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Development of constitutive equations for creep damage behaviour under multi-axial states of stress

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Abstract

A generic formulation for creep damage constitutive equations for multi-axial states of stress is proposed. First, the requirements of lifetime consistency and creep deformation consistency have been specified. Then two functions of states of stress were introduced to depict the influences of multi-axial states of stress on rupture lifetime and strain at failure, respectively; this is similar to that of separate yielding function and admissible function proposed and employed in the development of the generalised plasticity theory. Previous formulation and its applications are based only on lifetime consistency, and the fundamental requirement of creep deformation consistency has not been addressed, which results in incorrect prediction of creep strain and damage evolution. Furthermore, previous formulation is unable to consider the effect of mean stress on lifetime, which is controversy to general conclusion. This paper presents a theoretical development through new formulation. Its super effectiveness is demonstrated by and compared with previous formulation through predicted strain at failure and lifetime under bi-axial cases. It is further concluded that this new formulation and associated concept and methodology is generic and could be applied in the development of other damage constitutive equations.

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1 Introduction

Creep continuum damage mechanics, originally proposed by Kochavo in 1958, has been developed aiming at dealing with creep damage problem in engineering design [1-7, 13-14]. Creep continuum damage mechanics study is mainly of phenomenological approach, in which the observed phenomena are modelled by constitutive equations through state variables and internal variables. Normally, general constitutive equations for multi-axial states of stress are produced by extending or generalising the one for uniaxial conditions. But, how to conduct the extension or generalisation has not been properly studied, resulting flaws or deficiencies in the constitutive equations developed.

This paper presents a theoretical study of the development of creep constitutive equations for multi-axial states of stress by generalising the one for uni-axial conditions. It presents 1) a general description creep damage processes and uni-axial constitutive equations, 2) fundamental requirements, 3) new proposal and formulation, 4) demonstration and comparison, and 5) conclusions.

2 Creep damage process and uni-axial constitutive equations

The creep deformation is typically divided into primary, second, and tertiary creep; while the damage process is generally understood to be a process of nucleation, cavity growth and coalescence. For the fracture by classical grain boundary caviation process, Cane [9] relates the stress state dependence of this process to the von Mises equivalent stress, $\bar{\sigma}$, the hydrostatic stress, σ_m, the maximum principal stress, σ_1, and the maximum deviatoric stress, S_1. Thus, it is anticipated that lifetime and creep deformation are functions of stress states.

Constitutive equations capable of describing primary, second and tertiary creep have been developed. The second and tertiary creep was dealt first. Different constitutive equations have been proposed [4,6]. But, essentially, they are the same and use one damage variable ω. The creep strain rate $\dot{\varepsilon}$ and damage rate $\dot{\omega}$ are given by

$$\dot{\varepsilon} = f(\sigma, \omega) \qquad \text{and} \qquad \dot{\omega} = g(\sigma, \omega) \tag{1}$$

By appropriate selection of the function f and g, as well as the critical value of damage, it is possible to represent the tertiary creep and to produce stress-lifetime relationship consistent with experimental observations. One of the examples is

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$$\dot{\varepsilon} = G\left[\frac{\sigma}{(1-\omega)}\right]^n \quad \text{and} \quad \dot{\omega} = C\frac{\sigma^\chi}{(1-\omega)^\phi} \tag{2}$$

where G, C, n, χ and ϕ are material constants. The material is deemed to fail when the value of damage variable reaches its critical value equals 1.

Then primary creep could be dealt with by introducing another hardening function H. This has been proposed and used in [6]. For example, the constitutive equations for 316 stainless steel have adopted this approach and are given as [10]:

$$\dot{\varepsilon} = \frac{A}{(1-\omega)^n}\sinh(B\sigma(1-\mathrm{H})),\ \dot{\mathrm{H}} = \frac{\mathrm{h}}{\sigma}\left(1 - \frac{\mathrm{H}}{\mathrm{H}^*}\right)\dot{\varepsilon},\ \text{and}\ \dot{\omega} = \mathrm{D}\cdot\dot{\varepsilon} \tag{3}$$

where A, B, h, H*, and D are material constants. The critical value of damage ω_f is 0.33.

3 Phenomenological requirements

The two phenomenological requirements for the creep constitutive equations to be developed for multi-axial states of stress are lifetime consistency and creep deformation consistency. These consistency requirements mean that the lifetime and the creep deformation predicted by constitutive equations under various states of stress should be consistent with the experimental observations.

These consistency requirements can be expressed in differential form such as damage rate and creep strain rate. But the integration form is of particular use in the development of constitutive equations, as the information of lifetime and strain at failure is more easily ready to obtain.

