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Performance evaluation of network timetables using PETER

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Abstract

Efficient evaluation and comparison criteria of network timetables are not yet well-defined and planners usually have to rely on time-consuming simulations of (sub)networks. PETER (Performance Evaluation of Timed Events in Railways) is a new analytical tool that quickly assesses network performance indicators in a deterministic setting corresponding to the design times used in timetable construction. A timetable represents the steady-state according to which trains should operate. Timetable performance is then analysed as the effort of returning to the steady-state after disruptions. Systems of interconnected train services have a special structure that can be described by (max,+) recursions, resulting in a linear system in max-plus algebra. Efficient algorithms have been developed to analyse such systems in real-time. Performance indicators include cycle times, throughput, and stability margins, which are related to critical circuits in the train network. Timetable robustness is analysed by cumulative recovery times and delay propagation.

1 Introduction

Train operations are typically exposed to random variations in train running times and dwell times resulting in primary delays. Moreover, once a train is delayed this may produce severe delay propagation over the network when trains are highly interconnected. The sensitivity to delays of a train network timetable is managed by incorporating recovery times or buffer times. A robust timetable must be able to deal with a certain amount of delay without intervention of process operators. Timetable robustness is thus comparable to the amount of

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effort for schedule adherence after disruptions. Evaluating a train network timetable on robustness is an important part of the timetable design process and typically requires a computer-aided approach, as the (circular) interactions of train movements on a railway network are very complex.

Performance of a rail traffic network depends on many aspects. However, the operational traffic conditions are completely summarized in the timetable. A timetable implicitly includes the set of train lines, train characteristics, and rail infrastructure (railway network and signalling system) Alternative line systems or altered allocations of train types to lines (and thereby changed running times) result in distinct network timetables that can be evaluated separately.

Railway planning systems that are capable of analysing large-scale highly-interconnected timetables are rare. Simulation is usually pursued to analyse complex systems when no analytical tools are available. However, simulation is typically very time-consuming, which forces railway planners to concentrate on stations or rail corridors and thereby discarding network interdependencies. Recently some tools have been developed that analyse train traffic on large-scale railway networks. FASTA [1] is a Swiss stability analysis tool based on simulation. In the Netherlands, the simulation system SIMONE [2] has been developed for testing stability. This tool is compatible with the Dutch timetable design system DONS [3]. CAPRES [4] is a recent developed Swiss *analytical* capacity analysis system based on saturation of timetable variants.

This paper presents PETER (Performance Evaluation of Timed Events in Railways), an analytical tool based on a (system-theoretic) max-plus algebra performance evaluation approach [5]. The railway model and a first prototype has been introduced at COMPRAIL 2000 [6]. Based on this experience, Delft University of Technology decided to cooperate with ORTEC to develop a user-friendly decision support system for railway planners. PETER now combines state-of-the-art efficient mathematical algorithms with a graphical man-machine interface that suits railway planners. This system is presented in the current paper. For mathematical details we refer to Goverde & Soto y Koelemeijer [7].

The paper is outlined as follows. The next section introduces the PETER max-plus algebra modelling approach. Section 3 considers the PETER user-interface. The analysis opportunities and network performance indicators are explained in Section 4. Section 5 presents an application to the Dutch intercity network of 2000-2001. Finally some conclusions are presented.

2 Max-Plus Algebra

PETER is based on the max-plus algebra modelling of transportation networks [6],[8]. Here, the train network dynamics are described as a Discrete Event Dynamic System (DEDS), that is, train movements are modelled by the occurrence of discrete events and in particular train departures. The actual model is a timed event graph and its state representation consisting of (max,+)-recursions. This abstraction generates a mathematical structure that is capable to capture large-scale but tractable models whereas it keeps the essential features of complex train interactions. The (max,+)-recursions can be viewed as linear

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(dynamic) systems in max-plus algebra. A system theory has been developed for the analysis of max-plus linear systems analogue to conventional system theory of differential or difference equations, see Baccelli *et al.* [5].

