A new method for repairing railway track irregularities using levelling and lining machines
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Abstract

We have developed a new method for repairing railway track irregularities with heavy mechanical machines. Levelling and lining operations in these machines are based on the so called 3-point measurement system with asymmetric chord spacing. To repair effectively track irregularities over the wide-range of wavelength, an absolute reference frame method is excellent but it requires the measurement of the actual track geometry before the repair work.

In our new method such an extra measurement is not required. Instead the waveform restored or reconstructed by the inverse operation from the usual versine data is used. But a restoration is only able to be carried out in the limited range of wavelength and so not equal to the actual track geometry. Therefore the effectiveness of the use of such a partially restored waveform on the repair work is investigated. In the application of the new method several data processing techniques are required. In the curved track it is necessary to extract the component due to the fundamental track geometry which is different from a track irregularity, from the measured versine data, as precisely as possible.

A new method is already successfully applied to the actual repair work.

1 Introduction

1.1 Principle of levelling and lining
In most of Japanese railway companies a repair work of railway track irregularities in level, cant and alignment is for a large part carried out with modern mechanical tamping machines. In these machines the levelling and lining are based on the so called 3-point measurement system with asymmetric chord spacing. The principle of levelling and lining is explained schematically in Figure 1. In the repair work the trailing point A is considered to rest on the already corrected track, \( x_n(\xi) \). Whereas the leading
point B is still lying on the uncorrected old track, \( x_o(\xi) \), where \( \xi \) is a distance measured along the track. Under these assumptions we have two operating methods in the repair work. They are called the relative reference frame method and the absolute reference frame one, respectively.

![Diagram of levelling and lining](image)

**Figure 1: Principle of levelling and lining (Relative Reference Frame method)**

### 1.2 Two operating methods in the repair work

**Relative reference frame method** In this method the machine is operated simply based on the built-in measurement system. In Figure 1 the track at working point C is raised or aligned so that it comes to lie on the straight line AB. The result is expressed as follows (Esveld[1]).

\[
x_n(\xi) = \beta x_n(\xi - a) + \alpha x_o(\xi + b)
\]

(1)

where \( \alpha = a/l \), \( \beta = b/l \), \( l = a + b \).

Taking the Fourier transform of the above equation, we get the following frequency response function which gives the effect of the repair work.

\[
H(\omega) = \frac{\alpha e^{j\omega b}}{1 - \beta e^{-j\omega a}}
\]

(2)

where \( \omega = 2\pi/\lambda \). \( \lambda \) is a wavelength.

Figure 2 gives the example of \( H(\omega) \) for the Plasser machine. As seen from its value, track irregularities in the range of wavelength of up to 30 or 40 m are theoretically corrected. But the components of longer wavelength cannot be very corrected.

![Magnitude \( |H(\omega)| \)](image)

**Figure 2: Magnitude \( |H(\omega)| \)** for \( a = 4.64m \), \( b = 11.55m \)

**Absolute reference frame method** This method is effective to correct track irregularities of longer wavelength. But it requires to measure the actual track geometry, that is, its absolute ordinates against some external
reference frame, before the repair work. On the practice of the repair work, as shown in Figure 3, the leading point B on the old track is virtually moved on the ideal track position calculated from the above actual track geometry data. Then the track at the working point C is raised or aligned so that it comes to lie on the relocated straight line AE, although in the curved track a quantity due to the fundamental track geometry must be still left at the working point C.

![Figure 3: Absolute Reference Frame method](image)

Figure 3: Absolute Reference Frame method

Figure 4 shows the typical examples of the power spectral density analysis of track irregularities before and after the repair work done by the usual relative and absolute reference frame methods.

![Figure 4: Examples of power spectral analysis of track irregularities before and after the repair work done by two methods](image)

(a) for levelling by relative reference frame method
(b) for lining by absolute reference frame method

Figure 4: Examples of power spectral analysis of track irregularities before and after the repair work done by two methods

From Figure 4 the difference in performance between two methods will be obvious. By the absolute reference frame method track irregularities over the wide range of wavelength can be corrected very well compared with the case by the relative one. But, as previously described, the absolute
reference frame method requires to measure the actual track geometry beforehand which is a very laborious and time-consuming task.

A new method described in this paper operates in the same way as the absolute reference frame method but does not require the measurement of the actual track geometry. Instead it utilizes the waveform restored from the measured versine data.

