Computer-aided passenger train scheduling and car circulating

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Abstract

The recent operation practice and market survey in China railway show that passengers with different journey time have different favorite departure and arrival time windows. How to schedule train timetables according to this new market feature is of vital importance to the market share of railway passenger transport. However, in the computer-aided train scheduling systems developed recently, the determination of passenger train departure and arrival times depends fundamentally on the experience of train schedulers. This research is motivated by improving such method in the systems. The aim of this paper is to present some progress of the research. It will focus on the use of computer-aided techniques to optimize the departure and arrival time windows for all passenger trains on the network. The problem of scheduling train departure and arrival time windows to minimize the total passenger inconvenience is solved firstly. Given solutions found at this stage, they are used to determine a subset that minimizes the passenger cars needed to run the services. The techniques have been programmed and tested with the data from China railway. After inputting all data, it takes about 4 minutes on an ordinary 486 PC to output the results.

1 Introduction

The motivation of this research is to improve the computer-aided passenger train scheduling systems, by developing some computer-aided techniques to optimize the departure and arrival time windows for all passenger trains on the network.

The recent operation practice and transport market survey in China railway show that passengers with different journey time have different favorite
paper are described as they relate to finding optimal departure time windows at the origin station only.

3 Techniques

This problem is a complex multiple objective programming problem. According to qualitative analysis results and timetabling practice, the importance of the two objectives mentioned above is not the same. The first one is more important than the second. Therefore the problem can be transferred into a hierarchical single objective problem to find the satisfying solution.

Similarly, owing to the different importance of stations, the first important, second important, ..., kth important station on the relevant network can be selected in turn, optimizing the departure/arrival time windows of those trains that originate from or terminate at the selected station, until all trains in the same category are considered.

The flowchart of optimizing passenger train departure and arrival time windows is shown in Figure 1.

![Flowchart of optimizing train departure/arrival time windows](image-url)
departure and arrival time windows. The most of short-distance (less than 3 hours) passengers prefer to start at dawn and return at dusk. For those with middle-distance (7~18 hours), they prefer to start in the evening and to arrive in the next morning. While the long-distance (18~30 hours) passengers prefer to depart and arrive in the evening or morning. How to schedule train timetables according to this new market feature is of vital importance to the market share of railway passenger transport.

However, owing to the lack of relevant scientific way, the determination of passenger train departure and arrival times presently still depends on the experience of train schedulers. For example, in the computer-aided train scheduling systems developed in U.S. (Jovanovic and Harker [1]), German (Heber and Uhlmann [2]), and China (Guo [3]), the train schedulers have to choose train departure times from very wide time windows, testing one by one, until a satisfying result is obtained. This severely limits the number of options that could be examined and possibly introducing bias into the comparisons between options.

The train scheduling problem is receiving increased attention in the last few years. A survey of the literature may be found in Cordeau et al [4].

This paper aims to analyze the problem, describe the computer-aided techniques used to optimize passenger train departure and arrival time windows, from which train departure or arrival times can be chosen interactively by train schedulers to make train timetable.

2 Problem representation

Train timetabling is the process after train planning. Train planning involves the specification of the origin and terminal stations of trains, moving routes and stopping activities along the routes. Train timetabling involves the specification of train departure and arrival times at stations for the given train plan. The main objectives are to schedule train departure and arrival times so that (1) it is convenient for passengers to travel; (2) the number of passenger cars needed to operate trains is minimum.

Train timetable making is usually in the order of category of trains, i.e. lower grade trains after higher grade trains. For each category of trains the whole network must be considered simultaneously.

All stations on the network can also be graded according to their importance, which is judged by the number of passengers and trains involved, as well as station capacity.

For a planned train, the traveling times between stations are known, so the train arrival times at the destination and intermediate stations on the route can be found easily as the departure time at the origin station is determined. In other words, the favorite arrival time windows at the destination and intermediate stations can be satisfied by scheduling the departure time at the origin appropriately, and vice versa. Therefore, for convenience, the techniques in this
3.1 Minimize the total passenger inconvenience

To minimize the total passenger inconvenience is the first objective to be optimized and is the one related to the quality and level of service provided. The passenger convenience is affected by many factors, such as (a) buying tickets, (b) train departure and arrival times, (c) delays, (d) time spent connecting to other trains, (e) the number of passengers, etc. But (b) and (d) are the factors related to the objective in the timetabling process. The effect of (d) is weaker than (b) owing to small number of passengers involved, and can be considered by choosing the departure times at the origin station appropriately.

