substituting to the eqns (7-10) the value $\theta=53^\circ$ and taking into account equations (4-6), the retardation factor can be obtained:

$$\lambda_P = (0.1921)^{n/2}$$  \hspace{1cm} (26)

To obtain comparative results concerning the order of crack growth retardation factor due to overload crack branching and to correlate the estimations with available test results, a number of existing mixed-mode models [9,12-14] will be modified in a similar way.

According to the ref. [12], Tanaka has suggested the following mixed-mode model:

$$\frac{d\alpha}{dN} = C\Delta K_{\text{eff}}^n$$  \hspace{1cm} (27)

where:

$$\Delta K_{\text{eff}} = \left[ (K_{I_{\text{max}}} - K_{I_{\text{max}}})^4 + 8(K_{II_{\text{max}}} - K_{II_{\text{max}}})^4 \right]^{1/25}$$  \hspace{1cm} (28)

According to the eqns (18), (19) and (28), the eqn (27) takes the form:

$$\frac{d\alpha}{dN} = \left( 0.4744 \right)^{0.25n} C\Delta K_I^n$$  \hspace{1cm} (29)
Then, the retardation factor $\lambda_r$ obtained by the modification of the Tanaka’s mixed-mode model [12] takes the value:

$$\lambda_r = (0.4744)^{1.25n}$$ \hspace{1cm} (30)

With respect to the ref. [13], Yan et al. have suggested that the driving force $\Delta K_{eff}$ of the eqn (27) can be written:

$$\Delta K_{eff} = \frac{1}{2} \cos \frac{\theta}{2} \left[ \Delta K_{1} (1 + \cos \theta) - 3 \Delta K_{2} \sin \theta \right]$$ \hspace{1cm} (31)

According to the eqns (18), (19), (27), (31) and taking into account the value $\theta = 53^\circ$, the resulted retardation factor can be obtained:

$$\lambda_r = (0.2516)^n$$ \hspace{1cm} (32)

In the ref. [9], Suresh has suggested that the $K_{eff}$ has the form:

$$K_{eff} = \sqrt{K_{1}^2 + K_{2}^2}$$ \hspace{1cm} (33)

Then

$$\Delta K_{eff} = K_{eff max} - K_{eff min}$$ \hspace{1cm} (34)

or

$$\Delta K_{eff} = \sqrt{K_{1 max}^2 + K_{2 max}^2} - \sqrt{K_{1 min}^2 + K_{2 min}^2}$$ \hspace{1cm} (35)

According to the eqns (18) and (19), the eqn (35) can be written:

$$\Delta K_{eff} = 0.854 \Delta K_{I}$$ \hspace{1cm} (36)

and then the retardation factor can be obtained:

$$\lambda_{SR} = (0.854)^n$$ \hspace{1cm} (37)

According to the mixed-mode fatigue crack growth model proposed by Sih [14], the respective mixed-mode fatigue crack growth rate equation can be written:

$$\frac{d\alpha}{dN} = C_s \Delta S_{min}$$ \hspace{1cm} (38)

In this model the S-factor range can be written:

$$\Delta S_{min} = \alpha_{11} \left(K_{1 max}^2 - K_{1 min}^2 \right) + 2 \alpha_{12} \left(K_{min max}^2 - K_{min min}^2 \right) + \alpha_{22} \left(K_{2 max}^2 - K_{2 min}^2 \right)$$ \hspace{1cm} (39)

where:

$$\alpha_{11} = \frac{1}{16G} \left(3 - 4\nu - \cos \theta\right) \left(1 + \cos \theta\right)$$ \hspace{1cm} (40)

$$\alpha_{12} = \frac{1}{16G} \left[2 \sin \theta \left(\cos \theta - 1 + 2\nu\right)\right]$$ \hspace{1cm} (41)

$$\alpha_{22} = \frac{1}{16G} \left[4\left(1 - \nu\right)\left(1 - \cos \theta\right) + (3 \cos \theta - 1) \left(1 + \cos \theta\right)\right]$$ \hspace{1cm} (42)

Applying the conditions of mode-I fatigue crack growth $K_2 = 0$, $\beta = 90^\circ$, $\theta = 0^\circ$ into eqn (38) it takes the form:
According to the eqns (13) and (43) the Paris type parameters \( C, n \) can be obtained:

\[
\frac{da}{dN} = C_s \left( \frac{1 - 2\nu}{4G} \right)^{n_s} \left( \frac{1 + R}{1 - R} \right)^{n_s} \Delta K_i^{2n_s} \tag{43}
\]

or

\[
C_s = \left( \frac{4G}{1 - 2\nu} \right)^{\frac{n}{2}} C \tag{46}
\]

\[
n_s = \frac{n}{2} \tag{47}
\]

With the aid of eqns (18-19), (40-42), (46-47) and using the values \( \beta=45^\circ \), \( \theta=53^\circ \), the eqn (38) can be written:

\[
\frac{da}{dN} = \left[ \frac{1.518}{4(1-2\nu)} \right]^{n/2} C \Delta K_i^n \tag{48}
\]

