Corrugation on railway rails - a linear model for prediction
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Abstract

A model is presented to predict the initial phase of the formation of corrugation. The basic idea of the model is a feedback loop between high-frequency dynamics of wheelset and track coupled by the contact mechanics and a damaging process (wear). The model allows for the prediction of local corrugation growth rates, corrugation patterns along the rail with corresponding amplitude spectra and global corrugation growth rates. Parameter studies are presented to show the influence of track and wheelset on the formation of corrugation.

1 The corrugation model

Fig 1 shows the basic idea of the corrugation model following ideas of Valdivia [1] and Frederick [2]. The vehicle system dynamics are introduced into the model by a reference state which is quasi static for a linear high-frequency dynamics model. This reference state supplies reference normal and tangential forces as well as reference creepages, contact geometry and vehicle speed. These values are taken to be the point of linearization of the model. The structural dynamics are excited by the initial profile irregularities. They cause fluctuating forces and contact parameters. These fluctuating forces and contact parameters produce fluctuating wear assuming the proportionality between frictional power and wear (frictional power hypothesis). The fluctuating wear will change the profile irregularity filtering certain wavelengths to become what is known as corrugation. The initial profile irregularities are represented by Fourier series and each of the Fourier wavelengths $L_k$ is treated separately. Regarding only the vertical and
lateral dynamics the linear model of the high-frequency dynamics for each frequency \( f_k = \frac{v_m}{L_k} \) is then represented by a system of linear equations as given in equation (1). The upper three equations show the vertical kinematics, the Hertzian normal contact and the introduction of the vertical structural dynamics. The lower three equations denote the tangential contact mechanics, the lateral structural dynamics and the definition of the lateral creepage. The excitation is found in three terms on the right hand side of equation (1). The profile height \( \Delta z \) enters into the vertical kinematics. The changing longitudinal curvature \( \Delta A \) influences both the lateral and vertical contact forces.

\[
\begin{bmatrix}
-1 & 0 & -1 \\
\frac{\partial N}{\partial d} & -1 & 0 \\
0 & F_{\zeta}(f) & -1
\end{bmatrix}
\begin{bmatrix}
\Delta d \\
\Delta N \\
\Delta u_{\zeta}
\end{bmatrix}
= \begin{bmatrix}
-\Delta z \\
-\frac{\partial N}{\partial A} \Delta A \\
0 \\
\Delta T_{\eta} \\
\Delta u_{\eta} \\
\Delta v_{\eta}
\end{bmatrix}
\begin{bmatrix}
\frac{\partial T_{\eta}}{\partial d} \\
0 \\
0 \\
\frac{i2\pi f}{v_m} & -1
\end{bmatrix}
\begin{bmatrix}
0 \\
0 \\
0
\end{bmatrix}
\]
2 Dynamics Model

2.1 Wheelset

The dynamics of the wheelset are calculated by a special finite element program [3]. The wheel and the brake discs are modeled by Mindlin plates under the consideration of additional inplane deformations. The axle is modeled as a Timoshenko beam with torsion and elongation. The eccentric connection of the web and the rim are considered by rigid body elements. Using both the symmetry to the midplane and the rotational symmetry the computational effort was remarkably reduced. The wheelset dynamics are introduced into the high-frequency model by receptances. An example of the vertical receptances of a German high speed passenger coach wheelset type 92 with four brake discs fitted on the axle is shown in Fig 2, left. The first symmetric mode shape with a natural frequency of 80 Hz is shown in Fig 2, right.

Figure 2: Vertical direct receptance (left) and mode shape of a natural frequency of 80 Hz of a German passenger coach wheelset type 92.

2.2 Track

The track model of Ripke [4] considers the infinity of the track, the discrete support by the sleepers and the viscoelastic properties of pad and ballast. The costly modeling of the rail section gives a validity up to 3500 Hz.

It is used to identify track parameters like both stiffness and damping of ballast and pad by adapting measured track receptances. An example for such an adaptation is given in Fig 3. It is also used for track parameter studies.

2.3 Contact Mechanics

The contact mechanics model follows Groß-Thebing [5] using frequency dependent linear coefficient instead of steady state ones (Kalker’s coefficients).
This is necessary for wavelengths of excitation $L$ approaching the length of the contact patch $2a$.

The normal contact problem is solved using a complete linearization of the Hertzian contact mechanics. Here a changing curvature along the rail of the profile and the changing contact patch size and shape is considered.

### 3 Feedback Process

#### 3.1 Local corrugation growth rate

The only feedback process regarded is the wear of the rail tread. The wear is assumed to be proportional to the frictional power $P_{\text{frict}}$ i.e. creep force times relative velocity between wheelset and track (creepage). Linearized with respect to the reference state values $(T_0, \nu_0)$ the frictional power writes

$$P_{\text{frict}} = v_m \nu T = v_m \nu_0 T_0 + v_m (\nu_0 \Delta T + T_0 \Delta \nu)$$  \hspace{1cm} (2)

The fluctuations of creep force $\Delta T$ and creepage $\Delta \nu$ due to the profile height $\Delta z$ are taken from the high-frequency model. The change of the profile height due to wheelset passages $\frac{\partial \Delta z}{\partial n}$ gives

$$\frac{\partial \Delta z}{\partial n} = \lambda(x, v_m, \nu_0, L_k, \ldots) \Delta z$$  \hspace{1cm} (3)

Solving this differential equation gives an exponential growth for the profile height for each location $x$ in the sleeper bay. The real part of $\lambda$ denotes the growth and is called local corrugation growth rate.

