

Adaptation of the kinematic train model using the Interacting Multiple Model estimator

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Abstract

This paper presents an Interacting Multiple Model (IMM) estimator that performs a reliable train location based on soft-switching between two kinematic train models: one assuming constant velocity and the other assuming constant acceleration. Two sensors provide input to the system, namely an eddy current sensor device measuring the velocity and a Global Navigation Satellite System (GNSS) sensor unit delivering absolute position information. Simulations show that the presented location system offers robust and accurate position information, even in case of sensor failures or GNSS signal discontinuances.

1 Introduction

State-of-the-art location systems are mainly based on hardware installations along the track [1]. In contrast, the presented system necessitates only train-borne sensors to perform the train location. This prevents expensive investments in the modification of the railway infrastructure.

An eddy current sensor device provides an accurate measurement of the train velocity while a GNSS sensor unit supplies an absolute position information in world coordinates. These sensor data are fused in an Extended Kalman Filter (EKF) to obtain reliable and accurate position information. However to account for changes in the train kinematic, two models have to be implemented in parallel. The combination of the two kinematic models is conducted by the IMM estimator.

The remainder of this paper is organized as follows: sections 2 and 3 describe the sensors and the simulation scenario, respectively. The kinematic train models and the IMM estimator are discussed in section 4 and 5. Simulation results of the proposed system are presented in section 6, while section 7 concludes the paper.



2 Sensors

The location system consists of several sensors that collect the required data and estimate the actual position of the train. For the sake of high reliability and robustness, the sensors employ independent measurement principles and separate pre-processing units. Balancing all considerations, we have chosen an eddy current sensor device and a GNSS sensor unit.

2.1 Eddy current sensor device

The eddy current sensor device consists of two sensors, which are mounted at the bogie of the common railcar. Each sensor detects inhomogeneities in the magnetic resistance along the track caused by rail clamps or by irregularities of the rail [1].

The train velocity is obtained by applying the principle of a closed-loop correlator. The sensors are scanning the magnetic resistance to the rail, separated by a distance l . Ideally, both sensors capture signals that are identical up to a transit time T that resembles the position of the maximum of the cross-correlation function [2]. The train velocity is then readily obtained by $v = l/T$. This measurement is widely unsusceptible to environmental perturbations and other interferences. The accuracy of the eddy current sensor is specified to a relative standard deviation $\sigma_v/v < 2\%$ [2]. Clearly, a first estimate of the position can be computed by integration of the velocity signal. While this estimate will be accurate for small distances, it will be successively deteriorated by integration drift as time proceeds.

2.2 Global Navigation Satellite System

The main advantages of satellite navigation are its global availability and the high accuracy measurement that is not prone to drift. However, in navigation with GNSS other drawbacks and deviations occur and have to be considered, e.g. deviation through ionospheric and atmospheric delays, clock deviations, dilution of precision, multipath deviations and satellite shadowing [3].

An analysis of the usability of GNSS for train location reveals several disadvantages, such as shadowing of satellites in railway tunnels, forest aisles and urban environments resulting in none or an uncertain measurement. Besides, in an urban environment towers or big buildings may reflect the signal and thus cause multipath propagation. When at least four satellites are visible, C/A-code processing yields position measurements with a standard deviation of $\sigma_{\text{GPS}} = 5$ m. Hence, the eddy current sensor and GNSS exhibit orthogonal strengths thus motivating a coupled measurement system.

3 Simulation scenario

For designing and evaluating of the presented estimator a simulation scenario was developed to enable the imitation of different environmental and fault situations for validation of the algorithm, see figure 1 for a schematic diagram.



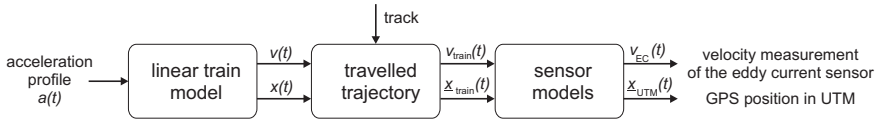


Figure 1: Structure of the simulation scheme.

