A study to estimate of effective coefficient of regenerative energy in electric railway

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Abstract

Regenerative vehicles are working even on less busy lines. The brake shoe will be heavily abraded. Therefore the resistor is mounted on the vehicles. It is desirable not to mount the resistor on the vehicle. Then, to mount or not should be decided with clever judgment.

Therefore, we studied the effective coefficient of regenerative energy which provides the criteria for judgment. We looked for an estimation formula of effective coefficient of regenerative energy. We proposed a technique of estimating manually and by a simple program based on that formula and confirmed that the calculation can be precise enough to be practically employed.

1 Preface

Recently, aged vehicles are being replaced with regenerative vehicles (vehicles that are equipped with regenerative braking system), which are being introduced even for infrequently used service lines. Not many trains are operated, which means that the regenerative energy is ineffective. Accordingly, excessive brake shoe wear must be considered especially. To avoid such problems, additional brake resistors are installed in order to absorb the excessive regenerated energy.

Of course, it is better not to install additional brake resistors which result in heavier vehicle weight. However, this fact may change according
to the actual service system and the track condition. The amount of energy which can not be absorbed by powering vehicles of other trains (herein after called "regenerative loss") is the key to decide whether or not the brake resistors should be installed. The amount of energy which can be regenerated is determined by the operation characteristics, the train performance and the track profiles. If the effective coefficient of regenerative energy (absorbed energy/potential regenerative energy) is calculated, the absorbed energy and the regenerative loss can be obtained. Accordingly, the effective coefficient of regenerative energy gives the index for the installment of the brake resistors.

At the beginning, the estimation formula for the effective coefficient of regenerative energy was obtained. Based on this formula and with simplified calculation or simplified program, the estimation method was proposed. This method was confirmed not only for the actual railway line application but also for other lines. In this report the estimation formula, the prediction method and prediction results are discussed.

2 Evaluation of Simulation Program

To establish the estimation method for the effective coefficient of regenerative energy, various cases including the number of trains, the operation diagrams and the electric facility conditions should be considered with the various data and their analyses.

Those can be easily changed with the simulation method and a number of data can be obtained. Therefore, this type of study can only be made with the simulation analysis.(2,3)

First the actual running test was conducted. Then, the simulation results were compared with the test results for improving the accuracy of the simulation analysis. The test was performed on February 16,(Friday), 1996, between Ohtsuki and Shiojiri on JR East, Chuo Line, East Part, by using Super Azusa train(4), (3M, 6M). During such test the powering energy consumption, the regenerative energy and the regenerative loss were measured.

The comparison between the test results and the simulation results for the confirmation is shown in Table-1. The effective coefficients of regenerative energy basically show similar tendencies, even though the number of trains is changed in this simulation. Therefore, it is considered acceptable that this discussion can be developed with the effective coefficient obtained by the simulation with the conditions applied for the vehicle, the operation and the electric facilities.
Table-1. Comparison between the measured result and the simulation result

<table>
<thead>
<tr>
<th>Train</th>
<th>Energy for powering (kWh)</th>
<th>Regenerative energy (kWh)</th>
<th>Regenerative loss (kWh)</th>
<th>Energy for powering (kWh)</th>
<th>Regenerative energy (kWh)</th>
<th>Regenerative loss (kWh)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 M</td>
<td>2685</td>
<td>453</td>
<td>263</td>
<td>2758(1.03)</td>
<td>477(1.05)</td>
<td>253(0.96)</td>
</tr>
<tr>
<td>6 M</td>
<td>2178</td>
<td>645</td>
<td>382</td>
<td>2238(1.03)</td>
<td>632(0.98)</td>
<td>379(0.99)</td>
</tr>
<tr>
<td>Total</td>
<td>4863</td>
<td>1098</td>
<td>645</td>
<td>4996(1.03)</td>
<td>1109(1.01)</td>
<td>632(0.98)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Train</th>
<th>Regenerative rate (%)</th>
<th>Effective coefficient of regenerative energy (%)</th>
<th>Potential regenerative energy (kWh)</th>
<th>Regenerative rate (%)</th>
<th>Effective coefficient of regenerative energy (%)</th>
<th>Potential regenerative energy (kWh)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 M</td>
<td>16.9</td>
<td>63.3</td>
<td>716</td>
<td>17.3</td>
<td>65.3</td>
<td>730(1.02)</td>
</tr>
<tr>
<td>6 M</td>
<td>29.6</td>
<td>62.8</td>
<td>1027</td>
<td>28.2</td>
<td>62.6</td>
<td>1011(0.98)</td>
</tr>
<tr>
<td>Average</td>
<td>22.6</td>
<td>63.0</td>
<td>1743</td>
<td>22.2</td>
<td>63.7</td>
<td>1741(1.00)</td>
</tr>
</tbody>
</table>