#### 3.2 Corrugation pattern, corrugation amplitude spectra and global corrugation growth rate

The local corrugation growth rates have the disadvantage to be only valid at the specific position $x$ of the sleeper bay. Hence the dynamics of the track...
change largely over a sleeper bay the local corrugation growth rates change as well. A Fourier decomposition of the local corrugation growth rates with respect to the sleeper periodicity allows for the calculation of **corrugation pattern** along the rail and their **amplitude spectra**. Assuming the same exponential growth as for the local corrugation growth rates **global corrugation growth rates** are calculated predicting the exponential growth of the amplitudes of the different wavelengths of the initial profile irregularity to corrugations after one mio wheelset passages.

### 4 Model restrictions

Due to the linear model only the initial phase of corrugation is predictable. The calculation is stopped at corrugation heights where limits of the linear model are reached such as loss of contact, stick–slip situations or limits of the Hertzian contact mechanics.

Low frequency vehicle dynamics are always acting as a quasi–static reference state.

Only wear is regarded in the feedback loop.

Excitation is only caused by the profile irregularity. No self excitation or parameter excitation is considered.

### 5 Results

First the influence of the pad stiffness on the corrugation growth is investigated. Fig 4 shows the global corrugation growth rates for different wavelengths for a variation of the pad stiffness from 0.25 to 4.0 times the nominal value of 280 $MN/m$ which was taken according to Grassie [6]. A German track UIC60 (concrete sleeper, 0.6 m sleeper pitch, ballast stiffness 180 $MN/m$), a lateral reference creepage of 0.05 % and a vehicle speed of 144 km/h was assumed. The global corrugation growth rates show one peak at about 3.0 cm wavelength which is almost not influenced by a decrease of the pad stiffness. It belongs to a track mode with nodes on the sleepers (pinned–pinned-mode). For excitation at a sleeper the vertical track receptances show a antiresonance which cannot be removed by pad changes. The second peak at about 10 cm wavelength belongs to a low frequency antiresonance of the vertical track receptances (the sleeper acts as a dynamic absorber) and is largely reduced by the use of softer pads.

The use of very stiff pads produces lower receptances at the low frequency antiresonance where the sleeper acts as a dynamic absorber. The use of softer pads removes this peak of the global corrugation growth rates completely. Global corrugation growth rates belonging to the pinned–pinned-mode antiresonance are almost not influenced by changes of the pad stiffness.
For a track construction which is usual in the eastern part of Germany (Russian rail R65 (65 kg/m), concrete sleeper, sleeper pitch 0.6 m, pad stiffness 700 MN/m, ballast stiffness 100 MN/m, vehicle speed 120 km/h, lateral reference creepage 0.05 %) an optimization of the track is performed to reduce the corrugation growth. The above investigation implied the use of a softer pad (280 MN/m) to reduce corrugation growth due the removal of the low frequency vertical antiresonance of the track. Than the antiresonance of the pinned-pinned-mode at about 1200 Hz was shifted to higher frequencies by reducing the sleeper pitch from 0.6 m to 0.4 m. The further reduction of the pad stiffness (200 MN/m) compensates the increased support stiffness. Fig 5 shows the original parameter set and the two steps of optimization. The first step removes the high global corrugation growth rates at wavelengths of about 10 cm caused by the low frequency vertical antiresonance of the track. The reduction of the sleeper pitch shifted the second critical wavelength range to such short wavelengths that the contact filter suppresses their growth.

Assuming that the original track parameters gave the need of grinding once a year the optimized track would only need grinding every 2.5 years.

6 Conclusion and open problems

The corrugation model shows that the initiating phase of the formation of corrugation is mainly influenced by vertical structural antiresonances of
the track with low receptances. At these antiresonances high normal force fluctuations occur and cause corrugation formation. The correct modelling of the contact mechanics provides a filter which suppresses the growth of extreme short wavelengths $\leq 2$ cm. Other critical parameters encouraging corrugation growth are the contact patch size, high lateral reference creepage or high wear rates of the rail steel. In order to reduce corrugation growth antiresonances in the vertical track dynamics with low receptances should be avoided for example by using resilient pads or continuously supported track. Also conformal profile combination (worn wheel on worn rail) should be avoided. Track misalignments will increase the corrugation growth by increasing the lateral creepage. The presented corrugation model and its components is used to identify parameters of existing track which cause the corrugation formation and is used to optimize these parameters in terms of a reduction of corrugation growth. It can also be used to investigate new track design in order to dismiss critical parameter combinations in the design phase.

Complete assessments of corrugated sites to supply realistic input data for the corrugation model are still rare. For this purpose a questionnaire for corrugated sites was developed by Technical University of Berlin.

References


A Notation

$A$ curvature of the profile along the rail

$d$ elastic approach of wheelset and track

$\Delta$ fluctuation of state variable

$f$ frequency

$F_{\xi\zeta}, F_{n\eta}$ vertical, lateral direct receptances of wheelset and track

$F_{n\zeta}, F_{\xi\eta}$ cross receptances of wheelset and track

$L_k$ wavelength

$n$ number of wheelset passages

$N, T_{\eta}$ vertical, lateral contact force

$\nu_{\eta}$ lateral creepage

$u_{\eta}, u_{\zeta}$ lateral, vertical relative displacement between wheelset and track

$v_m$ vehicle speed

$x$ location on the track

$z$ profile height