The passenger convenience, as mentioned in the introduction section, is different as trains depart/arrive in different time windows. The closer to their favorite time windows, the more convenient it is for passengers. Therefore, customer penalty can be introduced for every time window. The problem is how to schedule the trains to minimize the total customer penalty (passenger inconvenience). Obviously, it is a sort of open vehicle scheduling problem with soft time windows, that to our knowledge has not been presented before in the literature.

Let us divide 24 hours of every day into \( p \) time windows \( \{t_1, ..., t_p\} = W \), according to relevant passenger inconvenience. \( a_{ok} \) is the penalty of those passengers to depart from origin station if the departure time of train is within time window \( t_k \). \( b_{uk} \) is the penalty of those passengers to arrive at destination if the arrival time of train is within time window \( t_k \). \( d_{sk} \) is the penalty of those passengers to arrive at or depart from intermediate station \( s \) if the arrival or departure time of train is within time window \( t_k \). \( a_{ok}, b_{uk}, d_{sk} \geq 0 \). The less their value, the better the passenger convenience. For origin, intermediate stations and destination, the number of passengers involved are usually different, so the effect of passenger inconvenience is also different. The weight coefficients \( w_o, w_i, w_d \) can be introduced to describe the differences.

Let \( f \) represent the minimum train departure interval of the selected station. Then 1440 minutes of every day can be divided into \( n = 1440/f \) departure intervals. At most one train is permitted to depart within each interval.

Let \( m_1 \) denote the number of relevant higher grade trains, \( m_2 \) denote the number of trains of considering grade to depart from the selected station. If \( m = (m_1 + m_2) < n \), then \( (n-m) \) dummy trains are introduced. Thus a \((n \times n)\) matrix \([c_{ij}]\) can be obtained, where the columns represent the \( n \) departure intervals, the rows represent the \( n \) departure trains. The element \( c_{ij} \) denote the total passenger penalty when the departure time of train \( i \) is within departure interval \( j \). The less the value of \( c_{ij} \), the better the passenger convenience. Generally, for \( j=1, ..., n, \)
$$c_{ij} = \begin{cases} 
0, & \text{if departure interval } j \text{ is one of the optimal interval of train } i (i = 1, \ldots, m), \\
\infty, & \text{if departure interval } j \text{ is occupied by a predetermined train, or for } i = m + 1, \ldots, n. \\
& \text{or is not an optimal one of train } i (i = 1, \ldots, m_1), \\
& \text{or is an infeasible one of train } i (i = m_1 + 1, \ldots, m). \\
w_o \alpha_{sk} + \sum_{s \in S} w_s \delta_{sk} + w_r b_{sk}, & \text{if departure interval } j \text{ is an feasible one of train } i \\
& \text{and within time window } t_k (i = m_1 + 1, \ldots, m, t_k \in W). 
\end{cases}$$

The predetermined trains also include those higher grade trains with only one optimal departure interval. $S$ is the set of the intermediate stations of train $i (i = m_1 + 1, \ldots, m)$. The feasibility of every departure interval $j$ of train $i$ ($i = m_1 + 1, \ldots, m$) should always be checked by seeing if the constraints of train operations on the route are satisfied as the departure time of train $i$ is assigned to departure interval $j$. The constraints include all kinds of operational interval limits and station capacity limits. Let

$$x_{ij} = \begin{cases} 
1, & \text{if the departure time of train } i \text{ is assigned to departure interval } j. \\
0, & \text{otherwise.}
\end{cases}$$

Thus the problem of minimizing total passenger inconvenience can be transformed into following assignment problem.

