Modeling dynamic fracture following high shock compression

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Abstract

Constitutive-microdamage equations are developed that are capable of simulating thermo-mechanical material behavior following high shock compression, dilatation, microdamage evolution and fracture, caused by projectile-target impact at hypervelocity.

1 Introduction

Impact at hypervelocity of a target plate by a projectile can cause various types of damage, e.g. cratering and spall fractures, fragmentation of projectile and the impact area of the target, and formation of high velocity debris clouds that may include melting and vaporization of projectile and target materials [1-5]. The most dominant of the factors that determine the damage outcome are the impact velocity, the ratio of the projectile diameter to thickness of the target, and the thermomechanical properties of the target and projectile materials.

The authors have developed a constitutive-microdamage model that currently is appropriate for the lower range of hypervelocity impact, i.e., approximately 2-6 Km/s, over which fracture and fragmentation of projectile and target materials will most likely remain in the solid state. The indicated impact velocity range is not meant to be precise, as the specific impact velocity at which a highly compressed material releases into a molten liquid, or vapor, will depend upon the particular thermal properties of the material. The target and projectile materials under consideration are assumed to be polycrystalline metals.
2 Elastic rate of deformation

The rate of deformation $D$ has the elastic-viscoplastic decomposition of rates, $D = D^e + D^p$, where $2D = \text{grad} \ \mathbf{v} + \left(\text{grad} \ \mathbf{v}\right)^T$, $\mathbf{v}$ is the velocity and the superscript signifies the transpose of the matrix of the tensor grad $\mathbf{v}$. The high compression caused by hypervelocity impact produces large volume strains. The elastic shear deformations however are of small order because of limitations imposed by onset of plastic yield in polycrystalline materials. Expressing $D^e$ in terms of volumetric and deviatoric components,

$$D^e = \frac{1}{3} \left(\text{tr}D^e\right) \mathbf{1} + D'^e$$

where $\mathbf{1}$ is the identity tensor, $D^e$ is the deviatoric rate of elastic deformation and $\text{tr}D^e = D_{kk}^e = D_{11}^e + D_{22}^e + D_{33}^e$ when referring to a rectangular coordinate system. The elastic rate of volume strain is related to the mean stress and the elastic bulk modulus, $K$, while the elastic rate of shear is determined by the stress deviator and the shear modulus, $\mu$, so that the sum specifying $D^e$ is given by,

$$D^e = \frac{1}{3K} \sigma_{\text{m}} \mathbf{1} + \frac{1}{2\mu} \mathbf{T}'$$

$\mathbf{T}$ is the Cauchy stress tensor, $\sigma_{\text{m}} = (1/3) \text{tr} \mathbf{T}$ is the mean stress (pressure), $\mathbf{T}'$ is the stress deviator and $(\ldots)$ represents the Jaumann time rate applied to $\sigma_{\text{m}}$ and $\mathbf{T}'$.

All polycrystalline materials contain microvoids and microcracks. Since impact at hypervelocity can produce large adiabatic temperature increases, the corresponding dynamic fracture will be ductile, caused essentially by nucleation, growth and joining together of microvoids. The measure of ductile material microdamage can therefore be described by the microvoid volume fraction $\xi = V_v / V$, where $V_v$ is the void volume for a material element having volume $V$. The microdamage is viewed as a scalar valued continuous point function of position and time, i.e., $\xi(x,t)$ similar to the other field variables. The volume strain $\psi = (1 - V / V_0) = (1 - \rho_0 / \rho)$, where $V_0$, $\rho_0$ and $\psi$, $\rho$ are the reference and current values of the volume and density. The equation relating the mean stress to the elastic volume strain and temperature (equation of state) may be shown to have the Gruniesen form [6], modified to include nonlinear volume strain dependence of the Gruniesen coefficient. Thus during high shock compression and release

$$\sigma_{\text{m}} = \rho_0 c_0^2 \left[ 1 - \frac{1}{2} \left\{ \gamma_0 \left( \frac{\psi}{1+\psi} \right) + a_0 \left( \frac{\psi}{1+\psi} \right)^2 \right\} \right] + \left[ \gamma_0 + a_0 \left( \frac{\psi}{1+\psi} \right) \right] E(\psi, T)$$
for \( \psi > 0 \), where \( E(\psi, T) \) is the internal energy per unit volume, \( T \) is the absolute temperature, \( c_0 \) and \( \gamma_0 \) are the bulk sound speed and Gruniesien constant, respectively, \( a_0 \) and \( s \) are material constants the latter of which is obtained from shock compression test data. It is noted that the initial void volume fraction for polycrystalline metals range from \( \varepsilon_0 \approx 10^{-7} \) to \( 10^{-5} \). Because of the high compression following impact, the microvoid volume fraction will be reduced to insignificantly small values, and need not therefore be included in the equation of state (3). Following Steinberg et al. [7], the increase of the melt temperature with large volume reduction can be expressed by

