

Hybrid Petri nets as a new formalism for modelling railway systems

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Abstract

We suggest a new approach to railway systems modelling based on hybrid Petri nets. The background for this approach is the interpretation of railway systems as hybrid (continuous-discrete) systems. Our definition of hybrid nets is based on Coloured Petri Nets, which allows a smooth integration into existing discrete models as well as a straightforward approximation for the purpose of simulation. After outlining the considerations that lead to the definition of this particular formalism, we explore some of the implications with respect to modelling, simulation and analysis of railway systems.

1 Introduction

The purpose of this paper is to suggest the use of hybrid Petri nets in the area of railway systems modelling. Considering the current practice in this field, we assume that the introduction of this formalism may lead to improvements in terms of integrated modelling, simulation, and verification.

The general background of this approach is the study of hybrid systems, defined as systems possessing both continuous and discrete aspects. There is a growing interest in this relatively young area of research, since many real-world systems exhibit hybrid characteristics and there is a need to model these systems adequately, which is not possible in purely continuous or discrete terms. Railway systems

can be considered as hybrid systems in several respects. Their most prominent continuous and discrete aspects are the continuous train motion and its control by discrete signalling systems. On another, more abstract level, the flow of railway traffic can be considered continuous, with the infrastructure and control strategies as the discrete counterpart. At a more detailed level there are hybrid physical subsystems within the vehicles and in their interaction with the track. An important example for the interplay between continuous and discrete processes is the study of modern moving-block train control, where the continuous nature of the train movement immediately affects the (discrete) control system.

2 Railway Systems Modelling

Investigations on any aspect of railway systems generally require some form of modelling. In order to structure the various different approaches, we classify the models according to their level of abstraction.

We distinguish three levels: The top level is called the level of traffic flows. Here the traffic is viewed in analogy to continuous media [10]. At the next more detailed level, called the level of transport processes, the movements of the individual vehicles are considered. Railway simulators typically operate at this level, since it allows the detailed modelling of schedules. More specifically, the interaction between different trains and the influence of the train control system is studied at this level. The disadvantage here lies in the fact that the detailed simulation of large infrastructures can be too complex, such that for questions such as the strategic development of lines traffic flow models are resorted to. The lowest level considered here is that of components, where (technical) subsystems of vehicles and control systems are modelled.

In Figure 1, we added two surrounding levels at the top and bottom: At the level of traffic demand, the railway traffic as a whole is viewed as a black box, whereas the level of physical systems deals with general physical principles not specific to railway systems. Because of their generality, these two levels of abstraction are not considered in detail here.

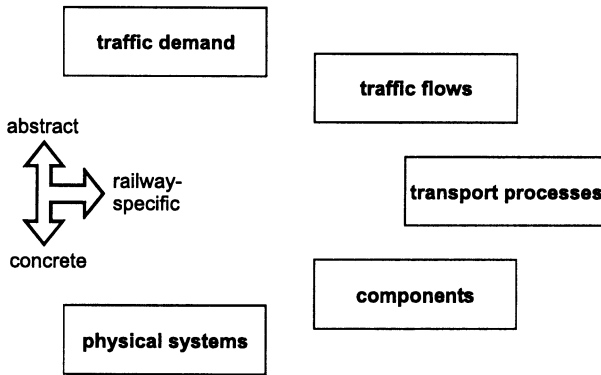


Figure 1: Levels of railway modelling.

In the following, we will focus on the level of transport processes and its main components, i.e. the actual operating processes and the control systems which interact with them. The control systems are the typical objects of formal analysis, since they have to guarantee the safety of the operating processes [3, 14]. The principal subjects of models of the operating processes are the train dynamics and the topology on which the transport processes take place.

Since we are considering railway systems as hybrid systems, we classify the formalisms and languages used at the different levels as being either continuous or discrete. Continuous modelling is used primarily in models of physical aspects of components and models of the traffic flow. The models at the intermediate level are mostly discrete, often including a discretized form of a continuous description of the train motion.

Discrete formalisms which are used for modelling railway systems include graphs, Petri nets, programming languages and logic-based formal methods. Graphs are typically used to model the track topology, where track sections correspond to edges and vertices represent specific points such as stations or switches. Similarly, Petri nets, which can be characterized as bipartite graphs with a dynamic marking [12], are used to model the topology [7]. They can, on the other hand, also be employed to model discrete processes in general, including train control systems [11]. Programming languages are important in order to build simulators, and formal methods are used for safety-relevant problems requiring verification, such as the design of interlocking software.

On the continuous side, there are primarily differential equations as a general means to describe the dynamics of physical processes, as well as traffic theory, which forms the basis for models of traffic flows.

3 Definition of the formalism

We chose Petri nets as the starting point for the development of our hybrid formalism since they seem to offer the broadest scope with respect to discrete modelling, uniting the abilities of programming languages in a formal means of description on the basis of graphs. This is underlined by the fact that different styles of discrete Petri Nets are already being used successfully in this application area [7, 11, 13]. The integration of continuous aspects, yielding hybrid Petri nets, would then lead to a formalism that in principle covers the capabilities of all present ones.