This paper concentrates on network timetable evaluation in a deterministic setting. All process times (running times, dwell times, transfer times) and headway times are assumed to be deterministic corresponding to the design times used in the timetable design. The max-plus model is a discrete-event system that can be represented as

$$x(k+1) = A \otimes x(k) \oplus d(k+1), \tag{1}$$

where $x(k)$ is a vector of the train departure times after k periods (hours), $d(k)$ is a vector of the *scheduled* departure times in the kth period, and A is the state matrix containing the process times between timetable (departure) events. The construction of the state matrix A also reflects train (departure) orders and the allocation of trains to the network, whereas the vectors x and d may be augmented to obtain the first order representation of eqn (1), see [6],[7]. The mathematical operators $\oplus$ (maximum) and $\otimes$ (addition) are the basic max-plus algebra operators, see e.g. [6],[8]. The first term on the right-hand side of eqn (1) is the ready-to-depart time vector based on the current state and the process times applied in the current period. Eqn (1) can then read as follows: the departure times in the next hour are the maximum of the ready-to-depart times (1st term) and the scheduled departure times (2nd term). Current delays are thus propagated if an entry of the 1st term exceeds the 2nd term and otherwise the trains wait on their scheduled departure time.

Network analysis methods usually concentrate on critical paths in the network (PERT/CPM), whereas analysis methods of dynamic systems focus on steady-state solutions. In the case of periodic operated train networks, the critical circuits in the network and the periodic timetable are the main focus points. PETER contains critical circuit analysis on the one hand and timetable robustness on the other hand. The critical circuit determines the least cycle time in which a periodic timetable may operate, whereas the timetable constitutes the steady state according to which trains should operate.

PETER has been developed to analyse periodic timetables in which all train departures repeat at a regular (hourly) interval. In the Netherlands and most European countries this timetable strategy is pursued. However, the method is not restricted to periodicity and can be adapted to suit other timetable concepts.

3 PETER Interface

PETER is a stand-alone stability analysis tool for periodic timetables of highly-interconnected large-scale train networks. The tool contains the following functionalities:

- data import and automatic model building,
- data view and editor,
- analysis computations,

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- results view and network visualization,
- data and results export.

PETER has a modular system architecture written in Delphi and runs on a standard PC under a Windows 95/98/NT/2000 operating system. The policy iteration algorithm of the critical circuit analysis is a callable routine implemented in C. The modular design facilitates future enhancements.

Networks can either be created using the build-in editors or imported from ASCII input files with a prescribed format. A *generic data* input file contains information on stations, train lines, and connections, and is easily generated by standard text editors or spreadsheets. After reading the input data, PETER automatically constructs the max-plus model and draws the network view. A build-in editor can also be used to edit data. After editing a feasibility check follows and a new max-plus model is constructed. Edited or manually inserted networks in PETER are also saved in the generic format.

The modular system architecture of PETER enables easy extension of functionalities. As an example, a module is provided that directly imports data files from the Dutch timetable design tool DONS [3]. Another envisaged extension is a direct access link to the DONS Data Market [9], by which railway planners at Railned or NS Reizigers can apply PETER directly with the full functionality of DONS.

The delay propagation module requires a set of initial delays, which can be inserted manually or imported from a *delay file*. Manually inserted initial delays can also be saved to the standard delay file for future usage. The delay files enable easy access to a range of scenarios for delay propagation.

In the menu Tools the analysis computations are activated. A disabled computation item means that computed results are still valid. In the Tools menu the user can also choose to include or exclude all transfer times. A toggle denotes the current status. By excluding transfer times the situation is analysed where transfer connections can be cancelled, the so-called *controlled* network. The analogue holds for layover times. Excluding layover times presumes that reserve trains are available when necessary or that train circulations are not fixed to train line routes.

Results can be shown in tables, visualized in the network (see Section 5), and saved in output files. Also the constructed max-plus model can be saved in an output file.

4 Stability Analysis

4.1 Introduction

The max-plus model of the train network consists of two parts: the *state matrix* constructed from process times between timetable events, train orders, and the allocation of trains to the network, and the *timetable vector* containing the scheduled event times. Analysis of the state matrix identifies critical circuits and network stability indicators associated to the train network *structure* (interactions, departure sequences, train allocation). This *critical circuit analysis*

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requires computation of the eigenvalue and eigenvector of the matrix A (in max-plus algebra), i.e. solving

$$A \otimes v = \lambda \otimes v \,. \tag{2}$$

A very efficient policy iteration algorithm [10] is implemented in PETER to solve the eigenvalue problem (2). Even large-scale networks of several 10,000 variables are solved within a few seconds. The eigenvalue λ is the minimum network cycle time and the eigenvector v the corresponding timetable [8]. This timetable gives the earliest possible departure times (with respect to the imposed train allocation and train orders) according to which the train network can operate with minimum cycle time. Applying this (feasible) timetable yields the *earliest operation mode (EOM)* of the train network. Critical circuit analysis is considered in more detail in Section 4.2.