\[
T_m = T_{m0} \exp \left( 2a_1 \psi \right) \left( \frac{1}{1 - \psi} \right)^2 \left( \gamma_0 - a_1 - \xi \right) \quad \text{for } \psi > 0
\]

where \( T_{m0} \) is the melt temperature at zero volume strain and \( a_1 \) is a material parameter. Following high compression and release (decompression), the elastic dilatation is limited to small volume expansion because of impeding fracture and, therefore, is given the linear form

\[
\sigma_m = K \psi, \quad \psi < 0.
\]

The volume reductions at high compression can be as high as thirty to forty percent, with corresponding temperatures of the order of \( 10^3 \) to \( 10^4 \) °K. Conversely during volume expansion under high tensile mean stress, material degradation due to increase of the microvoid population and the microvoid sizes leads ultimately to ductile fracture at sites of high stress [8]. Constitutive relations for the elastic moduli that reflect the effects of high compression hardening, temperature change and microdamage softening can be described by

\[
K = K_0 \left[ \frac{(1 - \psi)(1 + (s - \gamma_0) \psi)}{(1 - s \psi)^3} \right] \left[ 1 - a_2 \left( \frac{T}{T_0} - 1 \right) \right] \left[ \frac{4\mu_0 (1 - \xi)}{4\mu_0 + 3K_0 \xi} \right]
\]

\[
\mu = \mu_0 \left[ \frac{(1 - \psi)(1 + (s - \gamma_0) \psi)}{(1 - s \psi)^3} \right] \left[ 1 - \left( \frac{T - T_0}{T_m - T_0} \right)^\eta \right] \left[ (1 - \xi) \left( 1 - \frac{6K_0 + 12\mu_0 \xi}{9K_0 + 8\mu_0} \right) \right]
\]

\( K_0 \) and \( \mu_0 \) are the elastic moduli values at the reference state \( \psi = 0, T = T_0 \) and \( \xi = \xi_0 \) (i.e., the values that are determined by routine laboratory tests at room temperature). The temperature variations assume linear and near linear softening at elevated temperatures [9-11], with the shear modulus reducing to zero at melt. For shock compression the microdamage term, which is based on the hollow sphere micromodel [12,13], reduces essentially to unity, while for small dilatation the volume strain term makes no significant contribution.
3 Rate of viscoplastic deformation

An increment of dilatational microvoid volume strain defined by $-dV_v = (1/V_v)dv$, leads to the logarithmic expression

$$\psi_v = -\ln \left( \frac{1 - \xi_0}{1 - \xi} \right)$$  \hspace{1cm} (8)

Decomposition of the viscoplastic rate of deformation into volumetric and deviatoric components

$$\mathbf{D}^p = \left( \frac{1}{3} \right) \left( \text{tr} \mathbf{D}^p \right) \mathbf{1} + \mathbf{D}'^p$$  \hspace{1cm} (9)

allows for easy expression of the fact that the plastic volume strain stems only from the compressibility of the microvoids. With the dot signifying time rate, it may be shown that

$$\frac{1}{3} \left( \text{tr} \mathbf{D}^p \right) \mathbf{1} = \frac{1}{3} \left( \frac{\dot{\psi}_v}{1 - \psi_v} \right) \mathbf{1} = \left\{ \frac{\dot{\xi}}{3(1 - \xi)} \left[ 1 + \ln \left( \frac{1 - \xi_0}{1 - \xi} \right) \right] \right\} \mathbf{1}. \hspace{1cm} (10)$$

The constitutive equation for the deviatoric rate of plastic deformation, cf. [14,15], has the form

$$\mathbf{D}'^p = \frac{1}{\eta} \Phi \left( \hat{F} \right) \frac{\partial J'_2}{\partial \mathbf{T}} = \frac{1}{\eta} \left( \frac{J'_2}{\kappa} - 1 \right) \mathbf{1}$$  \hspace{1cm} (11)

where the yield condition requires that

$$\hat{F} = \frac{J'_2}{\kappa} - 1 > 0. \hspace{1cm} (12)$$

$J'_2 = \frac{1}{2} \text{tr}(\mathbf{T}' \cdot \mathbf{T}')$, $\eta$ is the material viscosity and the yield function

$$\kappa = f_1^2 \left( \mathbf{e}^p, \mathbf{D}' \right) f_2 \left( \mathbf{T} \right) f_3 \left( \xi \right) \hspace{1cm} (13)$$

