Statistical and probabilistic methods applied to dc traction system supply
S. Bertini*, M. Fracchia*, M. Garbero*, A. Mariscotti* & L. Pierratb
*aCentro Ricerca Trasporti, Via Opera Pia, 11a, 16145 Genova, Italy
*bINPG-IMG-LEGI, BP 53, 38041 Grenoble Cédex 9, France

Abstract

Considering a DC traction system connected to the medium voltage AC supply network by means of Electric Supply Substations (ESSs) equipped with power static converters, it is worth in general to adopt statistical analysis for both interfaces to power distribution (AC supply and DC distribution). The structure of the system and the values of the parameters are variable (random variables), with a specific range of values and a probabilistic distribution. Attention is focused onto two problems: the power quality (low frequency conducted harmonics and resonance issues) of the AC supply interface and the DC distribution interface. Some results are obtained by means of statistical and probabilistic approaches. Software tools and code used to this aim are described.

1. Introduction

The reference DC traction system can be represented as shown in Fig. 1. The Electrical Sub Station (ESS) usually includes two or three AC/DC conversion groups (rated 1.5MW for metrorail and 3.6-4.5 MW for rail

Figure 1: Reference DC traction system
way systems), 12-pulse configured with or without interphase reactor. The DC side structure of the system and the parameter values are variable due to the number of trains and the operating conditions (speed, torque, power demand, drive configurations, etc.). The authors consider the interface power quality (low frequency conducted harmonics and resonances) of the AC supply (from a statistical point of view) and of the DC distribution (from a probabilistic point of view).

2. AC interface statistical analysis

A set of measurements performed on a metrorail supply system (MetroGenova) are analysed to evaluate the harmonic distortion at the ESS and at the PCC produced by the ESSs rectifier groups Fracchia [1].

2.1 Metrorail system and measurement set description

The urban MetroGenova transport system includes only 3 stations and 2 main ESSs, located in PRINCIPE and BRIN respectively. The intermediate Via Venezia ESS feeds only the auxiliary services in the Di Negro station (the system configuration is detailed in Fracchia[1]). The two main ESSs are equipped with three 12-pulse rectifier groups (2 out of 3 for normal operation). The ESS is supplied from the national supply grid, at one single point in normal operating conditions. The main electrical features are reported in Table 1.

| Transformer (two secondary three phase windings) | Rated Power | 1.5 MVA |
| Converter | Rated primary voltage | 15 kV |
| | Rated sec. no load voltage | 590 V |
| | Short circuit reactances | $X_{12}=X_{13}=8\%, \ X_{23}=1\%$ |
| | Rated Power | 720 kW |
| | Interph. reactor inductance | 460 μH |
| | Rated Power | 300 kW |
| | Rated DC voltage | 750 V |

The headway is 5 min during rush hours and 10-15 min. for the rest of the day. The measurement records span over several working days, from 6am to 9pm. At present only two locomotives operates along the track, and the aim of these preliminary tests is to forecast future operations with several locomotives. Three main set of tests were performed all concerning the harmonic currents on the 15 kV AC side of the rectifier groups.
Test 1: two phase currents $I_R, I_S$ are recorded for about 9 operating hours. $I_R$ and $I_S$ currents are the input currents of a two secondary windings ($\Delta$ and $Y$ connected) transformer inside the ESS Principe. During this test only two of the three rectifier groups were operating and feeding the locomotives for all the working day (ESS Brin was not operating). The system is a three wire supply system without neutral connection.

Test 2: the two phase input currents $I_R, I_S$ of the ESS Brin main bus bar were recorded for about 9 hours. Two rectifier groups were operating, located in ESS Brin and ESS Principe.

Test 3: two phase output currents ($I_R, I_S$) of the point of common coupling (PCC) (derived from the public supply network transformation cabin) were recorded. Measurement conditions are the same as in Test 2.

2.2 Data statistical analysis and representation

The recorded data have been represented statistically by means of normalised amplitude histograms and scatter plots Morrison[2]. In the following only Test 1 results are presented and analysed to explain the statistical methods. The recorded data are represented in terms of fundamental and characteristic harmonics components ($h=11$ to $h=23$) of the phase current $I_R$ (see Fig.2a,b,c). The corresponding scatter plots including also the $I_S$ and $I_T$ vectorial components are depicted in Fig.3a,b,c.

