Optimal coating of fibers in composites

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Abstract
In some recent papers of the author, localization of damage of composites and fiber-reinforced concretes have been presented, using the BEM as an extraordinarily prospective tool for solving such problems, [4], [6].

The objective of this paper is an optimization of a coating of fibers in composite materials by transformation field analysis, [1]. It consists in a priori relating of stresses and eigenparameters (eigenstresses or eigenstrains) on phases. The eigenparameters can represent, among others, plastic strains, or relaxation stresses. The most endangered region in composites is the interfacial zone between phases. In order to suppress the impact of jump in material stiffness on the contact boundaries of the phases against the localized gradients of stresses in the aggregate, the material nature of the coating will be determined by applying the additional eigenparameters in such a way that the peak stresses are removed. As a consequence of this process the improvement of bearing capacity of the composite is expected and "the optimal" material properties of the coating can be suggested to producers of composite structures.

The relation stresses-eigenparameters is determined by the BEM. A special BEM procedure is proposed to simplify the computation. The optimization will be applied to 2D unit cell in a periodic structure of the composite. The macro-level behavior will be stated from the standard homogenization starting with given overall strain tensor.

1 Introduction
Conventionally, the optimization problems consist in minimization of an appropriate cost functional with certain constraints, such as equilibrium and compatibility conditions and design requirements. The formulation of the cost functional depends on the concrete intention of a designer. In our
case the most natural requirement is a reasonable and practical form of the cost functional respecting the minimization of the peaks of stresses wherever on a unit cell, i.e. the cost functional will describe the minimum stresses all over the domain of the unit cell. Such a problem can easily be formulated in terms usually used in the classical theory of composites structures, [5].

A similar problem was solved for laminated cylindrical structures where optimal distribution of stresses were sought, [2]. A solution of the problem proposed in terms of the FEM is straightforward and often used. When applying the transformation field analysis, the intended formulation in terms of the BEM seems more prospective and seems to be more suitable for such problems. On the other hand, the direct connection of the BEM with the variational principles is not seen at first sight and desires a deeper study.

2 Homogenization of periodic structures

Localization and homogenization is concisely described in [5], and theoretically discussed in [3]. Recall some basic consumptions which we use later in the integral formulation.

First, we denote quantities used in this text. Two different scales will be introduced. The macroscopic scale, the homogeneous law in which is sought, will be described in coordinate system \( \mathbf{x} \equiv \{x_1, x_2, x_3\}^T \) and the microscopic scale – heterogeneous – is characterized in the system of coordinates \( \mathbf{y} \equiv \{y_1, y_2, y_3\}^T \). The medium is generally heterogeneous, but locally – in the microscopic scale – is assumed to be periodic, thus a representative volume element may be cut out from the composite structure and the periodicity conditions can be introduced on the boundary of this element.

Let us distinguish the quantities under study in dependence of the macroscopic or microscopic scale in the following manner: The displacements in the macroscopic level will be denoted as \( \mathbf{U} \equiv \{U_1, U_2, U_3\}^T \) while in the microscopic level as \( \mathbf{u} \equiv \{u_1, u_2, u_3\}^T \). Moreover, in macroscopic level, let us denote strains as \( \mathbf{E} \equiv \mathbf{E}_{ij}, i, j = 1, 2, 3 \) and stresses by \( \mathbf{S} \equiv \mathbf{S}_{ij}, i, j = 1, 2, 3 \). In the microscopic level let us denote strains as \( \mathbf{\varepsilon} \equiv \mathbf{\varepsilon}_{ij}, i, j = 1, 2, 3 \) and stresses by \( \mathbf{\sigma} \equiv \mathbf{\sigma}_{ij}, i, j = 1, 2, 3 \).

Define also the microscopic-macroscopic relation of the average stresses and strains by

\[
\mathbf{S}_{ij} = \frac{1}{\text{meas } \Omega} \int_{\Omega} \sigma_{ij} \, d\Omega(\mathbf{y}) = \langle \sigma_{ij} \rangle ,
\]

\[
\mathbf{E}_{ij} = \frac{1}{\text{meas } \Omega} \int_{\Omega} \varepsilon_{ij} \, d\Omega(\mathbf{y}) = \langle \varepsilon_{ij} \rangle ,
\]

where \( \langle \cdot \rangle \) stands for the average, \( \Omega \) is the representative volume element, and \( \text{meas } \Omega \) is its volume, \( \Omega = \Omega^f \cup \Omega^c \cup \Omega^m \), where respectively \( \Omega^f \) describes the domains adjacent to fiber, \( \Omega^c \) to coat, and \( \Omega^m \) adjacent to matrix.

