A study on the criteria of multiaxial fatigue for aluminium structures

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Abstract

In the study of welded structures of a 6000-type aluminium alloy, equivalent structures (simulated HAZ) are tested in fatigue under in-phase, out-of-phase biaxial loads in the High Cycle Fatigue regime. Several existing multiaxial fatigue criteria (Dang Van, K.; Findley; Matake; McDiarmid) are evaluated under Castem2000. The results of all simulations are compared one to another and to the test data. Some criteria (Dang Van, K. & McDiarmid) are found to be somewhat conservative. Moreover, the comparison of the critical plan or direction of simulations is compared to test results. The criterion of McDiarmid is found to work the best for in-phase loads, but the worst for out-of-phase. While the criteria of Findley and Matake are found to give life predictions not precise enough, the proportion between test results and life assessments can sometime be larger than 10. The criterion of Dang Van always gives conservative life assessments.

Keywords: (HCF) High Cycle Fatigue, multiaxial fatigue criterion, aluminium welded joints.

1 Introduction

Despite the long history of the research on fatigue (Dang Van [1], Stulen and Cummings [2] and Robert [4] etc.), we are still far from solving the problem totally. The pieces or components in service are often submitted to multiaxial loading, caused by the physical load in service or residual stresses. Multiaxial or biaxial fatigue has attracted many attentions: specimen geometry study [16], life
assessment approach study [1~2][4][8~14], treatment to increase biaxial fatigue limit endurance [15]. Recently, aluminum is often used to reduce the weight of vehicles and to increase the capacity of transportation. But in the domain of multiaxial fatigue research, there are few results, especially for 6000 aluminum alloys. The main purpose of this paper is to test several existing multiaxial fatigue criteria for this aluminum structures. The criteria tested are the criteria of Dang Van [1], Stulen and Cummings [2], McDiarmid [8][9], and Matake [10],

**Notation**

\( \tau_{nm} \) Mean value of shear stress on the plane in question

\( \tau_{na} \) Shear stress amplitude on the plane in question

\( \tau_{na}(t) \) Instant shear stress amplitude value

\( P_H(t) \) Instant hydrostatic stress

\( \sigma_{nna} \) Normal stress amplitude on the plan in question

\( \sigma_{nnm} \) Mean normal stress on the plan in question

\( \sigma_{nn \text{ max}} \) Maximum normal stress on the plan in question

\( R_m \) Tensile strength

\( R \) Load Ratio

\( \tau_{-1} \) Fatigue limit of alternative torsion

\( \sigma_{-1} \) Fatigue limit of tensile and compression tests with mean zero value

\( D_n \) Parameter of damage on the plan

\( D_{DV1} \) Parameter of damage on the critical plan using the criterion of Dang Van. Similarly we have \( D_{MD2}, D_{FD}, D_{MA} \).

*Note: Here the method of the smallest circumscribed circle is used. [1][4][17]*
2 Fatigue experiments

Uniaxial tests are performed under 30HZ using a INSTRON 8501 hydraulic machine on both parent metal and the so-called 'Simulated Heat Affected Zone' [3]. The basic characteristics are listed in Table 1. The biaxial tests are carried out on a platform with four horizontal hydraulic jacks.

Table 1: Mechanical characteristics for simulated HAZ.

<table>
<thead>
<tr>
<th>Simulated HAZ</th>
<th>(R_m) (MPA)</th>
<th>(R_e) (MPA)</th>
<th>A%</th>
<th>HV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simulated HAZ</td>
<td>274</td>
<td>230</td>
<td>16</td>
<td>80</td>
</tr>
</tbody>
</table>
2.1 Uniaxial experiments and results

Fatigue tensile tests are carried out to get some basic information on the alloy we are studying. As the residual stresses in a welded structure are important, we carried out three different types of uniaxial fatigue experiments (R = 0.1, 0.3 and 0.5). For each R value, one iterative method [3] is used to carry out the tests and the theoretical curves are presented by using the formula of Bastenaire [5][3] (figure 3). Uniaxial results of welded structures show that the nucleation of fatigue cracks often happens in the Heat Affected Zone of the structure, especially at the toe of the weld. To take into account this phenomenon, a specific thermal treatment is proposed to simulate the HAZ structure and applied to the cruciform specimen, after which the control of hardness and other mechanical characteristics are made (Ferraton [3]). The uniaxial test results on 'Simulated HAZ' are shown in figure 3.

![Figure 3: Uniaxial fatigue test results for simulated HAZ.](image)

2.2 Biaxial experiments and results

The biaxial tests on 'Simulated HAZ structures' are carried out on the platform of test. RS-Consoles (INSTRON Software) "Modal Control" is used to make the center of the specimen maintained at the initial position in order to avoid unwanted bending. The cruciform specimen center is much thinner (1mm) than the four ends (8mm). The very similar geometry of cruciform specimen is used already by other authors with success [14][16]. In-phase load tests and out-of-phase load tests are carried out in different experiments. In the out-of-phase tests, phase differences such as 30°, 45°, 60° and 90° are used. All the biaxial tests results are available in Table 2.
2.2.1 Biaxial fatigue test results

Table 2: Biaxial Fatigue Test for simulated HAZ.

