The crash response of a train of masses connected by springs and of a single coach
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Abstract

The FEM is used to perform two quite different tasks. In the first a travelling line of masses joined by springs with different stiffness characteristics (elastic and elastic-plastic) is used to simulate the behaviour of a row of carriages suddenly brought to rest. This simulation of a train collision can be help gain an understanding of the forces involved in such a situation. However, it does not take into account any damage that may occur to each coach in the train. We therefore also apply the FEM to investigate the response of an idealised model of a single coach involved in such a collision.

1 Introduction

In recent years a great deal of work has been published in the literature concerning the loading of structures by impacts. The First International Symposium of Structural Crashworthiness was held in Liverpool 1983\(^1\), followed by a second at MIT 5 years later\(^2\) and a third again in Liverpool in 1993\(^3\). The publication of part of the proceedings of these meetings form an extremely useful source of additional and up to date information. Other books by Jones\(^4\) and Ammann\(^5\) provide significant contributions to structural impact. The Finite Element Method (FEM), has been extensively applied to these problems and its flexibility and ease of use has increased enormously since it was first used in the 1950s. Extremely versatile FE packages have been developed that are capable of providing solutions to a variety of non-linear problems. The non-linearities involved can be of material, geometry and/or boundary and the removal of elements can also be included to simulate wear and/or failure. In this paper we present some preliminary results on the use of the FEM to study the crash response of a train of carriages.
2 Methodology

The FE solution followed the general procedure:

1) Model discretisation;
2) material(s) property specification;
3) boundary condition definition;
4) applied loading definition;
5) solution analysis phase(s); and
6) result post processing and presentation.

which was implemented using the ABAQUS package.

3 Spring Connected Rigid Mass Motion

The dynamic behaviour of a series of connected carriages in a train collision has been modelled by Irvine using a one dimensional array of masses attached to each other by springs. This analysis can also be made using the FEM with all of the added advantages that that method brings - the ability to readily demonstrate the variation with time of various parameters of interest (such as velocity, acceleration, stress waves propagation, etc.).

The FE model is shown in Fig. 1. One hundred rigid mass elements are connected together by springs with particular elastic or elastic-plastic response characteristics. The entire assembly is moving as a rigid body along a straight line with a velocity of $v_0$. The motion is disturbed by bringing the end of the first spring attached to the first mass in the array to a sudden halt. The subsequent response of the model to this disturbance is then studied. Two models are analysed, corresponding to the cases where the springs are elastic or elastic-plastic.

The elastic and elastic-plastic response of a string of masses

The general situation is shown in Fig. 2. Here a string of $p+q$ masses are connected together by springs. The first $p$ masses are assumed to exhibit an elastic response and the remaining $q$ show an elastic-plastic response.
and the total number of springs in the system is \( n \). Fig. 2 also shows the schematic load displacement curves for the elastic and elastic-plastic springs, respectively. \( k_e \) and \( k_p \) are the elastic and plastic stiffnesses of the springs respectively (in Nm\(^{-1}\)), each of the masses is \( m \) kg and the line of masses is moving with a velocity \( v \) ms\(^{-1}\).

**Elastic Collisions** In this case the springs in the model are given material properties so that their stiffness remains constant at all times (see Fig. 2). The abrupt deceleration of the first mass in the array generates a compressive elastic wave along the spring. This disturbance travels along the spring with constant velocity to reach the next spring in the array. For linear elastic springs the displacement field in each spring increases by an equal amount as each mass is decelerated in turn. The resultant displacement field for each mass therefore is in the form of an arithmetic progression, i.e. \( \delta, 2\delta, \ldots n\delta \) for the 1st, 2nd \ldots and the last (\( n \)th) loaded spring in the array. The strain energy of the masses 1-2-3-4-\ldots-p-(p+1) involved in elastic collisions, \((SE)_{\text{elastic}}\), is given by:

\[
(SE)_{\text{elastic}} = \sum_{k=1}^{p} \frac{1}{2} k_e \delta (2\delta)^2
\]
Therefore, for all elastic collisions:

\[
(SE)_{\text{elastic}} = \frac{1}{2} (nmv^2)
\]

\[
= \sum_{i=1}^{n-1} \frac{1}{2} k_s (2\delta)^2 + \frac{1}{2} k_s \delta^2 n^2
\]

\[
= \frac{1}{6} k_s \delta^2 n (4n^2 - 3n + 2)
\]

from which:

\[
\delta = \left( \frac{3m}{k_s(4n^2 - 3n + 2)} \right)^{\frac{1}{2}} v
\]

When the first mass comes to rest:

\[
n\delta = \left( \frac{3m}{k_s \left( 4 - \frac{3}{n} + \frac{2}{n^2} \right)} \right)^{\frac{1}{2}} v
\]

This displacement field excludes the relative rigid body displacement of neighbouring masses from the interconnecting springs.