On the scatter plots the following consideration may be made: the range of variation of the phase angle is very large, practically 360° degree starting from the first characteristic harmonic. This phase behaviour can be easily verified applying the following relationship Kimbark[3]:

$$\phi_h = \frac{\angle -(h+1)\alpha - \angle -(h+1)(\alpha + \mu)}{h+1} - \frac{\angle -(h-1)\alpha - \angle -(h-1)(\alpha + \mu)}{h-1}$$

(1)

where

- $h$ is the harmonic order
- $\alpha$ is the firing delay angle ($\alpha=0$ here)
- $\mu$ is the overlap angle (variable from 0 to 30 degrees)

The amplitude histogram of the first characteristic harmonics follow strictly the fundamental component behaviour while the 23rd and 25th harmonic behaviour is more influenced by the measurement errors. The statistical analysis points out that the harmonic probabilistic model of the input current of the AC/DC conversion group in the ESS should be based mainly on the current amplitude probability density function (pdf); while above the 13th order harmonic (750 Hz) the phase angle cannot give any additional information and could not be suitably modelled. Analysing the
harmonic current amplitude histograms it has been noted that the real operating conditions lead to a large amount of "quasi zero current values", affected by noise and measure errors, which have to be separated by the harmonic currents related to the traction load. In fact, the expansion of the amplitude histograms represented in Figure 2 points out an underlying bimodal distribution of the current amplitudes. A first approach is to identify a threshold value to separate the current absorbed by the ESS auxiliary services from the current absorbed by the loco drives. Even small changes in the threshold level produce large variation of the statistical indexes (mean and standard deviation) of the amplitude and phase variables. The revised data according to the minimum value of the required traction current are represented in Fig. 4, as far as the 11th (characteristic) harmonic is concerned.

Figure 2: Amplitude histograms (a) fundamental, (b) 11th and (c) 23rd harmonic

Figure 3: Scatter plots (a) fundamental, (b) 11th and (c) 23rd harmonic

Figure 4: truncated harmonic amplitude histogram (a), scatter plot (b)
The statistical analysis and the recorded data preliminary elaborations have been made using the toolbox Statistics included in Matlab 5.1 and by a special program as described in section 4.

3. DC interface probabilistic analysis

The reference system is 3 kV DC electrified railway, consisting of a single track line, fed at both ends by two rectifier ESSs and a single locomotive (Fig. 1). The per-unit-length parameters reported in Table 2 refer to a two conductor representation of the traction line, which may slightly differ from the more accurate Multiconductor Transmission Line (MTL) model Fracchia[4].

Table 2 Electrical parameters per unit length

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$l = L/L$</td>
<td>1.37 $\mu$H/m</td>
</tr>
<tr>
<td>$r = R/L$</td>
<td>0.5 $\Omega/m$</td>
</tr>
<tr>
<td>$c = C/L$</td>
<td>20 $pF/m$</td>
</tr>
</tbody>
</table>

Other important line parameters may be derived (the characteristic impedance, the propagation constant and the speed wave respectively):

$$Z_0 = \frac{1}{\sqrt{r + j\omega l}}$$  
$$\gamma = \sqrt{(r + j\omega l) / j\omega c}$$  
$$v_0 = 1 / \sqrt{l c}$$  

(2,3,4)

The line impedance $Z$ at the pantograph terminals is

$$Z = Z_c \frac{\tanh(\gamma x L)\tanh[\gamma (1 - x) L]}{\tanh(\gamma x L) + \tanh[\gamma (1 - x) L]}$$  

(5)

where the parameters are reported in Table 2. The amplitude of the reduced impedance, as indicated by (5), is plotted in Fig. 5.

Figure 5: $|Z_{hp}|_{\text{max}} / |Z_{hp}|$ vs. freq. and loco position
The resonance and antiresonance frequencies are obtained as the poles and zeros of (5):

$$\omega_r(k, L) = \frac{k\pi\nu_0}{L}$$

(6)

independent on the locomotive position $x$ and

$$\omega_a(k, x, L) = \frac{k\pi\nu_0}{x} \quad \text{and} \quad \frac{k\pi\nu_0}{(L - x)}$$

(7)

dependent on the locomotive position $x$, following an hyperbolic law.

At these frequencies the value of $Z$ is

$$Z_r = Z(\omega_r) = \frac{Z_c}{2} \frac{1 - \cos(2k\pi x / L)}{\alpha L}$$

(8)

$$Z_a = Z(\omega_a) = \begin{cases} Z_c\alpha x \\ Z_c\alpha(L - x) \end{cases}$$

(9)

3.1 Probabilistic analysis of parameter variability

The main line parameters (the characteristic impedance and the impedance at the first resonance and antiresonance frequencies) are evaluated in the following as a function of the electrical parameters ($r$, $l$ and $c$) considered as random variables Pierrat[5]. The values reported in Table 2 are only average values and a suitable variation interval shall be considered to check for the validity of the above formulated expressions. The electrical line parameters are assumed to be uniformly distributed over a ±10 % interval around the average values of Table 1 Fracchia[6].

The amplitude of $Z_c$ is evaluated over the entire frequency range in Fig. 6; the scatter plots of real and imaginary parts of $Z_c$ are shown in Fig. 7 for four different frequencies (10, 30, 100 and 1000 Hz).
The dispersion of $\text{Im}(Z_c)$ is negligible above nearly 100 Hz and $\text{Im}(Z_c)$ itself may be considered zero above nearly 1000 Hz. $Z_c$ is high value and reactive at lower frequencies. The distribution of the values of the amplitude of $Z$ at the resonance and antiresonance frequencies is shown in Fig. 8 for uniformly distributed line electrical parameters.