The basis for the design is a linear train model that is given by

$$x(t) = x_0 + v(t)\Delta t + \frac{1}{2}a(t)\Delta t^2 \quad (1)$$

$$v(t) = v_0 + a(t)\Delta t, \quad (2)$$

where the acceleration profile $a(t)$ is provided as input, $x(t)$ is the travelled distance, x_0, v_0 the initial point and velocity and $v(t)$ is the velocity of the train. The simulated track was designed using the regulations for real track construction to yield a realistic course [4]. This track information is fed into the simulation to obtain the train velocity and position in world coordinates. The last section of the simulation integrates the sensor models to obtain realistic measurements. Both sensor signals are disturbed by white Gaussian noise.

The presented simulation environment also enables the analysis of different events and the imitation of sensor faults as illustrated in figure 2. Three different events are realized to validate the IMM estimator.

- **Event A:** Train stop at railway station 2; no GNSS signals available
- **Event B:** Railway tunnel, where no GNSS signals are available
- **Event C:** Eddy current sensor failure; no velocity measurement available

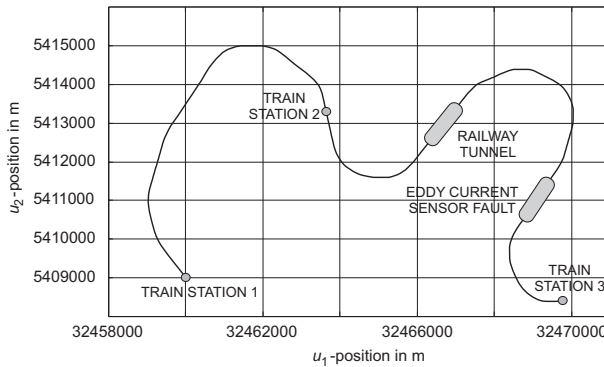


Figure 2: Schematic illustration of the track route in UTM coordinates.

4 Kinematic train model

Vehicle dynamics are considered by a discrete kinematic train model. Equation (3) presents the discrete kinematic train model in its general form:

$$\underline{p}_k = \underline{p}_{k-1} + \Delta t \cdot \dot{\underline{p}}_{k-1} + \frac{1}{2} \Delta t^2 \cdot \ddot{\underline{p}}_{k-1} + \frac{1}{6} \Delta t^3 \cdot \dddot{\underline{p}}_{k-1} + \dots \quad (3)$$

where $\underline{p}_k = [u_{1k}, u_{2k}]^T$ denotes position of the vehicle in two dimensional space at time step k and $\dot{\underline{p}}_k$, $\ddot{\underline{p}}_k$, $\dddot{\underline{p}}_k$ denote velocity, acceleration and jerk, respectively. The sampling interval is given as $\Delta t = t_k - t_{k-1}$.

4.1 Constant acceleration model

The constant acceleration model is based on an Extended Kalman Filter that represents the train movements adequately, where jerk is modelled as process noise [4]. With these assumptions, equation 3 can be rewritten as

$$\underline{p}_k = \underline{p}_{k-1} + \Delta t \cdot \dot{\underline{p}}_{k-1} + \frac{1}{2} \Delta t^2 \cdot \ddot{\underline{p}}_{k-1} + \frac{1}{6} \Delta t^3 \cdot w_{k-1}, \quad (4)$$

where w_{k-1} is the white Gaussian process noise. The general system model of the Kalman Filter is given by

$$\underline{x}_k = \underline{A} \underline{x}_{k-1} + \underline{B} \underline{u}_{k-1} + \underline{W} \underline{w}_{k-1}, \quad (5)$$

where the state vector $\underline{x}_k$ comprises the following state variables: the train position $\underline{p}_k = [u_{1k}, u_{2k}]^T$, the velocity $\dot{\underline{p}}_k$ and the acceleration $\ddot{\underline{p}}_k$. The location system assumes no information about the train driver commands and thus no statement about the desired acceleration can be made. Therefore, no input to the system model $\underline{u}_{k-1}$ is available and hence we set $\underline{B} = \underline{0}$.