When damage variable reaches its critical value, integration of constitutive equations will produce:

$$t = t_f^{multi} \quad \text{and} \quad \bar{\varepsilon} = \bar{\varepsilon}_f^{multi} \tag{4}$$

where $\bar{\varepsilon}$ is effective creep strain. Information about lifetime and strain at failure, either theoretical or experimental study, could be used in formulating and calibrating constitutive equations.

4 New proposal and formulation

4.1 New proposal

Currently there are mainly two types of multi-axial damage criterion relevant to creep damage problem, namely: the elastic energy density release rate criterion and stress invariant criterion [2]. The elastic energy density release

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criterion is based on instability of a damaged material. It is formally justified by thermodynamics [2, 13,14]. However, in certain cases, it can be at fault, particularly, for creep damage where it gives the same answer in tension and in compression [2] and possible improvement can be achieved by damage potential concept [14]. The stress invariant, within the framework of isotropic, is a more general, but more complex form consist in expressing the damage criterion in terms of the stress invariant of the stress tensor [2].

In the second method the creep strain rate is controlled by Von Mises stress and coupled with damage. The damage evolution is expressed as a function of stress invariant and the critical damage value at failure is assumed to be a constant. Several versions of this type constitutive equation has been proposed and calibrated against experimental observations of lifetime [5-6]. However, this methodology has not been formally justified by thermodynamic potentials. The theory and methodology of stress invariant damage criterion has not been formally justified by thermodynamic potentials and this is necessary in order to ensure its overall correctness and general applicability.

Within the stress invariant damage criterion, it is proposed to introduce another function of states of stress to provide the space to depict the experimentally observed phenomena of strain at failure:

$$\dot{\varepsilon}_{ij} = \frac{3\sigma'_{ij}}{2\bar{\sigma}} \frac{A}{(1-\omega)^n} \sinh(B\bar{\sigma}(1-\mathrm{H})) \cdot f_1 \tag{5}$$

$$\dot{\mathrm{H}} = \frac{\mathrm{h}}{\bar{\sigma}}\left(1 - \frac{\mathrm{H}}{\mathrm{H}^*}\right)\dot{\bar{\varepsilon}} \tag{6}$$

$$\dot{\omega} = \mathrm{D}\dot{\bar{\varepsilon}} \cdot f_2 \tag{7}$$

where f_1 is the newly introduced function of states of stress and $\dot{\bar{\varepsilon}}$ is effective creep strain rate. The concept of separate functions for damage evolution and creep strain rate is similar to that of separate yielding function and admissible function proposed by Lubliner in the development of generalised plasticity [1]. The introduction of a specific f_1 into equation (5) could be justified by the existence of a particular plastic potential as a function of effective stress and states of stress. One of the well-known example is Drucker-Prager type yielding function.

4.2 Specific form of f_1 and f_2

Firstly, dividing the integrated damage-strain equations for uniaxial and multi-axial states of stress gives the relation of strain at failure as

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$$\bar{\varepsilon}_f^{multi} = \frac{\bar{\varepsilon}_f^{uni}}{f_2} \tag{8}$$

Thus, information about this relationship could be used directly to formulate the function f_2. For example, the theoretical relationships developed by Rice & Tracey [11] can be used. Thus,

$$f_2 = \frac{\bar{\varepsilon}_f^{uni}}{\bar{\varepsilon}_f^{multi}} = \exp\left(\frac{1}{2} - \frac{3\sigma_m}{2\bar{\sigma}}\right)^{-1} \tag{9}$$

Its value is 1 for uni-axial conditions and it is a function of the ratio of mean stress over effective stress.

On the other hand, experimental work has indicated the influence of first principal stress on the creep deformation, and it is possible to propose a different function to include that. One of the possible forms could be

$$f_2 = \left(\exp\left\{p\left[1 - \frac{\sigma_I}{\bar{\sigma}}\right]\right\}\exp\left\{q\left[\frac{1}{2} - \frac{3\sigma_m}{2\bar{\sigma}}\right]\right\}\right)^{-1} \tag{10}$$

Its value is 1 for uni-axial conditions and coincides with equation (10) if $p = 0$ and $q = 1$. Mathematically, the constant p and constant q offer a possibility of better fitting with experimental observations.

Secondly, it is proposed to adopt Hudderson's formulation for strength theory [12] for f_I, which is given as:

$$f_I = \frac{3}{2}\left(\frac{2\bar{\sigma}}{3S_I}\right)^{a-1} \exp\left\{b\left[\frac{3\sigma_m}{S_s} - 1\right]\right\} \tag{11}$$

where $S_s = \sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2}$ and $S_I = \sigma_1 - \sigma_m$.