There are several approaches to describe hybrid systems on the basis of Petri nets. Apart from modelling the continuous subsystems with a different formalism and interfacing it with the (purely discrete) Petri net (e.g. [1, 2]), there are two general possibilities to introduce continuous processes into the nets themselves [6]: This can be done either on the basis of Place/Transition nets with anonymous tokens which "flow" through the net [4, 5, 8] or based on high-level nets [15], where real-valued attributes of complex tokens are manipulated continuously by special transitions. In order to be able to exploit the modelling capabilities of (discrete) high-level Petri nets, we prefer the latter approach, since the complexity of the various systems under consideration presents a problem in the absence of distinguishable tokens. This is supported by the experience gained at our institute in the modelling of a modern train control system using Coloured Petri Nets (i.e. high-level nets) [11].

In the standard design of railway simulators, the (continuous) differential equations are typically solved by stepwise integration, where the steps are made in terms of velocity, location, or, most often, as fixed time steps. Starting from the notion that the behaviour of a Petri net is discrete as determined by the occurrence of transitions, a similar stepwise approach is taken here in order to translate the (continuous) net design into an ordinary high-level Petri net.

Using real-valued attributes, it is not necessary to introduce a

new type of continuous places. Instead, we define continuous transitions as the limit case of a discrete approximation using Coloured Petri Nets (CPN) with time (Figure 2), where the notion of time is employed in order to manage the integration steps. Both **delta** and **rate** are ordinary places which can be arbitrarily accessed by other transitions, thus creating any trajectory of the value $x.a$ as well as giving each transition an individual step size, if needed for each token separately. The hybrid net itself is then defined on the basis of this emulation by assuming infinitely small step sizes.

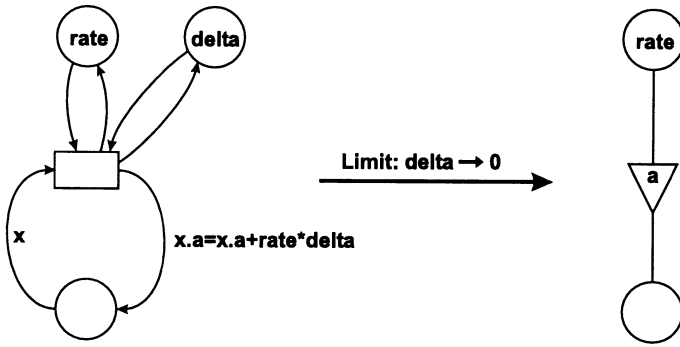


Figure 2: Definition of hybrid Petri nets.

This idea can serve as the pattern for a simple implementation of a simulator on the basis of conventional Coloured Petri Nets. It also allows a smooth integration with discrete nets, such as in adding continuous aspects to an existing discrete model. For our simulations, we used the tool Design/CPN as the discrete basis.

For a more formal presentation, we adopt the definitions of CPN given by Kurt Jensen in [12] p. 70, keeping the non-standard features to a minimum. To briefly summarize the definitions, a (non-hierarchical) CP-net is a tuple $CPN = (\Sigma, P, T, A, N, C, G, E, I)$, where the elements of the tuple are the set of colour sets, places, transitions, and arcs plus the node, colour, guard, arc expression and initialisation functions, respectively. We only add a finite set Δ of continuous transitions, and extend the definitions such that the union of discrete and continuous transitions ($T \cup \Delta$) takes the place of T (where $T \cap \Delta = \emptyset$).

For well-formedness of hybrid nets, we introduce some syntactic restrictions: We demand that each continuous transition has exactly

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one input place and one output place. Next, the type of the arc expression on an input arc to a continuous transition δ must be the set of real numbers (this is the rate of change effected by δ), and the expression on the corresponding output arc specifies a (real-valued) attribute of the place, which is the attribute to be manipulated by the transition (called the goal of δ).

The definition of the behaviour of CP-nets also carries over to our style of hybrid Petri nets. Again, we omit the standard definitions here and refer the reader to [12] (pp. 73-78). Just as for regular transitions, a binding $b(\delta)$ of a continuous transition δ maps each variable of δ to a value, respecting the type of the variable as well as the guard of the transition. A continuous binding element is a pair $(\delta, b(\delta))$, and a continuous step is a multiset of continuous binding elements. A discrete step is enabled if there are enough tokens on the input places for all transitions (i.e. binding elements) to fire concurrently. For continuous transitions this is slightly different, since they may share a common rate and also manipulate a particular token together. Hence, enabling a continuous step depends on the individual binding elements and not on their sum; the continuous bindings are enabled independently of each other. Whenever a continuous binding element is enabled, it becomes active, i.e. it fires continuously, changing the value of the goal attribute as indicated by the rate.