The observation model relates the state vector $\underline{x}_k$ of the system to the measurement vector $\underline{z}_k$, which consists of three components: u_{1k} the u_1 -position of the train, u_{2k} the u_2 -position of the train and v_k the train velocity. This leads to an observation model for the location system that is given by

$$\underline{z}_k = \begin{bmatrix} u_{1k} \\ u_{2k} \\ \sqrt{(\dot{u}_{1k})^2 + (\dot{u}_{2k})^2} \end{bmatrix} + \underline{v}_k, \quad (6)$$

where $\underline{v}_k$ is the measurement noise that is assumed being white Gaussian distributed. Since the observation model is nonlinear, an EKF offers itself [5]. For further details about this special EKF please refer to [4].

4.2 Simulation results obtained with the constant acceleration model

The simulation is based on the scenario outlined in section 3. During this scenario, three events occur: Event A describes a train standstill with loss of radio reception



of GNSS signals, Event B simulates a railway tunnel with another loss of GNSS signals, and Event C represents a failure of the eddy current sensor device.

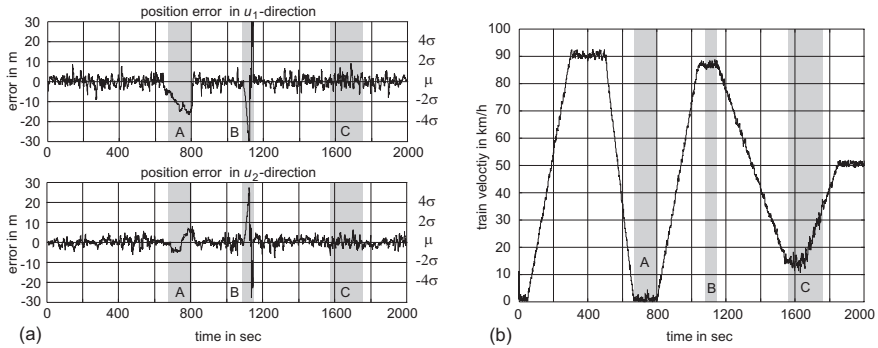


Figure 3: Simulation results using the constant acceleration model: (a) position errors of the EKF related to the σ -ranges of the GPS sensor; (b) estimated train velocity.

Figure 3 depicts the simulation results with the position error and the estimated train velocity are plotted versus time. It is clearly seen that the EKF using the constant acceleration model achieves better train state estimates as compared to using only one sensor information. This is due to the fact that the Kalman Filter performs a sensor data fusion and hence increases accuracy and reliability [4]. However, if the position or the velocity of the train is measured incorrectly or a sensor fault occurs, the high order of the kinematic train model results in a large drift of the position. On the other hand, a highly dynamical kinematic model is needed during normal operation to meet the train dynamics. This results in the idea to use two kinematic models and switch between the models depended on the train dynamics. An algorithm that performs such a switching between two or more models is the Interacting Multiple Model estimator.

4.3 Reduced order model

This section describes the second kinematic train model that is used in the IMM estimator. The conclusion of the simulation using the constant acceleration model performed in section 4.2 is that the order of the second kinematic train model should be reduced as compared to the first model to achieve better performance and reduce the position drift in case of sensor failures. Hence, the reduced order model is designed with train position and velocity as state vector and acceleration as noise, assuming constant velocity. This results in the following system model

$$\underline{p}_k = \underline{p}_{k-1} + \Delta t \cdot \dot{\underline{p}}_{k-1} + \frac{1}{2} \Delta t^2 \cdot \underline{w}_{k-1}, \quad (7)$$



where $\underline{w}_{k-1}$ is white Gaussian process noise. Designing the system model in the state space representation leads to the following overall system that is given by

$$\begin{bmatrix} u_{1k} \\ u_{2k} \\ \dot{u}_{1k} \\ \dot{u}_{2k} \end{bmatrix} = \begin{bmatrix} 1 & 0 & \Delta t & 0 \\ 0 & 1 & 0 & \Delta t \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_{1k-1} \\ u_{2k-1} \\ \dot{u}_{1k-1} \\ \dot{u}_{2k-1} \end{bmatrix} + \begin{bmatrix} \frac{1}{2}\Delta t^2 & 0 \\ 0 & \frac{1}{2}\Delta t^2 \\ \Delta t & 0 \\ 0 & \Delta t \end{bmatrix} \underline{w}_{k-1}, \quad (8)$$

where u_{1k}, u_{2k} represent the train position, $\dot{u}_{1k}, \dot{u}_{2k}$ the velocity and $\underline{w}_{k-1}$ is the process noise.

