Effect of intermediate beam in track on train induced ground vibration

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Abstract

This study theoretically evaluates the effect of intermediate beam in track on train induced ground vibration. It notices the force acting from track to elevated structures and the input to the ground vibration. Models of track on elevated structure ant of that directly supported on the ground are considered. The following results are obtained: (1) The method to evaluate the effect of track rigidity is given; (2) The effect of rail rigidity is successfully compared with measurements; (3) The effect of rigidity of the intermediate beam and its supporting spring is clearly given; (4) The acceleration of deck slab in the elevated structure as band-pass level including human sensitivity is compared in regard to ladder and ballasted tracks.

1 Introduction

The study aims at clarifying the effect of the intermediate beam in tracks on the elevated structure and on ground, on the ground vibration by theoretical studies. By referring the preceding study [1], the characteristics of Shinkansen ground vibration and the existing model for analysis are discussed in the former part of this paper. It shows that the peak of ground vibration is given by the axial load series and that the track and elevated structure work as low pass filters.

2 Measurement of Hankinson ground vibration

The basic characteristics of ground vibration along Shinkansen lines and its reducing measures are summarized in WAVE2000 by Yoshioka [2]. The band-pass characteristics of $VL_{bp-z}$ (Vibration Level including human sensitivity in the vertical direction) is given in Figure 1 for the Shinkansen
vibration measured at 10m from the center of elevated structure at 103 sites after normalizing them by putting the overall value to 0dB. The train speed in the measurement is about 200km/h. The conditions for the sites are the ballasted track on elevated structures set on alluvial ground. The solid line and two dotted lines in Figure 1 show the mean and the standard deviation range respectively. In the Figure 1 three peaks are noticed at 6.3, 16-20 and 40-50Hz. The intermediate peak is the most dominant. The range of the standard deviation is small enough even with so many measuring sites. It is noticeable that these features are independent of conditions of track, structure and ground.

Figure 1: 1/3-octave band VL-z spectra in vibrations along Tokaido Shinkansen [1].

3 Effect of axial load series

The effect of axial load in the frequency domain, \( r_F \), is given for the symbols of frequency \( f \) and speed \( V \) as follows;

\[
r_F \left( \frac{f}{V} \right) = \left\{ 1 + \exp \left( \frac{2 \pi f l_1}{V} \right) \right\} \left\{ 1 + \exp \left( \frac{2 \pi f l_2}{V} \right) \right\} \left\{ 1 + \exp \left( \frac{2 \pi f L}{V} \right) \right\} \prod_{i=1}^{n} \left\{ 1 + \exp \left( \frac{2 \pi f 2^{(i-1)} L}{V} \right) \right\}
\]  

\( (1) \)

The first term in the right-hand side is for the axes of bogie having the distance of wheelset, \( l_1 \); the second, for the bogie having the distance of bogie centers, \( l_2 \), under a car; the third, for the increase of cars having the length, \( L \), on the doubled base.

The effect of repetition of axles in a bogie is given in Figure 2 (a); that of bogies under a car, in Figure 2 (b); that of two cars, in Figure 2 (c); that of a train of 16 cars, in Figure 2 (d). In Figures 2 (a) to (c) the amplitude is doubled. In contrast, in Figure 2 (d), the amplitude is larger than 15 times by the repetition of loads of 16 cars. It means that the intensity of spectrum depends on the number of cars in a train.
Their composite effect is given in Figure 3. As the band-pass level is their integration in the 1/3 octave band, Figure 3 shows the three peaks in Figure 1. In Figure 2 (a) the center frequencies of these peaks are given by the distance between the two axles under a bogie. The first and second peaks are calculated as

\[ f_{1i} = \frac{v_i}{l_i} \]  

(2)

This is shown in Figure 4. It is linearly proportional to the train speed.

**Figure 2:** Effect of axial load series.

**Figure 3:** Composite effect of load repetition.
4 Discussion on model

Yoshioka sets the model for analyzing the $V_L$ spectrum at the deck slab in an elevated structure as shown in Figure 5 [2]. The model is for the rolling stock, for the interaction between rolling stock and track and for the loading on double beams of rail and elevated structure continuously supported by springs with structural damping between and under them. The excitation force is axle loads and the reactive vibration of rolling stock to rail due to track irregularities. The observation point BSL (Bridge SLab) is set at the top of the pillar of elevated structure. The connection from BSL to G$_r$ (ground surface of $r$ m away from the center of rigid frame structure) depends on another model.