Localization consists in solving the system of equilibrium equations on the representative volume element (or unit cell) for concentration factors.
$A^f, A^c$ and $A^m$ (concentration factors of fibers, coats and the matrix):

\[
\begin{align*}
\varepsilon_{ij}^f(u(y)) &= A^f_{ij,k}(u(y)) E_{kl}, \quad y \in \Omega^f, \\
\varepsilon_{ij}^c(u(y)) &= A^c_{ij,k}(u(y)) E_{kl}, \quad y \in \Omega^c, \\
\varepsilon_{ij}^m(u(y)) &= A^m_{ij,k}(u(y)) E_{kl}, \quad y \in \Omega^m. \tag{2}
\end{align*}
\]

Denoting the boundary of the unit cell by $\partial \Omega$, the periodic boundary conditions will be employed on $\partial \Omega$ ($\mathbf{n} = \{n_1, n_2, n_3\}$ is unit outward normal to $\Gamma$ with respect to fiber $\Omega^f$, see Fig. 1):

- stress: $p_i = \sigma_{ij} n_j$ are opposite on the opposite sides, $n_j$ is the $j$-th component of the unit outward normal,
- strains: the local strain $\varepsilon(u)$ is split into its average $\bar{\varepsilon}$, and a fluctuating term $\varepsilon$ as:

\[
\varepsilon(u(y)) = \bar{\varepsilon}(u(y)) = \bar{\varepsilon}(y), \quad <\varepsilon(u(y))> = 0. \tag{3}
\]

The fluctuating displacement $\tilde{u}$ may be considered as a periodic field, up to a rigid displacement that will be disregarded.

\[
\begin{align*}
\text{Figure 1. Unit cell used in this study}
\end{align*}
\]

Eq. (3) obeys the Hill’s energy condition, as proved, e. g., in [5]:

\[
<\sigma_{ij}\varepsilon_{ij}> = S_{ij} E_{ij}. \tag{4}
\]

The transformation field analysis relates e. g. stresses $\sigma$ and eigenstresses $\lambda$ as:

\[
\sigma_{ij}(y) = L_{ijkl}(y)\varepsilon_{kl}(u(y)) + \sum_{\alpha=f,c,m} F_{ijkl}(y)\lambda_{kl}(y), \tag{5}
\]

as three phases $\alpha = f, c, m$ (fibers, coats, and matrix) are considered. Matrix $F$ is influence matrix which may be created by unit impulses of
components of the eigenstress tensor successively imposed in the phases \( \alpha = f, c, m \).

It is necessary to point out that the eigenstresses (generally eigenparameters) are additional quantities in this case, do not influence averaging process, and their influence is applied after internal elastic material properties are made clear.

Using (2) and the generalized linear Hooke's law (5) we get:

\[
S_{ij} = \langle \sigma_{ij}(y) \rangle = \langle L_{ijkl}(y) \varepsilon_{kl}(u(y)) \rangle + \sum_{\alpha=1}^{3} F_{ijkl}^{\alpha}(y) \lambda_{kl}^{\alpha}(y) = \nonumber \\
\left( L_{ijkl}^{f} A_{k\alpha\beta}^{f}(y) \rangle_f + L_{ijkl}^{c} A_{k\alpha\beta}^{c}(y) \rangle_c + L_{ijkl}^{m} A_{k\alpha\beta}^{m}(y) \rangle_m \right) E_{\alpha\beta} \nonumber \\
+ \left( F_{ijkl}^{f}(y) \lambda_{kl}^{f}(y) \rangle_f + F_{ijkl}^{c}(y) \lambda_{kl}^{c}(y) \rangle_c + F_{ijkl}^{m}(y) \lambda_{kl}^{m}(y) \rangle_m \right) \nonumber \\
(6)
\]

where \( \langle . \rangle_f \) stand for average on fiber, \( \langle . \rangle_c \) is the average on coat and \( \langle . \rangle_m \) is the average on matrix. This averaging process is made in such a way that the integrals are taken over fiber, coat, and matrix, respectively, but the denominator remains \( \Omega \), see (1).