2 New method for repairing railway track irregularities

2.1 Partial restoration of track irregularities

A versine data can be easily obtained from the conventional track inspection car with symmetrical chord spacing or the repairing machine with asymmetric chord spacing. A versine, \( y(\xi) \) is mathematically related to the actual track geometry, \( x(\xi) \) through the following difference equation.

\[
y(\xi) = x(\xi) - \frac{bx(\xi + a) + ax(\xi - b)}{l}
\]

where \( l = a + b \), as shown in Figure 1.

So we can consider to estimate the actual track geometry from the versine data by the inverse operation and call it a restoration of track irregularities. But, from the frequency response analysis of the measurement system with symmetrical chord spacing, where the length of the chord is supposed to be 10 m, it is readily known that at the each wavelength of \( 10/2k \), \( k = 1, 2, \cdots \) of track irregularities the amplitude gain becomes zero and so its inverse cannot be taken.

On the other hand, for the measurement system with asymmetric chord spacing there are no finite wavelengths having amplitude gain of zero. So the inverse of the amplitude gain can be taken at any wavelength. But its value becomes very large at the longer wavelength. This causes the disastrous amplification of the noise components contained in the measured data in the inverse operation. Therefore a range of wavelength in the restoration cannot help being restricted. Thus, as a matter of fact, it is impossible to get a complete restoration of the actual track geometry.

However a partial restoration with the restricted range of wavelength, for example, such as a restoration over the range of wavelength from 5 m to 100 m which has a large influence on vibrations of vehicles and so should be corrected in the repair work, will be possible.

2.2 Effectiveness of partially restored waveform on the repair work

In the new method we apply a partially restored waveform of track irregularities at the leading point B. To explain its effectiveness illustratively, we consider an example of versine data of which Fourier spectrum is given in Figure 5. And we define the three kinds of ranges of wavelength of (a) \( 6m \sim 55m \), (b) \( 55m \sim 200m \), (c) \( 6m \sim 200m \), as shown in Figure 5.

Then we execute the restoration in these three ranges respectively and obtain three different restored waveforms, \( x_a(\xi) \), \( x_b(\xi) \), \( x_c(\xi) \), as shown in Figure 6. In this case, a following relation holds approximately.

\[
x_c(\xi) = x_a(\xi) + x_b(\xi)
\]
Figure 5: Fourier spectrum of example data of versine

Figure 6: Comparison of the waveforms restored in the three kinds of ranges of wavelength

Under these preparations, let us suppose that data $x_a(\xi)$ is fed to leading point B in Figure 3 in the repair work based on the absolute reference frame method. That is, the point B is virtually moved by the value of $-x_a(\xi + b)$. If, for simplicity, $x_c(\xi)$ is considered approximately as the actual track geometry, substituting it to the $x_b(\xi)$ of the equation (1) and also taking the above virtual operation into consideration, we obtain the following relation.

$$x_n(\xi) = \beta x_n(\xi - a) + \alpha x_b(\xi + b)$$  \hspace{1cm} (5)

This equation means that a new track resulting from the repair work is the one as if the repair work by the relative reference frame method were applied to the old track of $x_b(\xi)$. Of course, track irregularities, $x_a(\xi)$ in
Figure 6 will be almost disappear, that is, completely corrected in theory. On the other hand, track irregularities, $x_b(\xi)$ will remain to be almost still left in this case. Because the chord length of the tamping machine is about 15 m and $x_b(\xi)$ contains only components of longer than 60 m of wavelength, the value of versine along $x_b(\xi)$ is sufficiently small and so the effect of the repair work would be a little, as seen from Figure 2.

Thus we can conclude that a waveform obtained by a partial restoration is sufficiently effective for the repair work, even if it does not equal to the actual track geometry.

3 Special data processing in the curved track

3.1 Fundamental track geometry and its extraction

In the actual railway track there are many vertical and lateral curves. In the curve a versine data of 3-point measurement system contains not only a component due to the track irregularities but also one due to the fundamental track geometry which has been designed beforehand.

The fundamental track geometry is expressed with such parameters as curvature of the curve, cant and the length of the transition curve, etc. If there were no any track irregularities in the curve, its versine data would become approximately a trapezoidal one, as shown by (a) in Figure 7. But the measured versine data exhibits the trapezoidal waveform contaminated with the track irregularities, as shown by (b) in Figure 7.