The reduced order model implemented in the EKF has the same measurement vector $\underline{z}_k = [u_{1k}, u_{2k}, v_k]^T$ as the EKF representing the constant acceleration model. Hence, the observation model $\underline{z}_k = h(\underline{x}_k, \underline{v}_k)$ is almost identical apart from the measurement noise, see equation (6).

5 The IMM estimator

The train adapts its velocity several times through acceleration or braking during normal operation. This results in a continuously non-stationary train kinematic, which challenges state-of-the-art location systems, see section 4.2. We therefore propose different sets of state equations to account for the variable kinematics.

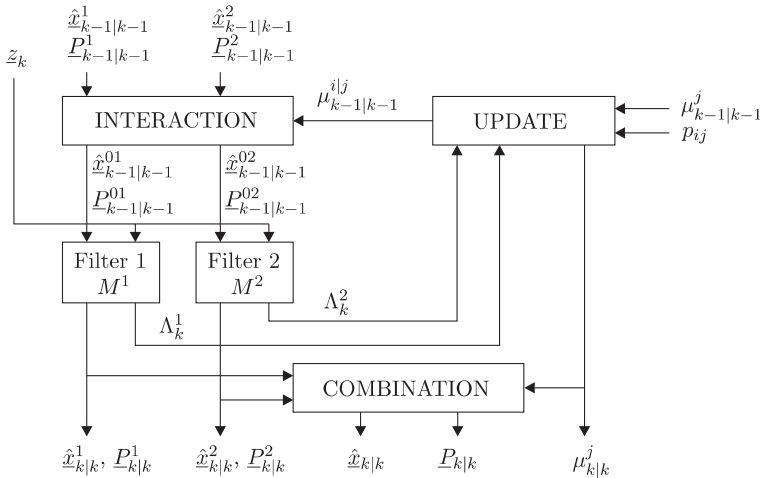


Figure 4: Block diagram of the IMM estimator for two Kalman Filters.

The IMM estimator is interacting between these models by a probabilistic switching, where the filters $M^j = \{M^1, M^2, \dots, M^r\}$ are in parallel operation. This exchange of information between the filters concludes in soft-switching,

where state estimates of several filters are fused by the use of model probabilities [6]. Furthermore, the IMM estimator is a suboptimal hybrid filter that was shown to achieve an excellent compromise between performance and complexity, i. e. it is computationally attractive [7].

Operation of the IMM estimator can be divided into four sections: interaction, update of the Kalman Filters, update of the model probabilities and the combination. The block diagram of the IMM estimator is depicted in figure 4. In our case, we use two Extended Kalman Filters as hypothesized models $M^j = \{M^1, M^2\}$, where each filter contains a different set of state equations. The filters have been discussed in section 4.1 and section 4.3, respectively.

Interaction

During Interaction of the IMM, the mixed estimates $\hat{\underline{x}}_{k-1|k-1}^{0j}$ and the state error covariances $\underline{P}_{k-1|k-1}^{0j}$ are computed for each filter M^j . At this step the estimates from the previous iteration are mixed to obtain the new filter input values.

$$\hat{\underline{x}}_{k-1|k-1}^{0j} = \sum_{i=1}^r \mu_{k-1|k-1}^{i|j} \cdot \hat{\underline{x}}_{k-1|k-1}^i \quad (9)$$

$$\begin{aligned} \underline{P}_{k-1|k-1}^{0j} = \sum_{i=1}^r \mu_{k-1|k-1}^{i|j} \{ & \underline{P}_{k-1|k-1}^i + [\hat{\underline{x}}_{k-1|k-1}^i - \hat{\underline{x}}_{k-1|k-1}^{0j} \\ & [\hat{\underline{x}}_{k-1|k-1}^i - \hat{\underline{x}}_{k-1|k-1}^{0j}]^T \} \end{aligned} \quad (10)$$