Then, the retardation factor derived by the Sih’s mixed-mode fatigue crack propagation model can be derived:

\[
\lambda_s = \left[ \frac{1.518}{4(1-2\nu)} \right]^{n/2} \tag{49}
\]

### 3 Comparison of theoretical estimations and correlations with test result

The retardation factor estimations derived by the modification of the models [9,10,12-14] will be correlated with test result obtained by the literature [11] for the aluminum-lithium alloy 2090-T8E41. For this alloy, experimental observations have indicated [11] that the application of an overload is concurrent with crack branching along shear bands at the tip and subsequent mixed-mode fatigue crack growth, which contributes to the magnitude of retardation. The material parameters, loading conditions and test result have been taken by the ref. [11] and are summarized in table 1.

<table>
<thead>
<tr>
<th>( G ) (GPa)</th>
<th>( \nu )</th>
<th>( C ) (m/cycle) ( \times 10^{-13} )</th>
<th>( n )</th>
<th>( R )</th>
<th>( R_{OL} ) (%)</th>
<th>( \Delta K_B ) (MPa( \sqrt{m} ))</th>
<th>( \lambda_{exp} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>26</td>
<td>0.33</td>
<td>9.18367</td>
<td>4.416</td>
<td>0.1</td>
<td>150</td>
<td>8</td>
<td>0.021</td>
</tr>
</tbody>
</table>

According to the values of table 1 and considering the eqns (26), (30), (32), (37) and (49), the estimations of the retardation factor \( \lambda \) derived by the modification of several existing models are summarized in table 2.
Table 2: Estimations of the retardation factor for the 2090-T8E41 alloy.

<table>
<thead>
<tr>
<th>Mixed mode model</th>
<th>(\lambda_{\text{theor}})</th>
<th>(\lambda_{\text{exper}})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pavlou [10]</td>
<td>0.026</td>
<td>0.021</td>
</tr>
<tr>
<td>Tanaka [12]</td>
<td>0.438</td>
<td>0.021</td>
</tr>
<tr>
<td>Yan et al. [13]</td>
<td>0.002</td>
<td>0.021</td>
</tr>
<tr>
<td>Suresh [9]</td>
<td>0.498</td>
<td>0.021</td>
</tr>
<tr>
<td>Sih [14]</td>
<td>0.285</td>
<td>0.021</td>
</tr>
</tbody>
</table>

These results indicate that despite the fact that crack branching is the main retardation mechanism for the 2090-T8E41 aluminum-lithium alloy [11], the estimations \(\lambda_{\text{theor}}/\lambda_{\text{exper}}\) vary between the values 1.23 and 23.71. The better estimation \(\lambda_{\text{theor}}/\lambda_{\text{exper}}=1.23\) has been achieved by the proposed model. The deviations between theoretical and experimental results could be interpreted by the following reasons:

a) The considered models take into account only one retardation mechanism (crack branching). However, the retardation is result of the contribution of several micromechanisms. The percentage of the contribution on retardation of each micromechanism can not be estimated up to now.

b) The used herein mixed-mode models take into account as driving force \(\Delta K_{\text{eff}}\) several formulations more or less suitable for the considered material. Moreover, the determined retardation factors depends on the crack branch angle \(\beta\), the subsequent mixed-mode fatigue crack growth direction \(\theta\) and the mode-I fatigue crack growth material parameter \(n\). However, the retardation factor due to the crack branching seems to be independent of the stage (early or late) of the fatigue life. This conclusion agrees with the phenomenological estimations of other researchers (e.g. [11]).

4 Conclusions

1. A methodology for the estimation of fatigue crack growth rate retardation following a single overload was developed. The proposed methodology was based on a recent mixed-mode fatigue crack growth model as well as on the relation of the mode I and II stress intensity factors of the branched crack with the mode I stress intensity factor of the main crack. Latter estimations were also extended using several existing mixed-mode models taken by the literature.

2. The estimations of the retardation factor were correlated with test results for the 2090-T8E41 aluminum-lithium alloy. It was found in the literature that the application of the overload on this alloy was concurrent with crack branching along shear bands at the tip and subsequent mixed-mode fatigue crack growth which results in crack growth rate retardation. Using several existing mixed-mode fatigue crack growth models the theoretical result \(\lambda_{\text{theor}}/\lambda_{\text{exper}}\) vary between the values 1.23 and 23.71.
3. The deviations between theoretical and experimental results could be interpreted by the following reasons: a) the retardation is influenced by several micromechanisms, while the proposed model takes into account only the crack branching, and b) the used mixed-mode models take into account formulations for $\Delta K_{eff}$ more or less suitable for the considered material.

4. The determined retardation factors are influenced by the angle of crack branch, the angle of the direction of crack growth following an overload and the Paris type material parameter $n$. However, it seems to be independent from the stage of fatigue life.

References

224 Damage and Fracture Mechanics VII