By definition, the homogenized stiffness matrix \( L^* \) is written as:

\[
S_{ij} = L_{ijkl}^* E_{kl} + \nonumber \\
+ \left( F_{ijkl}^{f}(y) \lambda_{kl}^{f}(y) \rangle_f + F_{ijkl}^{c}(y) \lambda_{kl}^{c}(y) \rangle_c + F_{ijkl}^{m}(y) \lambda_{kl}^{m}(y) \rangle_m \right) \nonumber \\
(7)
\]

Comparing (6) and (7), the overall stiffness matrix follows as

\[
L_{ijkl}^* = L_{ijkl}^{f} A_{k\alpha\beta}^{f}(y) \rangle_f + L_{ijkl}^{c} A_{k\alpha\beta}^{c}(y) \rangle_c + L_{ijkl}^{m} A_{k\alpha\beta}^{m}(y) \rangle_m \nonumber \\
(8)
\]

It is worth noting that the homogenized stiffness matrix is symmetric with similar properties as that of the classical stiffness matrix and is concerned only with elastic behavior of the material under consideration.

It can be easily proved that the following relation holds (\( v_i, i = f, c, m \) are volume ratios of the phases, \( I_{k\alpha\beta} \) is four order unit tensor):

\[
v_f < A_{k\alpha\beta}^{f}(y) \rangle_f + v_c < A_{k\alpha\beta}^{c}(y) \rangle_c + v_m < A_{k\alpha\beta}^{m}(y) \rangle_m = I_{k\alpha\beta} . \nonumber \\
(9)
\]

3 Localization using the BEM

Without lack of generality, let us consider a symmetric unit cell, Fig. 1. The overall strain \( E_{ij} \) is given independently of location in \( \Omega \). The loading of this unit cell will be given by unit impulses of \( E_{ij} \), i.e., we successively select \( E_{i_0j_0} = E_{j_0i_0} = 1 \); \( E_{ij} = 0 \) for either \( i_0 \neq i \) or \( j_0 \neq j \).
The procedure is split into two steps. Assume the above described surface displacements to be prescribed along the entire boundary $\partial \Omega$ and there are no body forces. In the first step, the cell obeys static equilibrium equations and linear homogeneous Hooke’s law:

$$\sigma_{ij}^0 = L_{ijkl}^0 E_{kl} \quad \text{in } \Omega, \quad u_i^0(y) = E_{ij} y_j \quad \text{on } \partial \Omega(y),$$

where $L_{ijkl}^0$ is the stiffness matrix (stiffness tensor) of a comparison medium. The meaning of $L_{ijkl}^0$ will be explained later.

The tractions easily follow from (10) as:

$$p_i^0(y) = \sigma_{ij}^0 n_j \quad \text{on } \partial \Omega,$$

where $n = \{n_1, n_2\}$ is the unit outward normal to $\partial \Omega$.

In the second step a geometrically identical cell with the same boundary conditions is considered. Define

$$\tilde{u}_i(y) = u_i(y) - u_i^0(y),$$

$$\tilde{\varepsilon}_{ij}(y) = \varepsilon_{ij}(y) - E_{ij},$$

$$\tilde{\sigma}_{ij}(y) = \sigma_{ij}(y) - \sigma_{ij}^0 \quad \text{in } \Omega,$$

The displacements $\tilde{u}_i$, strains $\tilde{\varepsilon}$ and stresses $\tilde{\sigma}$ are to be stated. Hooke’s law becomes:

$$\sigma_{ij} = L_{ijkl} \varepsilon_{kl} \quad \text{in } \Omega.$$  \hspace{1cm} (13)

Define the symmetric stress polarization tensor $\tau$ as:

$$\sigma_{ij} = L_{ijkl}^0 \varepsilon_{kl} + \tau_{ij},$$

or subtracting (14) and (10):

$$\tilde{\sigma}_{ij} = L_{ijkl}^0 \tilde{\varepsilon}_{kl} + \tau_{ij}.$$  \hspace{1cm} (15)

Remark 1: From the last relation immediately follows that

$$< \tilde{\sigma}_{ij} > = < \tau_{ij} >.$$  \hspace{1cm} 

This is the direct consequence of (3)$_2$.