The conditional model probabilities $\mu_{k-1|k-1}^{i|j}$ are given by

$$\mu_{k-1|k-1}^{i|j} = p_{ij} \frac{\mu_{k-1|k-1}^i}{\mu_{k|k-1}^j} \quad (11)$$

and the predicted model probability $\mu_{k|k-1}^j$ is computed by

$$\mu_{k|k-1}^j = \sum_{i=1}^r p_{ij} \cdot \mu_{k-1|k-1}^i \quad (12)$$

Update of the Extended Kalman Filters

For each filter M^j the update is given by the Extended Kalman Filter equations:

$$\hat{\underline{x}}_{k|k-1}^j = \underline{A}^j \cdot \hat{\underline{x}}_{k-1|k-1}^{0j} \quad (13)$$

$$\underline{P}_{k|k-1}^j = \underline{A}^j \cdot \underline{P}_{k-1|k-1}^{0j} \cdot (\underline{A}^j)^T + \underline{W} \underline{Q}_{k-1}^j \underline{W}^T \quad (14)$$

$$\underline{\nu}_k^j = \underline{z}_k - h(\hat{\underline{x}}_{k|k-1}^j, \underline{0}) \quad (15)$$

$$\underline{S}_k^j = \underline{H}_k^j \cdot \underline{P}_{k|k-1}^j \cdot (\underline{H}_k^j)^T + \underline{R}_k^j \quad (16)$$



$$\underline{K}_k^j = \underline{P}_{k|k-1}^j \cdot (\underline{H}_k^j)^T \cdot (\underline{S}_k^j)^{-1} \quad (17)$$

$$\hat{\underline{x}}_{k|k}^j = \hat{\underline{x}}_{k|k-1}^j + \underline{K}_k^j \cdot \underline{\nu}_k^j \quad (18)$$

$$\underline{P}_{k|k}^j = [\underline{I} - \underline{K}_k^j \cdot \underline{H}_k^j] \cdot \underline{P}_{k|k-1}^j \quad (19)$$

where $\hat{\underline{x}}_{k|k-1}^j$ is the predicted state estimate of model M^j , the corresponding prediction covariance is $\underline{P}_{k|k-1}^j$, $\underline{\nu}_k^j$ is the residual, $\underline{S}_k^j$ is the residual covariance, $\underline{K}_k^j$ is the Kalman gain, $\hat{\underline{x}}_{k|k}^j$ is the updated estimate, and $\underline{P}_{k|k}^j$ denotes the updated covariance matrix.

Update of the model probabilities

Assuming a Gaussian distribution, the likelihood of each filter M^j is computed using the filter residual and the residual covariances as

$$\Lambda_k^j = \frac{1}{\sqrt{\det[2\pi\underline{S}_k^j]}} \cdot \exp \left\{ -\frac{1}{2} \cdot (\underline{\nu}_k^j)^T \cdot (\underline{S}_k^j)^{-1} \cdot \underline{\nu}_k^j \right\} \quad (20)$$

and hence the model probability of each filter is updated by

$$\mu_{k|k}^j = \frac{\mu_{k|k-1}^j \cdot \Lambda_k^j}{\sum_{i=1}^r \mu_{k|k-1}^i \cdot \Lambda_k^i} \quad (21)$$

Combination

The last step of the IMM involves the linear combination of the individual results of each filter to an overall estimate. All individual results are weighted with the model probability. This leads to the combined state estimate $\hat{\underline{x}}_{k|k}$ and the state error covariance $\underline{P}_{k|k}$ as

$$\hat{\underline{x}}_{k|k} = \sum_{j=1}^r \mu_{k|k}^j \cdot \hat{\underline{x}}_{k|k}^j \quad (22)$$

$$\underline{P}_{k|k} = \sum_{j=1}^r \mu_{k|k}^j \{ \underline{P}_{k|k}^j + [\hat{\underline{x}}_{k|k}^j - \hat{\underline{x}}_{k|k}] \cdot [\hat{\underline{x}}_{k|k}^j - \hat{\underline{x}}_{k|k}]^T \} \quad (23)$$

6 Results

The parameter adjustment and performance evaluation of the IMM estimator have been carried out with data obtained by the simulation scenario proposed in section 3.