<table>
<thead>
<tr>
<th>No</th>
<th>Rx</th>
<th>$\sigma_{x_{max}}$</th>
<th>Ry</th>
<th>$\sigma_{y_{max}}$</th>
<th>$\phi$</th>
<th>Nr</th>
<th>Note</th>
</tr>
</thead>
<tbody>
<tr>
<td>01</td>
<td>0.1</td>
<td>70MPA</td>
<td>0.1</td>
<td>70MPA</td>
<td>0°</td>
<td>1540</td>
<td>R</td>
</tr>
<tr>
<td>02</td>
<td>0.1</td>
<td>60MPA</td>
<td>0.1</td>
<td>60MPA</td>
<td>0°</td>
<td>3300</td>
<td>R</td>
</tr>
<tr>
<td>03</td>
<td>0.3</td>
<td>60MPA</td>
<td>0.3</td>
<td>60MPA</td>
<td>0°</td>
<td>9237</td>
<td>R</td>
</tr>
<tr>
<td>04</td>
<td>0.3</td>
<td>50MPA</td>
<td>0.3</td>
<td>50MPA</td>
<td>0°</td>
<td>1141</td>
<td>R</td>
</tr>
<tr>
<td>05</td>
<td>0.1</td>
<td>50MPA</td>
<td>0.3</td>
<td>50MPA</td>
<td>0°</td>
<td>4165</td>
<td>R</td>
</tr>
<tr>
<td>06</td>
<td>0.1</td>
<td>70MPA</td>
<td>0.5</td>
<td>70MPA</td>
<td>45°</td>
<td>3484</td>
<td>R</td>
</tr>
<tr>
<td>07</td>
<td>0.3</td>
<td>70MPA</td>
<td>0.3</td>
<td>70MPA</td>
<td>30°</td>
<td>4928</td>
<td>R</td>
</tr>
<tr>
<td>08</td>
<td>0.1</td>
<td>60MPA</td>
<td>0.3</td>
<td>60MPA</td>
<td>45°</td>
<td>4308</td>
<td>R</td>
</tr>
<tr>
<td>09</td>
<td>0.1</td>
<td>60MPA</td>
<td>0.3</td>
<td>50MPA</td>
<td>90°</td>
<td>4467</td>
<td>R</td>
</tr>
<tr>
<td>10</td>
<td>0.135</td>
<td>57MPA</td>
<td>0.135</td>
<td>57MPA</td>
<td>60°</td>
<td>7650</td>
<td>R</td>
</tr>
</tbody>
</table>

3 Application of the existing criteria

3.1 Simple recall of some criteria

There are several families of fatigue criteria: (1) Critical Plane Criteria or Critical Plane Approach (2) Global Criteria (3) Energy Criteria etc. Only some critical plane criteria are reviewed here, because the existing criteria tested in this paper are all of this type. Such criteria calculate the path of shearing stress in the cyclical load for many planes of different directions, then on each plane the parameters of shear stress amplitude and average values are defined to calculate the damage value, at last find the plane which gives the most high damage value as the critical one. In this way, critical plane criteria give out both the life assessment and the direction of fatigue crack nucleation. The figure 2 shows definition of the amplitude and average values of shear stress on the plane in a general case; $\sigma_{nn_a}$, $\sigma_{nmm}$, $\sigma_{nmm_{max}}$ represent the normal stress on the plane in question: Amplitude, Average value, Maximal value respectively. By choosing some of these items, different critical plane criterions gives out different formula to evaluate the damage caused by multiaxial loading in the sense of equivalent stress.

The Dang Van criterion [1][4] is one of the most used criteria in French automotive industry. Dang Van thinks the instant amplitude of the shear stress on the plane in question is essential for the nucleation of fatigue cracks, and the hydraulic stress $P_H(t)$ effects the opening and closing of the cracks. So his formula for damage calculation is:
where

\[ P_H(t) = \frac{\sigma_{11}(t) + \sigma_{22}(t) + \sigma_{33}(t)}{3} \]

\[ \alpha = 3 \left( \frac{\tau_{-1}}{\sigma_{-1}} - \frac{1}{2} \right) \]

\[ \beta = \tau_{-1} \]  

The criterion of McDiarmid [8][9] takes the normal average stress and normal amplitude stress into account respectively:

\[ D_{MD2} = \frac{(1 - \frac{2\sigma_{nm}}{R_n})\tau_{na} + \alpha(\sigma_{nna})^{3/2}}{\beta} \]  

where

\[ \alpha = \frac{\tau_{-1}}{\sigma_{-1}} - \frac{1}{2} \left( \frac{\sigma_{-1}}{2} \right)^{3/2} \]

\[ \beta = \tau_{-1} \]  

The criterion of Findley takes the critical plane as the plane where the damage value is the largest.

\[ D_{FD} = \frac{\tau_{na} + \alpha\sigma_{nm\max}}{\beta} \]  

While in the Matake criterion, the author takes the plane where the amplitude of shear stress is maximal as the critical one.

\[ D_{MT} = \frac{\tau_{na} + \alpha\sigma_{nm\max}}{\beta} \]  

For this two criteria the coefficients are the same:

\[ \alpha = 2 \frac{\tau_{-1}}{\sigma_{-1}} - 1 \]

\[ \beta = \tau_{-1} \]  

\[ D_n = \max_i \left\{ \frac{\tau_{na}(t) + \alpha P_H(t)}{\beta} \right\} \]

\[ D_{DV1} = \max(D_n) \]
As in our uniaxial test results we don't have the results of alternative torsion fatigue and tensile compression fatigue, so the SN curves of $R = 0.1, 0.3$ and $R = 0.5$ are used to determine the coefficients in criteria instead of $\tau_{-1}$ and $\sigma_{-1}$.