The actual *periodic operation mode (POM)* of the train network that is governed by the given network timetable – modelled as the timetable vector – differs from the EOM by additional slack times between events. These slack or buffer times are necessary for robustness of the timetable. A second stage in the network analysis therefore focuses on the distribution of buffer times over the network timetable. This is referred to as *recovery time analysis*. Also *delay propagation* can be utilized to analyse timetable sensitivity to sets of initial delays. Recovery time analysis and delay propagation are considered in Section 4.3 and 4.4, respectively.

4.2 Critical Circuit Analysis

As already mentioned, analysis of the state matrix A gives the minimum cycle time in which the train network could operate. This minimum cycle time equals the average travel time of trains on the critical circuit in the network. This critical circuit is thus the 'slowest' circuit in the network. The critical circuit analysis computes the following network performance indicators:

- the minimum cycle time λ,
- the throughput $\rho = \lambda / T$, where T is the actual cycle time (usually $T = 60$),
- the stability margin Δ.

The implications of these indicators are presented in some detail below. Also the associated critical circuits are explicitly identified. Improving system stability then implies to present changes in the sequence of critical processes that make up the critical circuit. Possible actions are changing train orders, adding (or move) a train to one of the critical processes, or decreasing the critical process time by e.g. faster train units, shorter dwell times, shorter transfer times (cross-platform transfers), or infrastructure investments.

4.2.1 Cycle Time

The actual cycle time should clearly exceed the minimum possible one. This represents a stability test: if $\lambda \leq T$ then the system is stable. However, if λ is close to the cycle time then the system is sensitive to disruptions. Stability here means

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that there is enough slack on the critical circuit, so that delays will surely settle.

In practice, a network timetable contains several components (subnetworks) that are mutually connected only by the periodic timetable. Hence, the minimum cycle time corresponds to the critical circuit(s) of the most critical component. In a well-balanced system the critical cycle times of all components should be comparable, corresponding to an even distribution of trains to the network. PETER therefore computes the critical circuits and cycle times of each component in the train network. The *number of components* is an indication of the complexity or connectedness of the train system. The smaller this number is, the higher the system complexity. The associated range of critical cycle times quantify the *balance* between the various subnetworks.

4.2.2 Throughput

The *throughput* ρ is a network performance indicator denoting a trade-off between maximum performance (under ideal circumstances) and robustness. It presents the minimum cycle time relative to the actual cycle time. Obviously, a stable system requires $0 < \rho \leq 1$, where $\rho = 1$ is the saturated case in which the mean cycle time of trains on the critical circuit is just (a multiple of) T minutes. When $\rho < 1$ the system operates below its maximum (theoretical) performance and hence contains buffer times to compensate for delays.

4.2.3 Stability Margin

The *stability margin* Δ is a network performance indicator of robustness of the train network. The stability margin is defined as the maximum simultaneous increase of all process times such that the train network can still be operated with cycle time T. This margin may differ from the value $T - \lambda$ as another circuit may become critical when the process times are increased. In fact, the stability margin is computed by solving an auxiliary eigenvalue problem [7]. It corresponds to the circuit with the least average buffer time. This circuit is not necessarily the same as the critical circuit with the critical cycle time but depends on the amount of running trains in each circuit.

4.3 Recovery Time Analysis

A timetable generally contains slack to compensate for process time variations. This slack is provided in scheduled process times (e.g. running time margins) or between two directly interacting trains (buffer times). From a network point of view a sequence of train runs embraces an accumulation of slack times. These train sequences include train runs connected by train stops or transfers but also more abstract headway interactions. Note that we here use ‘train’ as a train run between two adjacent stations.

For any two trains we are interested in the amount of cumulative slack time over a train sequence going from one train to the other. However, in general multiple train sequences can be considered between two trains. Therefore we define the *cumulative recovery time* from train i to train j as the minimum cumulative slack time over all train sequences going from train i to j in the train

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network. Note that this is equivalent to the largest delay of train i that not reaches train j. We distinguish between 3 types of indicators based on recovery times:

- *delay impact* of a train to all trains over subsequent train sequences,
- *delay sensitivity* of a train w.r.t. delayed trains on incoming train sequences,
- *feedback recovery time* of a train: minimum recovery time over any circuit.

Delay impact refers to possible future delays and delay sensitivity depends on past delays that may have propagated over previous train sequences. The feedback recovery time gives the maximum delay of a train that will not return.