Substituting equations (10-11) into equations (5-7), gives

$$\dot{\bar{\varepsilon}} = \frac{A}{(1-\omega)^n}\sinh(B\bar{\sigma}(1-\mathrm{H}))\cdot\frac{3}{2}\left(\frac{2\bar{\sigma}}{3S_I}\right)^{a-1} \exp\left\{b\left[\frac{3\sigma_m}{S_s} - 1\right]\right\} \tag{12}$$

$$\dot{\mathrm{H}} = \frac{\mathrm{h}}{\bar{\sigma}}\left(1 - \frac{\mathrm{H}}{\mathrm{H}^*}\right)\cdot\dot{\bar{\varepsilon}} \tag{13}$$

$$\dot{\omega} = \mathrm{D}\dot{\bar{\varepsilon}}\left(\exp\left\{p\left[1 - \frac{\sigma_I}{\bar{\sigma}}\right]\right\}\exp\left\{q\left[\frac{1}{2} - \frac{3\sigma_m}{2\bar{\sigma}}\right]\right\}\right)^{-1} \tag{14}$$

The constant a & constant b could be determined by the experimentally determined isochronous surface.

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5 Examples and comparison

The characteristics of the newly developed constitutive equations will be demonstrated via lifetime and strain at failure for bi-axial cases. To simplify the numerical calculation the primary creep has been ignored by assuming H=H*. Thus, the newly developed constitutive equations are given as

$$\dot{\bar{\varepsilon}} = \frac{A}{(1-\omega)^{\mathrm{n}}} \sinh(B\bar{\sigma}(1-\mathrm{H}^*)) \cdot \frac{3}{2}\left(\frac{2\bar{\sigma}}{3S_I}\right)^{a-1} \exp\left\{b\left[\frac{3\sigma_m}{S_s}-1\right]\right\} \tag{15}$$

$$\dot{\omega} = \mathrm{D}\dot{\bar{\varepsilon}}\left(\exp\left\{p\left[1-\frac{\sigma_I}{\bar{\sigma}}\right]\right\}\exp\left\{q\left[\frac{1}{2}-\frac{3\sigma_m}{2\bar{\sigma}}\right]\right\}\right)^{-1} \tag{16}$$

The following constitutive equations will be used for comparison, which is produced by previous formulation (see references 4 & 7),

$$\dot{\bar{\varepsilon}} = \frac{A}{(1-\omega)^{\mathrm{n}}} \sinh(B\bar{\sigma}(1-\mathrm{H}^*)) \tag{17}$$

$$\dot{\omega} = \mathrm{D}\dot{\bar{\varepsilon}}\left(\frac{\sigma_I}{\bar{\sigma}}\right)^{\mathrm{k}} \tag{18}$$

where the k is stress sensitivity.

For bi-axial condition, with notation of $\xi = \sigma_2/\sigma_1$, the function f_2 for new formulation becomes

$$f_2 = \left(\exp\left\{p\left[1-\frac{1}{\sqrt{1+\xi^2-\xi}}\right]\right\}\exp\left\{q\left[\frac{1}{2}-\frac{3[1+\xi]}{2\sqrt{1+\xi^2-\xi}}\right]\right\}\right)^{-1} \tag{19}$$

while for the previous formulation it is

$$f_2 = \left(\frac{\sigma_I}{\bar{\sigma}}\right)^{-\nu} = \left(\frac{1}{\sqrt{1+\xi^2-\xi}}\right)^{-\nu} \tag{20}$$

The results from equation (19) are shown in Figure 1 to Figure 3 with various combination of p and q, while the results from equation (20) are shown in Figure 4.

The isochronous surface for new formulation with two set of material constants a, b and n are shown in Figure 5, while the isochronous surfaces for previous formulation are shown in Figure 6. Further demonstration of the wider range of isochronous surfaces predicted by new formulation is shown in Figure 7.

The following conclusions can be drawn:

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1. The new formulation for constitutive equations for multi-axial states of stress is capable to depict the ratio of strain at failure consistent with experimental observation; as shown in Figure1 to 3;
2. The previous formulation for constitutive equations for multi-axial states of stress is unable to do so, in fact the trend of its predictions is contrary to experimental observations;
3. The new formulation is able to predict a wider range of material behaviour. Particularly it is able to correctly predict material behaviour in tension-tension region, as shown by Figure 5 and further supported by Figure 7;
4. The limitation or deficiency of previous formulation are clearly shown by comparing Figure 5 to Figure 6;
5. The deficiency of previous formulation in lifetime consistency and deformation consistency is resulted from poor formulation and it should not be used any longer.

6 Conclusions

A theoretical study on the development of constitutive equations for multi-states of stress has been presented. In this study, the requirements of lifetime consistency and deformation consistency have been addressed, and a generic formulation for multi-axial states of stress is proposed. This new formulation employs two functions to depict the influences of multi-axial states of stress on rupture lifetime and strain at failure, respectively; and specific forms of two states of stress function have been introduced based on the previous knowledge of strength theory and strain at failure relationship. It has been demonstrated that this new formulation can predict a wider range of material behaviour, particular correctly predicts the behaviour in tension-tension region and the predicted strain at failure automatically satisfies a prescribed relation. This new model is far better than the previous one.