The response characteristics of the system obtained from the FEM are shown in Figs 3-5. The displacement-time and velocity-time dependencies are shown in Figs. 3 and 4, respectively, while Fig. 5 shows how the stress wave propagates through the elastically connected line of masses.

**Elastic-Plastic Collisions** The instantaneous deceleration of the first mass in the array has the same effect as for elastic collisions. In this case however, the linear elastic property of the springs is replaced by a bilinear force-displacement characteristic (see Fig. 2) and the springs have a much smaller plastic modulus \(E_p\) than an elastic one \(E_e\). For elastic-
plastic collisions between masses (q-1) and q:

\[
(SE)_{\text{elastic-plastic}} = \frac{1}{2} k_e (p \Delta)^2 + k_s (p \Delta) \left( \frac{k_s}{k_p} \right) (q-p) 2\delta + \frac{1}{2} k_p \left( \frac{k_s}{k_p} \right) (p+q) (2\delta)^2
\]

substituting (n-p-1) for q the expression can also be derived when going from p+1 to n. Then the elastic-plastic strain energy for all but the last spring in the line is given by:

\[
(SE)_{\text{elastic-plastic}} = (n-p-1) \frac{1}{2} k_e (p \Delta)^2 + \sum_{\delta=1}^{n-p-1} k_s \frac{p}{k_p} q (2\delta)^2 + \sum_{\delta=1}^{n-p-1} \frac{1}{2} k_p \left( \frac{k_s}{k_p} \right)^2 q^2 (2\delta)^2
\]

The energy of the last spring is:

\[
\frac{1}{2} k_e (p \Delta)^2 + \frac{1}{2} k_s (p \Delta) \frac{k_s}{k_p} (q \Delta) + \frac{1}{2} k_p \left( \frac{k_s}{k_p} \right)^2 (q \Delta)^2
\]

Therefore (using p+q=n), for the elastic plastic collisions:

\[
\frac{1}{2} (nmv^2) = \frac{1}{2} k_s (2\delta)^2 \left[ \left( \frac{p (p+1)}{6} + \frac{p (n-1-p)}{4} + \frac{p (n-p)}{4} \right) \delta^2 + k_s \left( \frac{n-p-1}{6} \right) \frac{n-p-1}{2} \right]
\]

This approaches the solution for the elastic case as \( E_p \) approaches \( E_e \). Displacement beyond the elastic limit is increased by a factor \( E_e/E_p \).

The response of the FEM mass-spring array to elastic-plastic collisions is shown in Figs. 6 and 7, which can be compared with Figs. 3 and 4 for the elastic collisions. Figs. 8 and 9 show how the stress wave propagates along the line of masses at different times of the analysis. To begin with
(when the collisions are still elastic - as in Fig. 8) the stress wave propagation is very much the same as in the previous case. However as the spring material starts to become plastic, the stress wave propagation characteristics change, the stress wave slows down and the situation becomes more complex (Fig. 9).

4 Stress Wave Propagation in a Single Coach

The simulation of a train collision by a series of masses connected by springs can be used to help understand the forces involved in a train collision, however, it does not take into account any damage that may occur to each coach in the train. We therefore also apply the FEM to investigate the response behaviour of a simplified model of a single coach involved in such a collision.
The discretised model of the coach is shown in Fig. 10 prior to and after the simulated collision. A vertical plane of symmetry is assumed, which means that only half of the coach is considered. Door and window openings are included and the presence of a chassis is represented in the model by a longitudinal reinforcement under the carriage as shown in Fig. 10. The problem is, in essence, very similar to work reported by Kormi et al. on the axial loading of thin-walled structures with various cross sections which has demonstrated how concertina buckling occurs as a consequence of the build up of stress intensity by the combination of incident and reflected travelling stress waves.

Instantaneous time-frozen deformed shapes of the coach, with superimposed contours of longitudinal stress contours, are shown in Fig. 11. The coach obviously absorbs a considerable amount of energy. Perhaps the most striking feature of the response of the coach is that the axial formation of wrinkles is much more pronounced in the region without an opening (i.e. the bottom and top part of the coach). It appears that the access doors and the windows are able to accommodate the axial displacement, thereby preventing the formation of wrinkles on the side panels.

References