Figure 5 (a) shows the position error in u_1 - and u_2 -direction as estimated by the IMM. To achieve a validation, the standard deviation of the GNSS is also shown. Figure 5 (b) depicts the velocity error of the train and the events A-C. Comparing both figures concludes that the position drifts as indicated in



figure 3 (a), where only the constant acceleration model has been used, are reduced and that the velocity deviation is also acceptable with a standard deviation $\sigma_v=0.7416$ km/h. The standard deviations of the estimated position are $\sigma_{u1}=1.8569$ m and $\sigma_{u2}=2.0891$ m, even though the events A-C representing GNSS signal discontinuances and sensor failures, have occurred.

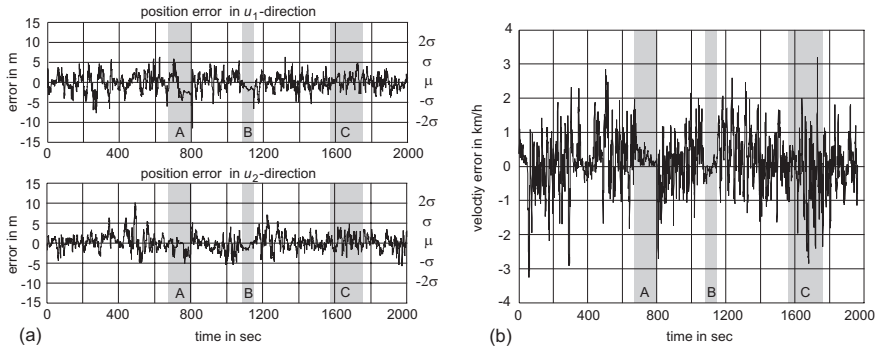


Figure 5: Simulation results estimated by the IMM: (a) position errors of the IMM related to the σ -ranges of the GPS sensor; (b) train velocity error.

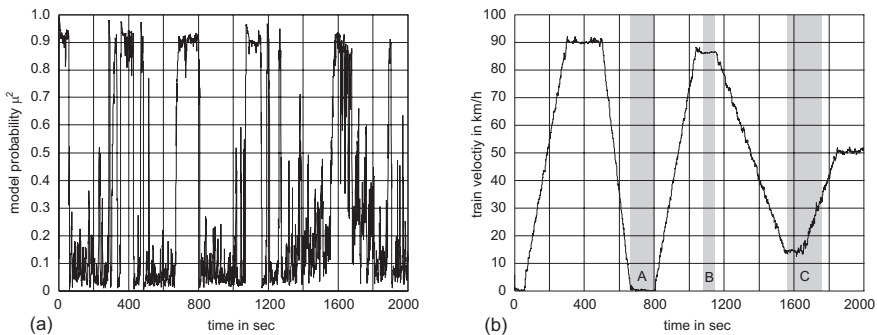


Figure 6: (a) Model probability μ^2 of the EKF based on the reduced order model; (b) velocity estimated by IMM.

Figure 6 (a) indicates the model probability μ^2 of the reduced order model. A comparison with figure 6 (b) shows that the constant acceleration model is used, when the train is changing its travelling speed and that the less dynamical reduced order model is used during constant speed. This concludes that the IMM estimator is switching exactly between the two models as intended and hence reduces the position error in case of sensor failures, thus yielding a reliable and accurate location.



7 Conclusion

This contribution has proposed an accurate and robust location system. It combines the slip-free velocity measurement of the eddy current sensor with the absolute position information of a GNSS sensor unit in a fusion approach. Furthermore, this method adjusts the employed kinematic train model to the actual train dynamics and thus reduces the position drift in case of sensor failures. Simulation results have shown that the presented algorithm is capable of providing excellent results, which have to be verified in future field tests.

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