Some metals exhibit large increase of the yield stress for strain rates in excess of $10^3$ s$^{-1}$. Among these are copper and aluminum [16,17]. To model this effect the Johnson-Cook equation has recently been revised, [18], and is incorporated as the function $f_1(\mathbf{e}^p, \mathbf{D}')$. Thus
\[ f_1 = \left( c_1 + c_2 e^{p} \right) \left( 1 + c_3 \ln|\mathbf{D}| \right) \left( 1 + c_4 \left( \frac{1}{c_5 - \ln|\mathbf{D}|} - \frac{1}{c_5} \right) \right), \]  
\[ (14) \]

where \( \mathbf{D}^* = \mathbf{D}^p / \mathbf{D}_0 \), \( \mathbf{D}_0 = 1 \) s\(^{-1} \), \( c_4 = 0 \) for \( \mathbf{D}^p < \mathbf{D}_0 \), \( c_1 = T_{y0} \) the uniaxial yield stress at reference temperature and strain-rate equal to \( \mathbf{D}_0 \), and the strain-rate sensitivity factor has the upper bound

\[ \left( 1 + c_3 \ln|\mathbf{D}| \right) + c_4 \left( \frac{1}{c_5 - \ln|\mathbf{D}|} - \frac{1}{c_5} \right) < c_6 \]

\[ (15) \]

The first bracket of eqn. (14) gives the isotropic strain hardening in nonlinear concave power form. The equivalent plastic strain invariant

\[ \varepsilon^p = \int_0^1 2 \sqrt{3} \left( \left| \Pi_\mathbf{D}^p \right| \right) dt, \quad \Pi_\mathbf{D} = \frac{1}{2} \left[ \left( \text{tr} \mathbf{D}^p \right)^2 - \text{tr} \left( \mathbf{D}^p \cdot \mathbf{D}^p \right) \right], \]

\[ (16) \]

The temperature and damage softening functions have the form

\[ f_2 = \left[ 1 - c_7 \left( \frac{T - T_0}{T_m - T_0} \right)^6 \right], \quad c_7 = \begin{cases} 1, & T > T_0 \\ -1, & T < T_0 \end{cases} \]

\[ (17) \]

and

\[ f_3 = \left[ 1 - \left( \frac{\xi - \xi_0}{\xi_F - \xi_0} \right)^6 \right], \]

\[ (18) \]

where \( \xi_F \) is the critical void volume fraction, the attainment of which at any point causes a microelement of material enclosing that point to separate. At \( T = T_m \) \( f_2(T_m) = 0 \), and at \( \xi = \xi_F \) \( f_3(\xi_F) = 0 \), whereupon \( \mathbf{D}^p \to \infty \), i.e., these represent the conditions for local melt and local fracture. For high compression at high strain rates the material viscosity varies with pressure (volume strain) and temperature [19]. Assuming that the lower bound value for the viscosity at melt temperature remains constant at \( \eta_m \approx 10^{-2} \) Poise, a viscosity-temperature-volume strain relation has been obtained for copper from limited impact data [20-22],
The rate of viscoplastic deformation eqn. (9) will now have the explicit form

\[
D^p = \left\{ \frac{J_2'}{f_1^2(\varepsilon^p, D^*)f_2(T)f_3(\xi)} - I \right\} T' + \left\{ \frac{\dot{\varepsilon}}{3(1-\xi)[1 + \ln(1-\frac{\psi_0}{\psi})]} \right\} I \tag{21}
\]

The yield condition eqn. (12) and inversion of eqn. (21) leads to the dynamical flow condition

\[
J_2' = f_1^2(\varepsilon^p, D^*)f_2(T)f_3(\xi) \left( 1 + \left\{ \frac{\text{tr}(D^p \cdot D^p) + \frac{\psi_0}{1-\psi}}{2J_2'} \right\}^{\frac{1}{2}} \right)^{\frac{m}{n}} \tag{22}
\]

which illustrates the effects of strain hardening, strain-rate hardening, temperature and microdamage softening on the plastic flow process. The yield condition (12) may also be used to derive following inequality

\[
|D^p| > \frac{1}{\sqrt{3}} \frac{\psi_v}{1-\psi_v}, \tag{23}
\]

i.e., during viscoplastic deformation the viscoplastic rate of deformation must always exceed the microvoid volume strain-rate.