4 Application

Considering again the level of transport processes, the hybrid nature of railway systems becomes visible within the models of the operating processes (as opposed to the control system). The dynamics of vehicle motion are inherently continuous, hence we suggest the use of continuous transitions here. The track topology is quite naturally modelled with graphs, such that the use of Petri nets as graphs is again rather straightforward, where the repetitive nature of the infrastructure calls for some sort of building blocks [7]. It is the combination of these different aspects where problems arise. This is due to the fact that on one hand the infrastructure is quite extended, but on the other hand the processes taking place on it are similar across the entire topology. This is true for the train control system, but of course also for the laws of train motion.

This is mirrored by the fact that Petri net models of railway sys-

tems are either topological or logical, in other words track-oriented [7] or process-oriented [11]. Topological models allow animated simulation as well as a graph theoretic analysis, the disadvantages are their size and the redundancy of structures repeated all over the net. Process-oriented models contain a "collapsed" representation of the topology as tokens, leading to more compact, less repetitive models, but the absence of a net representation of the topology implies that it is inaccessible for Petri net analysis as well as for the human reader.

In order to solve the problem of integrating topological and process-oriented models, we use fusion sets, which belong to the standard repertoire of Petri net theory ([12], pp. 97ff.). A fusion set is a collection of places; all places in a fusion set behave as if they were just one place, which implies that they carry the same marking at any given moment. Conceptually, the introduction of fusion sets does not change the modelling power of Petri nets since any net with fusion sets can be converted into a net without fusion sets by merging the places that belong to the same set into one place and redrawing their arcs correspondingly. Using this approach, the model of a simple topology (without switches) plus continuous train motion can be built as shown in Figure 3.

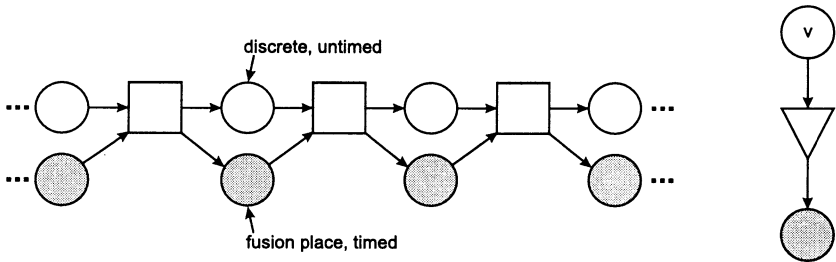


Figure 3: Hybrid topology.

The shaded places form one fusion set connecting the net of the topology to the net which handles the continuous train motion. The clear places are discrete in the sense that they just record the presence of a train in a particular section, whereas the exact position is kept in the fusion places. This approach can be generalized in the sense that all relevant processes are modelled just once, leading to a model with no unnecessary redundancy and a clear separation of concerns.

In order to simulate the hybrid Petri nets, we can recur to the initial idea underlying their definition, in other words taking the step back from the right hand side of Figure 2 to the standard Coloured Petri Net on the left. As an example, Figure 4 shows the implementation of a part of the net in Figure 3 using the tool Design/CPN. The integration steps are realized using the time concept of Design/CPN which is based on time stamps. The accuracy of the approximation can be adapted to the particular problem under consideration, where the step sizes can be different for individual vehicles as well as track sections.

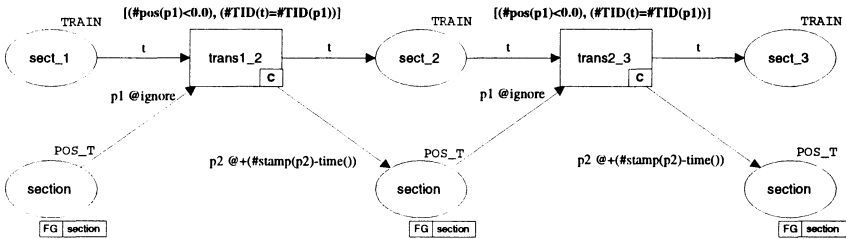


Figure 4: Simulation of the hybrid topology.

5 Summary

We are suggesting a flavor of hybrid Petri Nets for the modelling of railway systems which is based on an approximation using high-level Petri nets with time. This entails a straightforward semantics for the hybrid Petri nets which can at the same time be used to simulate the nets with standard (discrete) Petri net tools.

One benefit of this approach is that the granularity employed in the simulation can be controlled directly, such that it can be adapted to different purposes using the same model. The common features of high-level nets such as hierarchy and modularity are preserved. The approach is largely independent of the particular flavour of high-level nets or the tool with which they are implemented. A prototypical implementation was built using Design/CPN.

Due to space constraints, we were only able to give a brief overview of the formalism and its application. Another important aspect which has not been treated here is the analysis of hybrid nets, using invariants [9] and evolution graphs. The limits imposed by the com-

putability with respect to hybrid systems in general leads to considerable problems for the verification, which have to be addressed using domain-specific knowledge. This is one of the main subjects for future research.

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