(Shiojiri~Ohtsuki)(Feb.16,1996)

3 Estimation of Effective Coefficient

3.1 Relation between Effective Coefficient and Energy Consumption of Other Trains

When the regenerative braking is applied by regenerative vehicles, the potential regenerative energy turns into the regenerative energy if it can be absorbed and used by other vehicles running near by. Therefore, it is considered that there is a certain relation between the effective coefficient and the energy consumption of other trains. For this investigation, the energy consumption of other trains which could be receptive and located within a certain distance from the regenerative vehicles was calculated during the simulation. And the potential regenerative energy through the operation from Ohtsuki to Shiojiri (134.3 km) was calculated. Then, the relation between the factor; the total energy consumption of all vehicles in operation divided by the potential regenerative energy, (herein after temporarily called “the regenerative energy ratio”), and the effective coefficient of regenerative energy was obtained.

Therefore, the relation between the consumption/regeneration energy ratio and the effective coefficient of the regenerative energy is considered to be the approximate curve having some range, but not to be the simple value of the effective coefficient, at the certain consumption/regeneration energy ratio. When the consumption/regeneration energy ratio is small,
the effective coefficient also becomes small, because the consuming energy is only for the auxiliary equipment. As the consumption/regeneration energy ratio increases, the effective coefficient becomes saturated to 100%. Therefore the approximate curve can be substituted with the saturated curve.

3.2 Estimation of Approximate Saturation Curve

The approximate saturation curve was estimated from eight (8) trains of Super Azusa now being operated. However, eight (8) trains were too few to obtain enough data, and therefore, data of eighty eight (88) trains were intentionally composed by the following manner. From the existing operation diagram, ten additional trains from each original operation were considered by the differences of the headway; plus and minus, 2.5 minutes, 5 minutes, 7.5 minutes, 10 minutes and 20 minutes; a total of eleven (11) cases. With these eighty eight (88) train diagrams, the consumption/regenerative energy ratio and the effective coefficient of regenerative energy was calculated by the simulation (this simulation value is herein after called "equivalent measured value"). Then, the factors 'a' and 'b' of the equation for the approximate curve (1) were obtained from the consumption/regeneration energy ratios and the effective coefficients of regenerative energy by means of the least square method.

\[ Y = 100 \times (1 - \exp(-a \times X - b)) \]  
Where, \( Y \): Effective coefficient of regenerative  
\( X \): Consumption/regeneration energy ratio  
\( a, b \): Factors obtained by least square method (-)

<table>
<thead>
<tr>
<th>Distance range for calculation of consumed energy (km)</th>
<th>a</th>
<th>b</th>
<th>Coefficient of correlation (-)</th>
<th>Average square error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.001094</td>
<td>0.676010</td>
<td>0.51</td>
<td>4.7</td>
</tr>
<tr>
<td>20</td>
<td>0.000669</td>
<td>0.630685</td>
<td>0.50</td>
<td>4.8</td>
</tr>
<tr>
<td>30</td>
<td>0.000513</td>
<td>0.595425</td>
<td>0.48</td>
<td>4.8</td>
</tr>
<tr>
<td>40</td>
<td>0.000405</td>
<td>0.586103</td>
<td>0.48</td>
<td>4.8</td>
</tr>
</tbody>
</table>

When the distance range for calculation of consumed energy is 10 km, the
effective coefficient of regenerative energy, the consumption/regeneration energy ratio and the approximate saturation curve are as shown in Figure-1.

There are 67% of events within the average square error (±4.7%) and 94% of events within twice of the average square error (±9.4%).

In this estimation, for the effective coefficient of regenerative energy on the approximate saturation curve, 72% of the estimated values are within ±5% of the equivalent measured values and 94% within ±10% of the equivalent measured values. Such accuracy is useful for determining whether or not the braking resistors should be installed.