### 4 Model for microdamage evolution

The microdamage processes generated by impact at hypervelocities are very rapid, and consist essentially of microvoid nucleation and growth. Nucleation
has been modeled as a stress-activated rate type process, [15,21-23], where the nucleation rate

\[
\dot{\xi}_N = \frac{m_4}{(1 - \xi)} \exp \left( \frac{m_3 (\sigma_m - \sigma_N)}{kT} \right) - 1, \quad \sigma_m > \sigma_N \tag{24}
\]

The inner bracket is the mean stress driven actuation energy, where \( k \) is the Boltzman constant, \( m_3 \) and \( m_4 \) are parameters and the mean stress is positive in tension. The values for the nucleation threshold stress found in high velocity impact literature are mostly obtained from high velocity impact tests [8]. These values are designated by \( \sigma_{N0} \). The high compression associated with impact at hypervelocities can cause substantial hardening of the material, particularly at the impact sides of the target and projectile. This effect is indirectly exhibited by the higher nucleation thresholds for the harder metals, e.g., Ni-Cr-Steel, as compared to softer materials such as Copper or Aluminum, and may be expressed by assuming the microvoid nucleation stress to be proportional to the current yield stress, i.e.,

\[
\sigma_N = \frac{\sigma_{N0}}{c_1} f_4 \left( \varepsilon^p, D^* \right) \tag{25}
\]

\[
f_4 = \left( c_1 + c_2 |\varepsilon^p|^m \right) \left( 1 + c_3 \ln |D^*| \right). \tag{26}
\]

The microvoid rate of growth is calculated using the hollow sphere micromechanics model in a viscoplastic material that strain hardens nonlinearly [24,25]. The void growth rate \( \dot{\xi}_G \) requires solution of the nonlinear differential equation

\[
\tau \left[ A(\xi) \frac{d^2 \xi}{dt^2} + B(\xi) \left( \frac{d\xi}{dt} \right)^2 \right] + \eta F(\xi, \xi_0) \frac{d\xi}{dt} = \left[ \sigma_m(t) - \sigma_G \right] g(\xi), \tag{27}
\]

where

\[
\tau = \frac{\rho r_0^2}{3} \left( \frac{1 - \xi}{\xi_0} \right), \quad A(\xi) = \frac{\left( \xi^{-\frac{\nu}{\nu'}} - 1 \right)}{(1 - \xi)^{\frac{\nu}{\nu'}}} \tag{28}
\]

\[
B(\xi) = \frac{2}{(1 - \xi)^{\frac{\nu}{\nu'}}} \left[ \left( \xi^{-\frac{\nu}{\nu'}} - 1 \right) - \frac{1}{12} \left( \xi^{-\frac{\nu}{\nu'}} - 1 \right) \right], \tag{29}
\]
The threshold for void growth

\[
\sigma_G = \frac{2}{\sqrt{3}} \left(1 - \xi^3\right) \left[ c_1 \ln \left(\frac{1}{\xi}\right) - \frac{2}{3} c_2 I(\xi, \xi_0) \right] f_2(T) \tag{31}
\]

The material viscosity \( \eta \) is given by equation (19) and the temperature function by eqn. (17). The void growth threshold stress is seen to be dependent upon the current void volume fraction, the yield stress, strain-hardening and temperature. The positive valued material function \( g(\xi) = e^{a\xi} \) is meant to simulate the effect of void interaction on the growth process, while \( r_0 \) is the average initial microvoid radius. The integral

\[
I(\xi, \xi_0) = \int_{\beta_2(\xi, \xi_0)}^{\beta_1(\xi, \xi_0)} \left(1 + \frac{2}{3} x^2\right)^{5/3} \xi \left[ \frac{(1 - \xi_0) \frac{d\xi}{(1 - \xi)(\xi_0 - \xi)}}{dt}\right]^{\frac{m_0}{\xi}} \frac{1}{\xi^{\frac{5}{3}}} \, dx \tag{32}
\]

\[
\beta_1 = \frac{(\xi_0 - \xi)}{(1 - \xi_0)}, \quad \beta_2 = \frac{(\xi_0 - \xi)}{\xi(1 - \xi_0)} \tag{33}
\]

The microvoid nucleation rate \( \dot{\xi}_N \) given by eqn. (24) when added to the solution of the system of equations (27) – (33) for the void growth \( \dot{\xi}_G \), gives the microvoid volume fraction growth rate

\[
\dot{\xi}(x, t) = \dot{\xi}_N + \dot{\xi}_G \tag{34}
\]

at any point \( x \) of the material at time \( t \). Time integration then provides the microdamage \( \xi(x, t) \) appearing in the viscoplastic and elastic constitutive equation above.