Moreover, eliminating $\sigma_{ij}$ from (14) and (13), we get a possible definition of polarization tensor:

$$\tau_{ij} = [L_{ijkl}](\tilde{\varepsilon}_{kl} + E_{kl}),$$

where

$$[L_{ijkl}] = L_{ijkl} - L_{ijkl}^0.$$
Remark 2: Similarly to Remark 1, from (14), (12) and Remark 1 we get 
\( \sigma_{ij} = L_{ij}^0 E_{kl} + \sigma_{ij}^\circ \). If \( L^0 = L^* \), from the last relation follows 
\( \sigma_{ij}^\circ = 0 \) and the introduction of the overall stress \( S_{ij} \) instead of the overall 
strain \( E_{ij} \) leads to the same stiffness (or more precisely compliance) overall 
material matrix, providing the same boundary conditions are considered.

Since both \( \sigma_{ij} \) and \( \sigma_{ij}^0 \) are statically admissible, it holds (the following 
equations must be defined in the sense of distributions):

\[
\frac{\partial (L_{ijkl}^0 \varepsilon_{kl} + \tau_{ij})}{\partial y_j} = 0 \quad \text{in } \Omega, \\
\bar{u}_i = u_i - u_i^0, \bar{p}_i = p_i - p_i^0 \quad \text{on } \partial \Omega,
\]

Owing to constant distribution of \( L_{ijkl}^0 \) in \( \Omega \), the equivalent integral 
formulation can be written as

\[
c_{mn}(\xi)\bar{u}_n(\xi) = \int_{\partial \Omega} P_{mi}^*(\gamma; \xi)\bar{u}_i(\gamma) \, d\gamma(\gamma) - \int_{\partial \Omega} u_{mi}^*(\gamma; \xi)\bar{p}_i(\gamma) \, d\gamma(\gamma) + \\
+ \left( [L_{ijkl}^f - L_{ijkl}^0] \int_{\Omega_t} + [L_{ijkl}^c - L_{ijkl}^0] \int_{\Omega_c} + [L_{ijkl}^m - L_{ijkl}^0] \int_{\Omega_m} \right) \times \\
\times \{\varepsilon_{mij}(\gamma; \xi)(\tilde{\varepsilon}_{kl}(\gamma) + E_{kl})\} \, d\Omega
\]

where \( c_{mn} \) depends on the position \( \xi \in \partial \Omega \) and the quantities with asterisks 
are known kernels.

Differentiating the last equation provides

\[
\varepsilon_{mn}(\xi) = \int_{\partial \Omega} P_{mi}^*(\gamma; \xi)\bar{u}_i(\gamma) \, d\gamma(\gamma) - \int_{\partial \Omega} U_{mi}^*(\gamma; \xi)\bar{p}_i(\gamma) \, d\gamma(\gamma) + \\
+ \left( [L_{ijkl}^f - L_{ijkl}^0] \int_{\Omega_t} + [L_{ijkl}^c - L_{ijkl}^0] \int_{\Omega_c} + [L_{ijkl}^m - L_{ijkl}^0] \int_{\Omega_m} \right) \times \\
\times \{\varepsilon_{mij}(\gamma; \xi)(\tilde{\varepsilon}_{kl}(\gamma) + E_{kl})\} \, d\Omega(\gamma) + \text{convected term}
\]

First, let e. g. \( L_{ijkl}^f \equiv L_{ijkl}^f \). Eliminating unknown boundary values 
from (20) and (21) we obtain two types of equations for unknowns \( \varepsilon_{ij}^p(\tilde{u}) \) 
and \( \varepsilon_{ij}^m(\tilde{u}) \) as:

\[
\varepsilon_{ij}^p(\tilde{u}(\gamma)) = \varepsilon_{ij}^p(\tilde{u}(\gamma)) E_{kl},
\]

where \( p = c, m \).

Similarly, for \( L_{ijkl}^0 \equiv L_{ijkl}^c \) we get the relation (22) for \( p = f, m \) and, 
eventually, for \( L_{ijkl}^0 \equiv L_{ijkl}^m \), (22) holds for \( p = f, c \). Owing to (9), only two 
of the above types of equations need to be employed. It is sufficient draw 
centration on the fiber and coat, when dealing with concrete composites 
(the fiber ratio is very small), or on the matrix and coat, if the fiber volume 
ratio is large and the material behavior of stresses on fibers is close to 
uniform distribution of stresses.
This process leads to a fourth-order "concentration factor tensor" $A^p_{ijkl}$ defined as

$$
epsilon_{ij}^p(\bar{\mathbf{u}}(y)) = e_{ij,kl}^p(y)E_{kl}, \quad e_{ij}^p(\mathbf{u}(y)) = [I_{ijkl} + e_{ijkl}^p(y)]E_{kl} = A^p_{ijkl}(y)E_{kl},$$

where the superscript $p \equiv f$ for $y \in \Omega^f$, $p \equiv c$ for $y \in \Omega^c$, and $p \equiv m$ for $y \in \Omega^m$.