![Approximate saturation curve with effective coefficient of regenerative energy](image)

**Fig. 1** Approximate saturation curve with effective coefficient of regenerative energy

### 4 Estimation by means of Simplified Calculation

#### 4.1 Method by means of Simplified Calculation

The consumed energy was estimated by a simplified calculation, and the estimation method for the effective coefficient of the regenerative energy which could be obtained from the approximate saturation curve in the above chapter was investigated. From the operation diagram, the consumed energy of other trains which are running within certain distance in front and rear directions is calculated. First, as shown in Figure-6, the lines of 10 km and 20 km from the said regenerative train are drawn on the operation diagram. Then, the total running distance of other trains running in the track sections is calculated and further multiplied by the energy consumption ratio, so that the energy consumption amount can be calculated from the equation (2).

\[ P = A \times I \times W \]  \hspace{1cm} (2)

Where, \( P \): Energy consumption of other trains located within a 10 km range from the regenerative train (kWh)
4.2 Estimation Results by means of Simplified Calculation

The effective coefficient of regenerative energy for eight(8) Super-Azusa
trains now being operated was estimated by the simplified calculation. The calculation result when the energy consumption rate of each vehicle is constant (5)(actual values) is as shown in Figure-3. The average square error is 4.2% and the maximum error is 7.7%, which means that those errors are larger than the result from using the energy consumption rates for each type of vehicle.

![Figure-3](image)

**Fig.-3 Manual estimation and its computerized calculation by simple program**

### 5 Estimation by means of Simple Computer Program

#### 5.1 Method by means of Simple computer Program

The simplified calculation discussed in the above chapter takes excessive time and manpower. Therefore, a calculation by using personal computer and simple program was investigated. For this purpose, the operation pattern of each train was simplified to three parts, powering, constant speed run and braking, as shown in Figure-4.

![Figure-4](image)

**Fig.-4 Pattern of constant speed operation**
First, the establishment of the pattern for constant speed operation is discussed. The location of the starting point for the constant speed run and the location of the starting point for the braking are determined from the distance between stations and the time for the operation. Then, the equation (4) is realized for the distance between stations and the operation time.

\[ L = (T - \left( \frac{1}{\alpha + 1} \right) \times V) \times \frac{1}{3.6} + V^2 \times \left( \frac{1}{\alpha + 1} \right) \times \frac{1}{7.2} \]  

(4)

Where, \( L \): Distance between stations (m)
\( T \): Operation time between stations (sec)
\( V \): Speed at constant speed run (km/h)
\( \alpha \): Acceleration (km/h/s)
\( \beta \): Deceleration (km/h/s)
3.6, 7.2: Conversion factors for units of distance and time

The equation (4) is the quadratic equation for speed \( V \), and the speed for the constant speed run can be obtained by solving the equation. The location of the starting point for the constant speed run is obtained by the equation (5). The location of the starting point for the braking is obtained by the equation (6).

\[ L_1 = \frac{V^2}{1/\alpha} \times \frac{1}{7.2} \]  

(5)

\[ L_2 = L - \frac{V^2}{1/\beta} \times \frac{1}{7.2} \]  

(6)

Where, \( L_1 \): Location of the starting point for the constant speed run (m)
\( L_2 \): Location of the starting point for the braking (m)

The general description of the simple program is explained by the flow chart, Figure-5, as follows.

![Flow Chart](image)

Fig. 5 The flow chart under simplified program
5.2 Estimation Results by means of Simplified computer Program

As performed for the simplified manual calculation, the effective coefficient of the regenerative energy was estimated by means of simplified computer program. The results of the manual calculation, when the energy consumption rate of each type of vehicle was considered, are as shown in Figure-7.

The results of the manual calculation when it was assumed that the energy consumption rate of all types of vehicles was constant are as shown in Figure-3. The average square error was 2.9% and the maximum error was 4.7%.

6 Estimation in Other Track Route

6.1 Approach in Other Track Route

It is necessary to estimate the effective coefficient of the regenerative energy in other track route. For this purpose, the energy consumption/regenerative energy ratio can be obtained by the simplified calculation, and that, the possible amount of the regenerative energy and the factors, ‘a’ and ‘b’ for the approximate saturated curve should also be calculated.

The factors ‘a’ and ‘b’ for the approximate saturated curve can be obtained by the following method.