![Figure 8 Amplitude of Z at the resonance ($Z_r$) and antiresonance ($Z_a$)](image)

The antiresonance frequency is quite sensible to changes of the electrical parameters of the line and, in particular, observing (7) and (8), it can be desumed that the proportionality is nearly hyperbolic. The resonance frequency is less disperse around its average value, yet the value $Z_r$ is quite influenced by changes in all the line parameters (8).

### 3.2 Evaluation of line voltage

A more general formulation for the evaluation of line voltage waves is presented; the case of two locomotives on the line section between two ESSs is considered and the behaviour of the line voltage wave is evaluated as a function of locomotives position. Here, it is considered a traction line configuration with a $\ell=20\text{km}$ line between the two ESSs (see preceding sections) and two locomotives at $x_1$ and $x_2$ positions.

$$0 < x_1 < x_2 < \ell$$

(10)

Each locomotive is considered as a current source feeding the line impedance $Z_c$ as seen at the pantograph terminals.

For the locomotive 1, given the origin in $x=x_1$ and considering as positive direction of propagation the axis towards locomotive 2, the voltage wave is

$$v_1(x) = V_+ e^{-\gamma(x-x_1-x)} - V_- e^{\gamma(x_1-x)}$$

(11)

Resolving (11) for the two conditions
it is obtained

\[ n_1(x) = Z_1I_1 \frac{1 - e^{2\gamma(x-x_1)}}{1 - e^{2\gamma(x-x_1)}} e^{\gamma(x-x_1)} \]  

Analogous relationships may be found for the voltage wave of locomotive 2, given the origin in \( x=x_2 \) and considering as positive direction of propagation the axis towards locomotive 1.

\[ n_2(x) = V_+ e^{-\gamma(x-x_2+x)} - V_- e^{-\gamma(x-x_2+x)} \]  

Resolving (14) for the two conditions

\[ V_+ - V_- = Z_2I_2 \quad V_+ e^{-\gamma x_2} - V_- e^{-\gamma x_2} = 0 \]  

it is obtained

\[ n_2(x) = Z_2I_2 \frac{1 - e^{2\gamma x}}{1 - e^{2\gamma x_2}} e^{-\gamma(x-x_2)} \]  

The voltage along the line between the two locomotives may be expressed as the sum of the voltage waves (13) and (16).

\[ n(x) = n_1(x) + n_2(x) \]  

Eq. (17) is plotted in Fig. 13-17 for different locomotive positions over the frequency range [0–30 kHz], assuming a −20dB/decade locomotive current spectrum starting from 100 Hz (derived from measurements of the current absorbed by railway and metro vehicles during acceleration). This assumption is valid, as a tool to investigate line behaviour; slightly different voltage amplitude may be found if the real spectra (amplitude and phase) are considered.

Figure 9 Voltage wave \( n(x) \) over [0, 30 kHz] for \( x_1=0.1L \) \( x_2=0.9L \) 

Figure 10 Voltage wave \( n(x) \) over [0, 30 kHz] for \( x_1=0.3L \) \( x_2=0.7L \)
It can be noted that the line voltage amplitude at resonances is particularly high; this is in fact due to the assumption implicit in the locomotive current source model, that is the loco impedance is larger than the line impedance at pantograph terminals. Near the line resonances a voltage source and series impedance model should be used to improve these results.

Some comments follow on Figs 9-13. In Figs 9, 10, 11 the locomotives are positioned symmetrically with respect to the line ends, so the line voltage closely resembles the impedance plot of Fig. 5, in particular when the locomotive positions are close to the ends of the line. The locomotives in Figs 12, 13 are positioned unsymmetrically in the same line half with respect to the centre of the line. New resonances occur, contributed by the different zero conditions (at the end of each line portion) imposed on $v_1(x)$ and $v_2(x)$ voltage waves. All the line configurations must be accurately evaluated, as a function of real operating conditions and loco positions pdfs, in order to evaluate the relative importance of the above described phenomena.

4. Software tools

In the following, the software tools used and developed for the above described activities are briefly described. In section 2 the amplitude/phase
representation of harmonic components was obtained from the recorded waveforms using “convbin”, a Fortran 77 purposely developed executable. Statistical analysis was performed using Matlab 5.0™ and Statistical Toolbox™. In section 3 the expressions were evaluated using Matlab 5.0™; the sensitivity analysis was performed using random number generators.

Conclusions

Among DC traction systems the power quality analyses several aspects, in particular the low frequency disturbances conducted in the AC interfaces (supply system) and DC one (pantograph). Regarding the AC interface the main analysis difficulties are related to the circuital complex topology and the locomotive operating features. Regarding DC interface the main problems are related to the distributed characteristics of the line and to the locomotives random operating conditions. The AC interface analysis can be performed by means of the statistical measurements of the harmonic components and by using adequate representations. The DC interface analysis implies the deterministic and probabilistic modelling with the aim of forecasting the harmonic distortion levels on the catenary.

References