The parameters of track and elevated structure are determined so that the computed spectra can almost agree with measurements. They are given in Table 1 together with their symbols [3].
The approximation of elevated structure with a continuously supported beam depends on the same idea to use the Winkler model for the track.

Table 1: Parameters [3].

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Axial distance in bogie $(l_1)$</td>
<td>2.5m</td>
</tr>
<tr>
<td>Bogie distance under car $(l_2)$</td>
<td>17.5m</td>
</tr>
<tr>
<td>Car length $(L)$</td>
<td>25m</td>
</tr>
<tr>
<td>Cars in train</td>
<td>16</td>
</tr>
<tr>
<td>Rigidity of rail $(E_1I_1)$</td>
<td>$1.28 \times 10^7 \text{Nm}^2$</td>
</tr>
<tr>
<td>Mass of rail $(m_1)$</td>
<td>$1.2 \times 10^2 \text{kg/m}$</td>
</tr>
<tr>
<td>Track spring $(k_1)$</td>
<td>$1.0 \times 10^8 \text{N/m}^2$</td>
</tr>
<tr>
<td>Ditto, loss factor $(\eta_1)$</td>
<td>0.3</td>
</tr>
<tr>
<td>Rigidity of deck slab $(E_2I_2)$</td>
<td>$7.8 \times 10^8 \text{Nm}^2$</td>
</tr>
<tr>
<td>Mass of deck slat $(m_2)$</td>
<td>$7.0 \times 10^3 \text{kg/m}$</td>
</tr>
<tr>
<td>Deck slab spring $(k_2)$</td>
<td>$1.6 \times 10^8 \text{N/m}^2$</td>
</tr>
<tr>
<td>Ditto, loss factor $(\eta_2)$</td>
<td>0.3</td>
</tr>
</tbody>
</table>

5 Effect of rail rigidity

As the mass of the rail is 1/60 of that of deck slab in the elevated structure and the effective length of rail continuously supported on springs is maintained to be as the same as that in the static state up to about 100Hz [1], the deflection of rail is considered just as that of Winkler as in Figure 6. This is consistent even if the intermediate mass of tie and ballast is included.

![Figure 6: Model of track](image)

The deflection of rail, $z$, is given as

$$z = \frac{P}{8E_1I_1B^3} \exp\left(-|\beta(x - Vt)| \left| \cos(\beta(x - Vt)) + |\sin(\beta(x - Vt)) | \right| \right)$$

(3)

where $\beta = \sqrt{k_1/(4E_1I_1)}$

Here, the load is moving at the speed of $V$. The load on the deck slab is given by multiplying it by $k_1$. 

As the running distribution of deflection around the load corresponds to the load variation at BSL set on the deck slab of elevated structure, the variation of load at the speed of $V$ is calculated in the frequency domain as

$$ F \{z\} = \frac{P}{E_i I_1 \left( \frac{2 \pi f}{V} \right)^4 + k_i} $$

The results are given in Figure 7. It gives the response of 60kg rail having parameters in Table 1 and that of 50N rail having the rigidity of $0.81 \times 10^7 \text{Nm}^2$. In the Figure the difference looks not so large as in the actual measurement shown in Figure 8 [3], but it should be considered that the difference depends on its integration.

### 6 Effect of track rigidity

As the mass of the track is 1/40 of that of deck slab in elevated structure, the movement of track can be discussed independent of that of the latter. That is, the track could act as a distributor of axial load. The track is considered as double beams having distributed springs between and under them when the case of floating ladder track in Photo 1 [4] is considered. The model of track is shown in Figure 9.
When the load runs at the speed of $V$, the Fourier transform of the distributed force acting at BSL of elevated structure is given as

$$F(f) = \frac{1}{V} \left\{ -c_1 f \left( \frac{2\pi f}{V} \right) + k_1 \right\} \begin{bmatrix} \left( \frac{2\pi f}{V} \right)^2 + k \end{bmatrix} + \left( \frac{2\pi f}{V} \right)^2 \right\} \begin{bmatrix} \left( \frac{2\pi f}{V} \right)^2 + k \end{bmatrix}$$

As the standard, the ladder track has a section of 46*18.5cm for the intermediate beam. The parameters of track for it are given in Table 2 [5].

Table 2: Parameters of track.