The approximate saturated curve was investigated when the number of trains was changed. When the number of trains is decreased, the average energy consumption (average energy consumption per unit time and unit distance for regenerative train) is also decreased, the feeder voltage is increased, the effective coefficient of the regenerative energy is decreased, and the approximate saturated curve is moved downward. Thus, there is a close relationship between the average energy consumption and the factors, ‘a’ and ‘b’ of the approximate saturated curve. And the linear expressions for the factors ‘a’ and ‘b’ and the average energy consumption were obtained by means of the least square method, as shown in the equation (7) and (8), and Figure-6 and 7.

\[ a = -6 \times 10^{-6} \times Z + 0.0016 \] (7)
\[ b = 0.005 \times Z + 0.2586 \] (8)

Where, \( Z \): Average energy consumption (kW/km)

It is also possible that the average energy consumption in other track route is obtained by the simple calculation or the simple computer program. By substituting the average energy consumption into the equations, (7) and (8), the factors, ‘a’ and ‘b’ for the approximate saturated curve can be obtained.
Since the potential amount of the regenerative energy and the energy consumption of the regenerative vehicle within a certain range can be obtained by the simple calculation or the simple computer program, the effective coefficient of the regenerative energy can be estimated by substituting these values into the approximate saturated curve.

![Approximate equation for factor 'a' from the average energy consumption](image)

**Fig. 6** Approximate equation for factor ‘a’ from the average energy consumption

![Approximate equation for factor 'b' from the average energy consumption](image)

**Fig. 7** Approximate equation for factor ‘b’ from the average energy consumption

### 6.2 Estimation Results in Other Track Route

The train operation on the JR Chuo East line was modified so that other track route could be simulated for further study. That is, the estimation of the effective coefficient for the regenerative energy was also carried out with the method described above and the comparison was made with the results which were obtained by the precise simulation shown in chapter 2. In other words, the cases of non-operation of freight trains, local service
trains and super express trains (other than Super-Azusa) were investigated, as was the case when the track gradient is changed. The comparison of the effective coefficient of the regenerative energy for the eight (8) trains in the average value and the mean square error for the individual effective coefficient of the regenerative energy are as shown in Table-3. When all the local service trains are eliminated, the difference between the equivalent measured value and the estimated value is somewhat larger, but the mean square error is within 10% difference. And therefore, overall, the estimated effective coefficient for the regenerative energy on the other track route is reliable based on such error range.

Table-3. Computerized calculation by simple program assuming a different line

<table>
<thead>
<tr>
<th>Different condition to simulate other track route</th>
<th>Mean effective coefficient for regenerative energy (%)</th>
<th>Mean square error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>JR Chuo-East line (Present condition)</td>
<td>59.0</td>
<td>2.9</td>
</tr>
<tr>
<td>When freight trains are not operated</td>
<td>55.0</td>
<td>5.5</td>
</tr>
<tr>
<td>When local service trains are not operated</td>
<td>42.0</td>
<td>7.2</td>
</tr>
<tr>
<td>When super express trains except Super-Azusa are not operated</td>
<td>50.1</td>
<td>3.8</td>
</tr>
<tr>
<td>Gradient multiplied by 0.8</td>
<td>59.9</td>
<td>3.6</td>
</tr>
<tr>
<td>Gradient multiplied by 0.6</td>
<td>58.9</td>
<td>4.2</td>
</tr>
</tbody>
</table>

7 Conclusion

The following have been determined based on the estimation of the effective coefficient for the regenerative energy principally conducted by the simulation:

(1) There is a close relationship between the energy consumption/regenerative energy ratio and the effective coefficient of regenerative energy, which can be estimated with the saturated curve.

(2) When the effective coefficient of regenerative energy is estimated by simple calculation or simple computer program with the basis of the saturated curve, the mean square errors are 4% for the simple calculation and 3% or slightly smaller for the computer program. Such minor range of error means that these results can be used to determine whether or not the braking resistors should be installed.
(3) The factors for the approximate saturated curve in the simulation for other track route can be approximated by using the saturated curve, and the estimation results showed approximately 7%. Therefore, it is considered that this result can be used for the determination in other track route.

8 Reference

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(3) “Study on the output optimization of the control system for the large capacity electric locomotive” by Takeshi Sugimoto and Shinobu Yasukawa
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