It is expected that this new formulation will be assessed with a wider range of states of stress (for example, including plane strain cases) and then be implemented in finite element package for engineering design. Research progress will be reported in due course.

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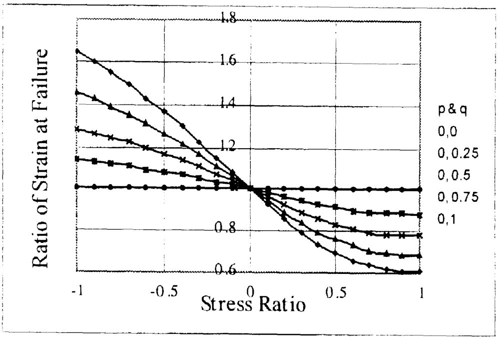

Figure 1 Ratio of strain at failure for new formulation

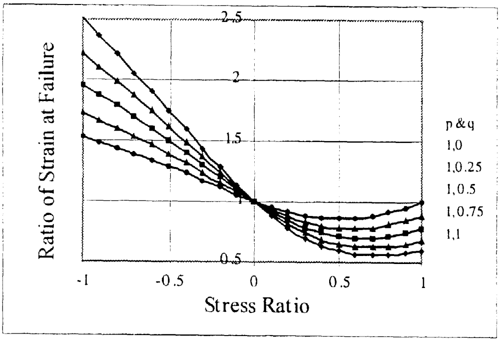

Figure 2 Ratio of strain at failure for new formulation

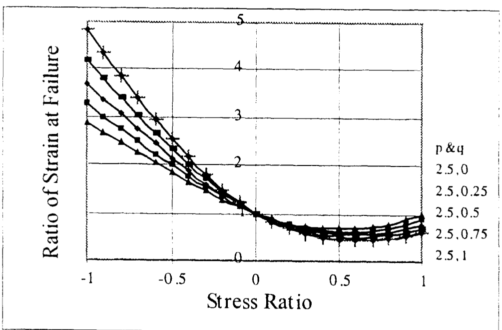

Figure 3 Ratio of strain at failure for new formulation

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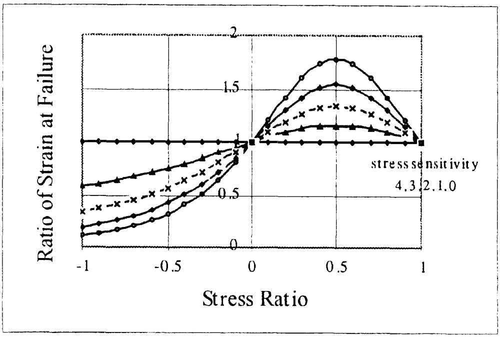

Figure 4 Ratio of strain at failure for previous formulation

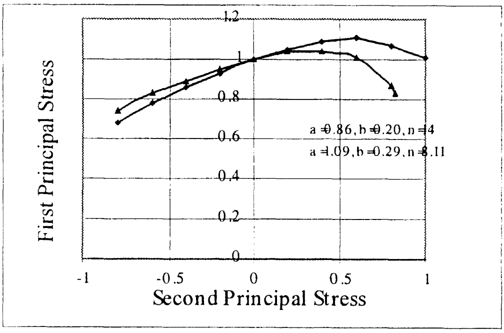

Figure 5 Isochronous surface for new formulation

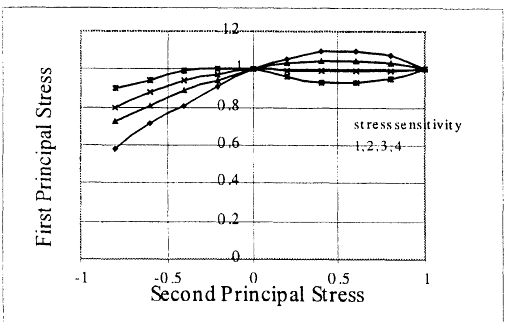

Figure 6 Isochronous surface for previous Formulation

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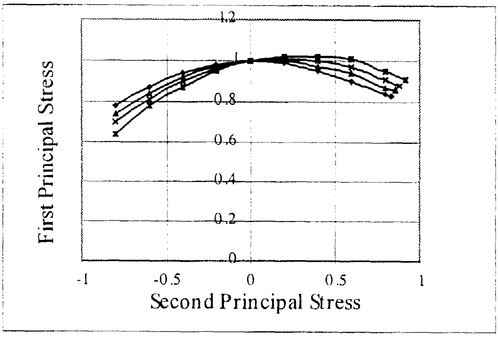

Figure 7 Isochronous surface for new formulation