### 4 Optimization

A natural question for engineers dealing with composites could be: determine such distribution of stresses that the "jump" in stresses along the interfacial lines fiber-coat and coat-matrix are as small as possible (in order to avoid damage, as undesirable localized stresses occur in the vicinity of the above mentioned lines), and also the bearing capacity of the entire composite structure increases in this way and attain its maximum. This is a problem of optimization structures and can be formulated for composites as follows: Let the uniform strain field $E_{ij}$ be applied to the domain $\Omega$ (in our case, a periodic distribution of fibers is considered). After adding design parameters (eigenstresses, or alternatively eigenstrains) into the Hooke's law, the optimization problem may by defined in this case as: let $\Pi(\mathbf{\lambda})$ be a real functional of $\lambda_{ij}, i,j=1,2,3, i \geq j$ in $\Omega$. The problem then consists in finding such design parameters from a class $O$ of admissible tensors, which minimize $\Pi$. This may symbolically be written as

$$\min \{\Pi(\mathbf{\lambda}); A(\mathbf{u}, \mathbf{\lambda})\} = 0,$$

where $A$ is an operator which for each $\mathbf{\lambda} \in O$ uniquely determines the displacement field $\mathbf{u}$ (and, consequently strains and stresses, i.e. the deformation method is applied).

Since there is no external loading in our solution (the load is due to unit impulses of strain tensor and eigenstresses, or equivalently, of prescribed displacements and eigenstresses), one of a practical requirements of designers is assumption of minimum variance of stresses all over the domain of unit cell:

$$\Pi(\mathbf{\lambda}) = \frac{1}{2} < S_{ij}S_{ij} > .$$

Let us analyze the functional $\Pi$. Section 3 was devoted to localization and homogenization of the elastic part of the composite unit cell. The philosophy will have to be slightly improved, when introducing the eigenstresses. The first step in Section 3 will remain unchanged, i.e. eqs. (10), (11), and (12) will remain valid. Starting with eq. (13), we have to consider also effect of eigenstress. The generalized Hooke's law now reads as:

$$\sigma_{ij} = L_{ijkl}e_{kl} + \lambda_{ij} \quad \text{in } \Omega .$$
The polarization tensor $\tau_{ij}$ is defined in the same manner as that in (14) and (15). The explicit definition of $\tau_{ij}$ may follow from (26) and (16) as:

$$\tau_{ij} = [L_{ijkl}]\varepsilon_{kl} + \lambda_{ij}.$$  

(27)

Remark 2 will then be improved as:

**Remark 3:** The average of the stress tensor is now rewritten as:

$$S_{ij} = <\sigma_{ij}> = L_{ijkl}^0 E_{kl} + <\bar{\sigma}_{ij}> + <\lambda_{ij}>.$$  

(28)

Formulas (26), (27), and (28) appear in deriving the influence matrices $F$.

It is worth noting that for purely elastic behavior of the composite $<\bar{\sigma}_{ij}> = 0$ iff $L_{ijkl}^0$ are the average components of material stiffness matrix of the composite aggregate, i.e. \( \int_\Omega L_{ijkl}^0 d\Omega = \int_\Omega L_{ijkl} d\Omega \). This means that $\sigma_{ij}^0$ may be considered as $S_{ij}$.

If some of components of eigenstress tensor are different from zero, the above assertion is no longer valid and (28) holds.

In order to use the boundary element method, we have to start with (14) rather than (26), when expressing stresses in (25) with addition of eigenstresses. Using (122) it holds for stresses:

$$\sigma_{ij} = L_{ijkl}^0 (\varepsilon_{kl} + E_{kl}) + \tau_{ij}.$$  

(29)

The process of derivation of influence matrices $F$ then follows procedure, described in Section 3.