![Figure 7: Extracting the component due to the fundamental track geometry in the curve](image-url)
Thus in the curve, when carrying out the restoring operations of track irregularities for measured versine data, it is required to extract and remove in advance the component due to the fundamental track geometry. In fact, even if we execute a restoration directly without removing this component, a restored waveform contains not only true track irregularities but also an extra pseudo-waveform which does not make sense. This is explained as follows.

In the 3-point measurement system a versine data is expressed by the difference equation (3) defined on the coordinate system taken along the fundamental track geometry itself. Also an independent variable $\xi$ is a distance measured along it. From these definition the restoring operation can make sense only for track irregularities. A trapezoidal component contained in the versine data cannot be restored into the actual curved track geometry because a distance variable differs from one measured along the global coordinate fixed on the ground. The difference equation does not have the same meaning for the fundamental track geometry as it for track irregularities.

### 3.2 Data processing techniques for extraction

Thus in the curve the following data processing are needed.

- step.1 extracting the component due to the fundamental track geometry from the measured versine data
- step.2 restoring the true track irregularities from the remaining data

On the practice of the repair work in the curve the above extracted component of the fundamental track geometry is fed at the working point C and used as a quantity to be still left in the corrected track.

The extraction should be done as accurately as possible. Figure 8 shows Fourier spectra of both the ideal trapezoidal data (a) and the actual measured versine data (b) given in Figure 7. According to this spectral analysis it is seen that their spectra are overlapped each other in the wavelength region and so they can not be exactly separated by the usual linear filter, though we can attain the purpose of the extraction approximately by designing a low-pass digital filter appropriately.

![Figure 8: Fourier spectra of versine data in the curve](image-url)
This difficulty will be solved only by some special non-linear filter or some optimum filter. We have been studying the application of both a non-linear filter and the Kalman optimum filter. In this case an optimum filter is fitted to the removal of external noise contained in the measured versine data. Therefore it helps to raise a reliability or quality of the data processing in not only the present extraction but also the restoration. But in general it needs a priori informations concerning probabilistic and statistic characteristics of noise contained in measured data.

On the other hand, nonlinear filtering techniques vary with each other. For the present purpose of the extraction we have applied the nonlinear filtering method called the running median filter which has been studied by Rabiner et al. in the field of speech signal processing (Rabiner[2]). The output \( y(n) \) of the running median smoother is simply the median of the \( L \) numbers of the input signal, \( x(n), \cdots, x(n - L + 1) \). In practice, a combination of running medians and linear smoothing is used, as shown in Figure 9.

![Figure 9: Combination of nonlinear smoothing and linear smoothing](image)

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![Figure 10: Fourier spectra of nonlinear smoothing data](image)
4 Methods of restoring track irregularities

We have been studying the various methods of restoring the true track irregularities from the measured versine data, based on taking the inverse of the frequency response of the 3-point measurement system (Yoshimura[3],[4]).

They are as follows.

1. Filtering by FIR (Finite Impulse Response) inverse digital filter
2. Filtering by ARMA (Auto Regressive Moving Average) filter
3. Analysis and synthesis by Fourier series expansion of versine data

In the method (1), given the range of wavelength for the restoration, a FIR filter is designed using the so-called frequency sampling method. For the system with symmetrical chord spacing a FIR filter have a symmetric impulse response and so a complete linear phase characteristic can be realized. It means there is no any distortion due to the phase delay in the restored waveform. This method gives the very precise restoration and is fitted to executing a restoring operation continuously for a large number of measured versine data.

The method (2) enables the filtering operation with the lower order having a smaller delay comparing with the method (1). It is fitted to a real-time restoring operation. But it has some drawbacks such as distortions in phase and the unstability in the system.

The method (3) differs with the former two methods at the point of it assumes that the input versine data has a discrete line spectrum. In other words, a spectrum of input data must be sufficiently smooth. This method has been mainly developed for the case the length of measured data is of very short distance, as so in the most cases a measurement of track irregularities is carried out by the repairing machine itself.

Figure 11 is an example of the restoration for the data in Figure 7. The restoration has been carried out using the method (3) described above. The range of wavelength in the restoration is the wide one of from 6 m to 125m. This restored data is applied to the practice of the repair work.

5 Conclusion

We have developed a new method for repairing railway track irregularities using levelling and lining machines. The method described here is applied in the field in East Japan Railway Company and very excellent results are already obtained.

References

Figure 11: Example of the restoration for the data in Figure 7