<table>
<thead>
<tr>
<th>Type</th>
<th>Coefficient (Mpa)</th>
<th>Section area (m²)</th>
<th>2nd moment (m⁴)</th>
</tr>
</thead>
<tbody>
<tr>
<td>60jg rail</td>
<td>2.1*10</td>
<td>7.75*10⁻³</td>
<td>3.0*10⁻³</td>
</tr>
<tr>
<td>Rail pad</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Inter-beam</td>
<td>4*10⁴</td>
<td>8.51*10⁻⁴</td>
<td>2.41*10⁻⁴</td>
</tr>
<tr>
<td>Beam pad</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ballasted track</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Photo 1: Floating ladder track laid in JR Hokkaido [4].

### Figure 9: Model of track.
The results for the case 1 of the intermediate beam supported with or without damping and that with a half mass are given in Figure 10. The Figure shows that there is no significant difference in between.

The result for the case 2 where an intermediate beam without rigidity is compared with that with rigidity is given in Figure 11. It shows that the rigidity of intermediate mass significantly decreases ground vibration. Without the rigidity of intermediate mass the bandwidth spreads to 1.5 time that with rigidity.

As the case 3, the effect of supporting spring constant of intermediate mass is checked in Figure 12. With a rigid spring corresponding to ballast with ballast-mats, the bandwidth spreads to 1.4 time that in the case with rubber support in the ladder track.

As the case 4, the effect of speed is checked in Figure 13. As the value at 0Hz in eqn (5) is $1/V$, the value in the case of 300km/h decreases to 200/300 of that of
200\text{km/h}. On the other hand, the bandwidth has increased to 1.4 times that at 200\text{km/h}. these correspond to the decrease of loading duration.

7  Band-pass level considering human sensitivity and acceleration of BSL

The band-pass level is given by integrating the density function in the frequency domain to the extent of 1/3 octave, but an approximate value can be given by multiplying the following value if the variation is not so large.

\[
\Delta f = \left(10^{\frac{\log_{10} 2}{6}} - 10^{\frac{\log_{10} 2}{6}}\right)f = 0.23156 f
\]  

The human sensitivity is given by ISO2631 [1].

By multiplying these to eqn (4), the force corresponding to \(VL\) to BSL is given for ladder and ballasted tracks in Figure 14. Since the intermediate mass of ladder track 0.0851*2500=213kg/m is nearly the same as that of ballasted track 235kg/m in Table 2 and is not sensitive to response in Figure 8, the mass of the ladder is used here. Figure 14 clearly shows the effectiveness of ladder track compared with that of the ballasted track through the actions of rigidity and soft spring.

The effect of intermediate beam in the track on the ground is given in Figure 15 by calculating it for the supporting spring of ballast on the ballast-mat which corresponds to a good roadbed. Here, the peak is nearly the same, but the bandwidth of ladder track decreases to 1/1.25 of that of ballasted track. This means that the use of longitudinal tie is effective to decrease the ground vibration.

![Figure 14: Load to BSL as VL.](image)
The acceleration response of BSL to load series corresponding to $VL$ is given in Figure 16 by regarding the elevated structure as a continuous beam with mass supported on springs according to the preceding paper [1]. This modeling is supported by the fact that the frequency of p-p vibration of elevated structure is large enough as 276Hz [1]. The peaks at 6.3, 16-20 and 40-50 in Figure 1 are observed in the Figure, but it shows a rapid decline in acceleration as frequency increases, strongly depending on the low-pass filter effect of elevated structure. The difference may depend on the process of propagation of ground vibration in the ground which is not included in the analyses so far. The acceleration of ballasted track prominently decreases to that of the ladder track in the frequency domain higher than 10Hz corresponding to the difference in Figure 14.
As for the rigidity of intermediate beam, its effectiveness for ground vibration has been clearly demonstrated so far. However, it should be checked for the noise of about $10^3\text{Hz}$ because it has been considered to have reverse effect [6].

8 Concluding remarks

Through the above studies, the following results are obtained.

(1) The damping and mass of track do not have significant effect for ground vibration.
(2) The rigidity of intermediate beam and its supporting spring have significant effect.
(3) Speed-up decreases the density function of forces at low frequencies, but increases it at high frequencies corresponding to the decrease of loading duration.
(4) The force acting to BSL distributed from axial load by track significantly decreases under the ladder track compared with that under the ballasted track.
(5) The acceleration of BSL has peaks predominant in measurement, but it decreases rapidly as the frequency increases showing a prominent decrease from that of ballasted track to that of the ladder track in the frequency domain of more than 10Hz.
(6) The use of longitudinal tie is effective on ground vibration.
(7) It is clearly shown that the rigidity has significant effect on ground vibration, but the effect could be reverse to noise. It must be checked carefully.

References