Without loss of generality, the uniform or step distribution of eigenstresses will be considered in the next text. The expression (7) can then be written for uniform distribution of eigenstresses as:

$$S_{ij} = L_{ijkl}^* E_{kl} +$$

$$+ <(F_{ijkl}^f(y))>_f \lambda_{kl}^f + <(F_{ijkl}^c(y))>_c \lambda_{kl}^c + <(F_{ijkl}^m(y))>_m \lambda_{kl}^m.$$  

(30)

Differentiation of II by $\lambda_{st}^\alpha$ provides a condition of its minimum:

$$(L_{ijkl}^* E_{kl} +$$

$$+ <(F_{ijkl}^f(y))>_f \lambda_{kl}^f + <(F_{ijkl}^c(y))>_c \lambda_{kl}^c + <(F_{ijkl}^m(y))>_m \lambda_{kl}^m) \times$$

$$\times <F_{ijst}^\alpha(y)>_\alpha = 0 , \quad \alpha = f, c, m.$$  

(31)

The last conditions create system of linear algebraic equations for unknown eigenstresses. The absolute term is:

$$(L_{ijkl}^* E_{kl} F_{ijst}^\alpha(y)) >_\alpha.$$  

(32)
Having once the solution of the linear system for eigenstresses, we can deduce the material properties of coats needed. Recall that eigenstresses may stay for residual stresses, or under some circumstances in numerical analysis, the values of material constants in elastic state may even be determined.

5 Optimization of coats

Our main interest should be concentrated on optimization of the coating of fibers. From (31) one easily obtains:

\[
(L^*_{ijkl}E_{kt} + (F^c_{ijkl}(y)) >^c \lambda^c_{kt} < F^c_{ijst}(y)) >_c = 0, \quad s, t = 1, 2, 3, \quad (33)
\]
as the eigenstresses are introduced only in coats. The last expression can be written as a linear algebraic system of equation for unknown \(\lambda_{kl}; k, l = 1, 2, 3, k \leq j\), because of symmetry of eigenstress tensor.

6 Example

Unit cell is considered with fiber volume ratio equal to 0.6 and 0.036 is coat volume ratio.

The material properties of phases are as follows: Young’s modulus of fiber \(E^f = 210\) MPa, Poisson’s ratio \(\nu^f = 0.16\), \(G^f = 90.52\) MPa; on the matrix \(E^m = 17\) MPa, \(\nu^m = 0.3\), and \(G^m = 9.8\) MPa; the coat material properties are \(E^c = 35\) MPa, \(\nu^m = 0.2\), and \(G^m = 14.6\) MPa.

The homogenized matrix \(L^*\) in this case attains the following values:

\[
(L^*)^T = \begin{bmatrix} 150.16 & 29.5 & 0.0 \\ 29.5 & 150.16 & 0.0 \\ 0.0 & 0.0 & 57.1 \end{bmatrix}
\]

As the prevailing ”jump” in stresses is registered in hoop direction, they are preferred from the other components of eigenstress tensor and also only the hoop stress is recorded.

The following table will demonstrate the change of average hoop stresses:

<table>
<thead>
<tr>
<th>Interface</th>
<th>fiber-coat</th>
<th>coat-matrix</th>
</tr>
</thead>
<tbody>
<tr>
<td>case 1</td>
<td>0.3173</td>
<td>0.1520</td>
</tr>
<tr>
<td>case 2</td>
<td>0.3169</td>
<td>0.2949</td>
</tr>
<tr>
<td>case 3</td>
<td>0.3119</td>
<td>0.2985</td>
</tr>
<tr>
<td>case 4</td>
<td>0.3118</td>
<td>0.3012</td>
</tr>
</tbody>
</table>

Table 1

In the Table 1 the first case depicts hoop stresses without effect of eigenstresses, the second case describes interfacial hoop stresses for uniform eigenstress, the third case is for equidistant division of the coat into two parts.
and the fourth case describes the state with three equidistant division of the coat. Hence, the general distribution of eigenstresses is substituted by step function. The adjacent values obviously converges to each other. This was our goal.

7 Conclusions

In this paper the improvement of interfacial conditions in composite aggregate has been proposed using transformation field analysis in connection with proper coating, avoiding localized damage. The boundary element method has been applied to the homogenization and solution of optimal coating on a unit cell of periodic composite structure. The BEM procedure possesses many advantages in comparison with the finite element method and is very prospective not only for applications to homogeneous media, but also for partly homogeneous materials, such as composites and soils.

References


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