Minimize $\sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$.

subject to $\sum_{j=1}^{n} x_{ij} = 1, \quad j = 1, \ldots, n.$

$\sum_{i=1}^{n} x_{ij} = 1, \quad i = 1, \ldots, n.$

$x_{ij} \in \{0, 1\}, \quad i, j = 1, \ldots, n.$

The optimal solution of this assignment problem is $m$ different optimal departure intervals of $m$ trains. Usually the problem has alternate optimal solutions. All of them consist of the optimal departure time windows of the first objective.

For the trains terminated at the selected station, the optimal arrival time windows can also be found by the similar process.

### 3.2 Minimize the number of passenger cars needed to operate trains

This objective is the one related to transportation cost. Generally the set of passenger cars of every pair of trains are special-purpose. So we focus on this case.

The different combinations of departure and arrival times of forward trip train and return trip train will lead to different number of passenger cars needed. The problem is, within the optimal solutions of the first objective, to specify the
combinations of train departure and arrival intervals to minimize this objective.

Let $T_{ABi}$ and $T_{BAi}$ denote the travel time of train $i$ from station $A$ to $B$ and from $B$ to $A$ respectively. $L_{Ai}$ and $L_{Bi}$ are the minimum retention time of passenger cars of train $i$ at station $A$ and $B$ for technical operations respectively. Their value depends on whether the station is its home depot or turn-around depot. $t_{Ai}$ and $t_{Bi}$ represent the actual retention time of passenger cars of train $i$ at station $A$ and $B$ respectively. $k_i = e_i / 24$ is the number of passenger cars of train $i$ needed. $S_i = (t_{Ai} + t_{Bi} - (L_{Ai} - t_{Bi}))$ is the surplus retention time of passenger cars of train $i$ within $Q_i$.

For an optimal departure interval $j = [f_j, f_{j+1})$ of train $i$ from a selected station $A$, let $f_j$, the lower bound of the interval, be the train departure time, then the passenger cars can return to station $A$ at least in $T = (T_{ABi} + L_{Bi} + T_{BAi})$ hours (see Figure 2). Its earliest arrival time $t_i = f_j \oplus T$, where $\oplus$ is the addition with 1440 as modulus. Its maximum surplus time window at station $A$ is $[t_1, t_2)$, where $t_2 = t_1 \oplus S_i$. For its corresponding return trip train, there are following two cases.

Case 1 There is at least one such optimal arrival interval $[f_{j'}, f_{j'+1})$ that $f_{j'}$ is within time window $[t_1, t_2)$. Then $[f_j, f_{j+1})$ and $[f_{j'}, f_{j'+1})$ is such a combination that both the passenger inconvenience and the number of passenger cars needed of train $i$ are minimum.

Case 2 There is no any such optimal arrival interval $[f_{j'}, f_{j'+1})$ that $f_{j'}$ is within time window $[t_1, t_2)$. Then $[f_j, f_{j+1})$ is such a departure interval that the passenger inconvenience is minimum but one more set of passenger cars is needed.

Let $m$ represent the number of all kinds of trains from the selected station every day. The number of departure intervals is $n = 1440 / 1$. If $m < n$, then $(n-m)$ dummy trains are introduced. For $j = 1, \ldots, n$, let

$$c_{ij} = \begin{cases} 0, & \text{if departure interval } j \text{ is the one in case 1 } (i = 1, \ldots, m), \\ 1, & \text{or for } i = m+1, \ldots, n, \\ \infty, & \text{if departure interval } j \text{ is the one in case 2 } (i = 1, \ldots, m). \end{cases}$$

$$x_{ij} = \begin{cases} 1, & \text{if the departure time of train } i \text{ is assigned to departure interval } j, \\ 0, & \text{otherwise.} \end{cases}$$

![Figure 2](image-url)
Thus the problem can also be transformed into a similar assignment problem as above. The optimal solution of this assignment problem is \( m \) different optimal departure intervals of \( m \) trains, which satisfy both objectives (1) and (2). Usually, the problem has alternate optimal solutions. Let the total number of them is \( N \). All of them consist of the optimal departure time windows of the two objectives.