5 Internal energy and temperature

The extreme rapidity of the impact fracture process that occurs causes adiabatic thermal conditions. If furthermore there are no external heat sources, the instantaneous internal energy and temperature at any location can be calculated.
by time integration of the following energy rate balance equations. For the internal energy

$$\dot{E} = \left| \sigma_m \right| \left( \frac{\psi}{1 - \psi} \right) + \text{tr} \left( \mathbf{T} : \mathbf{D}'^e \right) + \text{tr} \left( \mathbf{T} : \mathbf{D}^p \right),$$  \hspace{1cm} (35)

and for temperature

$$\rho_0 c_v \dot{T} = \left| \sigma_m \right| \dot{\psi} + (1 - \psi) \omega \text{tr} \left( \mathbf{T} : \mathbf{D}^p \right).$$  \hspace{1cm} (36)

Depending on whether there is compression or dilatation, $\sigma_m$ is given by eqn. (3) or (5), respectively. \( \mathbf{D}'^e = \left( \frac{1}{2\mu} \mathbf{\nabla} \right) \mathbf{T}' \) and \( \mathbf{D}^p \) is given by eqn. (21). The term $\omega$ is the fraction of plastic work that is dissipated as thermal energy. The elevated temperature local to areas of high material damage concentration (cf. [13]), can be calculated from the microvoid damage model and is expressed by means of the thermal energy rate

$$\dot{T}_L = \frac{4\omega}{9Q c_v} \sqrt{3J'_2} \frac{\dot{\xi}}{\xi(1 - \xi) \left[ 1 + 1/2 \left( \frac{1}{\xi^{1/3}} - 1 \right) \right]^3}. \hspace{1cm} (37)$$

6 Numerical simulations

Calculations of the pressure, volume-strain and temperature at a representative point behind a passing shock wave are shown below. The experimental shock pressure volume-strain Hugoniot for Copper up to 142 GPa [14], is shown together with the calculated curve in fig. 1. Fig. 2 illustrates the calculated temperature rise with increasing pressure, where the solid curve is determined from the energy rate balance equation (36), while the open circle curve was calculated by others using the equations of classical thermodynamics [26]. We note that at 0.3 volume strain corresponding to 135 GPa pressure, the melt temperature for Copper increases to 4,312 °K according to the melt temperature-volume strain model employed. The calculated temperature at 135 GPa is approximately 4,200 °K, indicating that the shock-compressed material would be close to melt upon release.

Figure 3 shows a copper plate 13mm thick after impact by an aluminum 1100 spherical projectile 3.18mm in diameter traveling at 6.01 km/s [27]. Figure 4 shows the corresponding simulated impact. The simulation accurately predicts the depth and width of the crater compared to the crater appearing in the test specimen. Several features of the simulated spall fracture need improvement.
The back lip of the spall is too wide, and does not bulge out sufficiently. Also the top edge of the simulated spall fracture does not conform to the almost straight edge of the test specimen. These aberrations will be rectified with further study.

We note that the current hypervelocity work is an outgrowth of earlier work that used constitutive-microdamage equations to model spall fracture induced by high velocity plate impact [25].

Material parameter values for high purity Copper and Aluminum-1100 are shown in the tables below. Space limitation preclude discussion of how they were obtained, i.e., data tables, the work of others, specific test programs.

Table 1. Material parameters for OFHC Copper.

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<td>( K_0 )</td>
<td>13.2 \times 10^7 MPa</td>
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<tr>
<td>( \mu_0 )</td>
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<td>( c_0 )</td>
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<td>( T_u )</td>
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<td>( T_m^{(0)} )</td>
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<td>( \gamma_0 )</td>
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<td>( a_0 )</td>
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Table 2. Material parameters for Aluminum-1100.

<table>
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Fig. 1. Calculated and experimental (circles) shock pressure versus volume strain for copper.

Fig. 2. Calculations of temperature versus shock pressure using thermal energy rate balance (solid line) and classical thermodynamics (circles).

Fig. 3. Laboratory specimen of a 13.0 mm copper target after impact by a 3.18 mm aluminum 1100 projectile at 6.01 km/s [27].

Fig. 4. Grid configuration 20μs after impact of a 13.0 mm copper target by a 3.18 mm aluminum 1100 projectile at 6.01 km/s.

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References