4 Biaxial life assessments and conclusions

The Total life approach is used here for life assessment, i.e. the stages of initiation and propagation are not separated here.

4.1 Algorithm of life assessment

1. Calculation of the stress path during the cyclical load, $\sigma_y(t)$.
2. Calculation of the terms of stresses normal to plane in question and the share stress on the plane using an interval as $10^\circ$ for both $\phi$ and $\gamma$ in a hemisphere (figure 1).
3. Using the smallest circle which include the path of the shear stress on the plane in question, terms like $\tau_{na}$, $\tau_{nm}$, $\tau_{na}(t)$, $\sigma_{na}$ and $\sigma_{nm}$ etc. are determined for the criteria.
4. Between all the planes in question, the so-called critical plane is determined using the terms in step 3. For example, the plane where $\tau_{na}$ is maximal is considered as one critical plane according to the criterion of Matake.
5. The coefficients $(\alpha(Nr), \beta(Nr)$ etc.) in the criteria are determined as functions of the number of cycles to rupture using uniaxial test results.
6. The life assessment is the number of cycles which gives out the coefficients so that $D = 1$ on the critical plane. An iterative method is used to determine the fatigue life.

4.2 Life assessments and conclusions

Life assessments are normalized using the proportions between the assessments and test results, and the results of comparison are shown in figure 4. Let $P$ to be the ratios between life assessment and test result, i.e.:

$$P_0 = \frac{\text{life assessment}}{\text{test result}}$$

if $P_0 < 1$, $P = \frac{1}{P_0}$, $P_0 > 1$, $P = P_0$

4.2.1 Comparison between the four criteria

In the figure 4, the loads are separated into two groups: In-phase (black points) and Out-of-phase. It is clear that all the four criteria work much better when the loading is in-phase. For the in-phase loads we have: Dang Van: $3.0 \sim 7.0$ with $P_0 < 1$ in all cases; Findley: $3.48 \sim 15.5$ with $P_0 > 1$ in all cases; Matake: $3.48 \sim 15.7$ with $P_0 > 1$ in all cases; McDiarmid: $3.93 \sim 5.48$ with $P_0 > 1$ in all cases. However for the out-of-phase loads, the predictions are very different form the test results: Dang Van: $5.0 \sim 19.0$ with $P_0 < 1$ in all cases; Findley: $1.24 \sim 64.7$
with $P_0 < 1$ in most cases; Matake: $1,52 \sim 50.7$ with $P_0 < 1$ in most cases; McDiarmid: $1.38 \sim 300$ with $P_0 < 1$ in most cases. So the three criteria Findley, Matake, McDiarmid give out non-conservative predictions for in-phase loads and conservative predictions for out-of-phase loads. Meanwhile the criterion of Dang Van always gives out conservative life assessments. So, the criterion of McDiarmid works the best among them for in-phase loads. The effects of $\sigma_{nnm}$ and $\sigma_{nna}$ are different from each other on fatigue crack propagation. Only the criterion of McDiarmid treats them differently in its formula. However, it seems to be the worst one for out-of-phase loads. And all the four criteria give out too non-conservative predictions for out-of-phase loads. The criterion of Dang Van seems to be the best one for out-of-phase loads, and its predictions for in-phase loads are better than those of the criteria of Findley and Matake. The criterion of Dang Van seems to be the best criterion, if one wants to choose a criterion for both in-phase and out-of-phase loads on structures made in this alloy.

![Figure 4: Comparison of life assessments using different criteria.](image)

### 4.2.3 Critical directions

No significant difference between the four criteria is found in this aspect. The critical directions of cracks at the center of specimen are studied. Both the angles of $\phi$ and $\gamma$ are measured and compared to computed values. For each one of the four criteria, about 83.33% of the predictions $\gamma$ do not differ from the tests more than $10^\circ$, while, for $\phi$, only 50%.
5 Conclusions and perspectives

This study has made it possible to compare the four criteria on the prediction of fatigue life and critical directions of fatigue crack on the aluminum alloy based on the biaxial test results. (1) Among the four criteria studied, the two criteria of Dang Van and McDiarmid work well for our aluminum alloy as life assessment is concerned. (2) Regarding the critical directions fatigue crack, no preference among them is found. (3) Modifications can probably used to make better predictions for those criteria, especially when the loads are out-of-phase. (4) Tests on welded cruciform structures will be carried out to verify the criteria with residual stress taken into account.

References