Every optimal departure interval in each solution just found corresponds to one or more return trip train arrival interval of case 1 or 2. Although all optimal departure intervals are different each other, some corresponding return trip train arrival intervals of case 1 or 2 may be the same. Therefore, those arrival intervals must be further optimized. The problem again can be transferred into \( N \) similar assignment problems, where

\[
c_{ij} = \begin{cases} 
0 & \text{if arrival interval } j \text{ is the one in case 1 (} i = 1, \ldots, m), \\
1 & \text{if arrival interval } j \text{ is the one in case 2 (} i = 1, \ldots, m), \\
\infty & \text{if arrival interval } j \text{ is not an optimal one of objective (1) (} i = 1, \ldots, m). 
\end{cases}
\]

\[
x_{ij} = \begin{cases} 
1, & \text{if the arrival time of return trip train } i \text{ is assigned to arrival interval } j. \\
0, & \text{otherwise.}
\end{cases}
\]

Let \( Z_k \) denote the objective function value of \( k \)th assignment problem, \( k = 1, \ldots, N \). \( Z^* = \min \{ Z_1, \ldots, Z_N \} \). Then those arrival intervals that their objective function values are equal to \( Z^* \) are the optimal ones which satisfy both objectives (1) and (2). Thus the train schedulers can choose train departure or arrival times from these narrow time windows to make train timetables.

### 3.3 Solution algorithms

The modification of the Hungarian algorithm by Wright [5] is adopted in solving the assignment problem. The branch-and-bound technique by Fu and Xiao [6] is used to find alternate optimal solutions for assignment problem. The algorithms have been programmed using Turbo Pascal.

### 4 Experimental results

Let \( I = 20 \) min. Take the data of 20 domestic middle and long distance express trains originated from Western Station of Beijing in the 2001 National Passenger Train Timetable of China Railway as input. If both the departure and arrival time window is set to be between 07 and 24 for all kind of passengers, then the allowable departure time windows for these trains are shown in Figure 3. As mentioned in the Introduction section, the train schedulers have to choose train departure times from these very wide time windows for the computer-aided train scheduling systems.
Figure 3: Allowable departure time windows

Suppose the level of inconvenience for the passengers on departure/arrival times within different time windows is shown as Table 1. The optimal departure time windows for these trains can be found by the techniques proposed in this paper in about 4 minutes with an ordinary 486 PC, see Figure 4. The predetermined trains in this context are the two international trains. In the actual timetable the departure times of 15 trains are within these optimal departure time windows, 2 trains are very close to them (less than 7 minutes).

Table 1. The level of inconvenience for passengers

<table>
<thead>
<tr>
<th>time window</th>
<th>originating time</th>
<th>arrival/departure time at stops</th>
<th>terminating time</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-5</td>
<td>M</td>
<td>M</td>
<td>M</td>
</tr>
<tr>
<td>6</td>
<td>M</td>
<td>M</td>
<td>8</td>
</tr>
<tr>
<td>7</td>
<td>8</td>
<td>6</td>
<td>0</td>
</tr>
<tr>
<td>8-9</td>
<td>3</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>10-12</td>
<td>2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>13-15</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>16-18</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>19-21</td>
<td>0</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>22</td>
<td>0</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>23</td>
<td>1</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>24</td>
<td>8</td>
<td>M</td>
<td>M</td>
</tr>
</tbody>
</table>

M is a sufficiently large positive integer.
Further optimization can be made together with the lower grade trains. From these narrow time windows train departure times can be chosen interactively by train schedulers to make train timetable.

5 Summary

Based on the fact that the importance of different category of trains, different stations on the network, and different objectives to be optimized are different, and passengers with different travel distances have different favorite departure and arrival time windows, the passenger train timetabling problem is formulated as a multi-objective programming problem. A hierarchical single objective solution to the problem is proposed. In the order of category of trains and importance of stations, the problem of scheduling trains to minimize the total passenger inconvenience is solved firstly. Given solutions found at this stage, a subset that minimizes the number of passenger cars needed is determined. Both problems are formulated as assignment problem to solve. The algorithms have been programmed and tested with the data from China Railway. The proposed computer-aided techniques can always give feasible solutions and can be used to optimize passenger train departure and arrival time windows in train timetabling.

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References