A matrix of the cumulative recovery times between any two trains can be computed numerically using an all-pair shortest path algorithm. In PETER, the efficient Johnson repeated single-source shortest path algorithm for sparse networks is implemented [11]. This method uses the well-known Dijkstra algorithm as a subroutine, where the priority queue is implemented as a Fibonacci heap. The delay impact vector of a train i equals the i-th column of the recovery matrix, the delay sensitivity vector of train i equals the i-th row of the recovery matrix, and the feedback recovery times of all trains are given by the diagonal entries of the recovery matrix.

4.4 Delay Propagation

The max-plus model and an initial delay vector – containing the departure delays at a certain reference time – are used to simulate the delay propagation of initial delays over time and space. In fact, the max-plus (sparse) matrix equation generates a particular efficient single-run discrete-event simulation. The computation time is a fraction of a second, even for large-scale networks.

The initial delay vector summarizes all train delays in the network at a certain reference time. The max-plus model then computes the delay propagation over the network and over future periods. The simulation output is given in different levels of detail. The most refined output gives the delay of each train in terms of the departure delay and its period-of-occurrence. The result can be given in tabular form or in a network visualization where the amount of delay is indicated by a colour scheme, see Figure 2. Aggregated output is more convenient for comparison of different scenarios or distinct timetables. The presented aggregated results are the total secondary delay, number of reached trains, average secondary delay, number of reached stations, and the settling period.

By using nominal process times the emphasis of the delay propagation model is on analysing the interconnection structure as dictated by the timetable. Computations of many scenarios are achievable to find structural shortcomings in a timetable design or to evaluate the effect of control strategies. Different control strategies result essentially in distinct delay scenarios represented by an initial delay vector. Using the max-plus model the implications of the different delay scenarios are computed and compared. This scenario analysis is useful in deriving all kinds of standard control strategies in case of e.g. maintenance works and track obstructions. Also online decision support systems may benefit from quickly evaluating several delay scenarios to resolve detected conflicts.

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5 Case Study: Dutch Intercity Network 2000/2001

The Dutch Intercity (IC) Network of 2000-2001 contains 19 IC train lines serving 70 (IC) stations. This network is obtained from DONS and imported into PETER. The max-plus model consists of 317 departure events (nodes) and 361 line segments (arcs), including 44 connections between train lines. A minimum of 112 trains is necessary to cover all IC train circulations. The model contains 137 tokens, which is a measure of the problem dimension.

The policy iteration algorithm computes 10 critical circuits associated to 10 network components. Figure 1 shows the most critical circuit and its component. The critical circuit contains IC1600 and IC500 trains between Schiphol and Groningen, including 8 stops, 2 transfers at Amersfoort, and 2 turns at Groningen and Schiphol, respectively. The minimum cycle time is 55:12 minutes. The train network is operated according to an hourly timetable, giving a throughput of 92%. The stability margin on this critical circuit is 1:45 minutes.

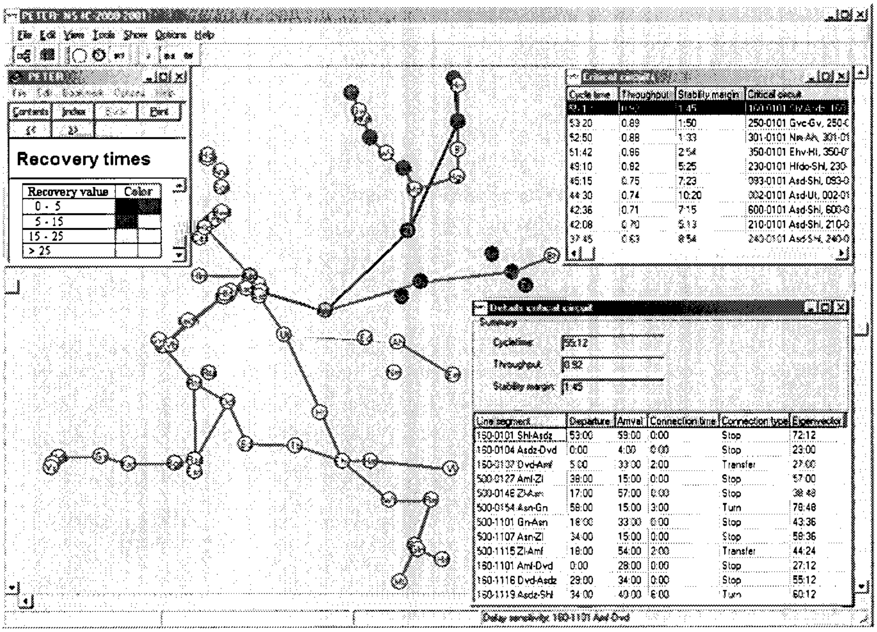

Figure 1: IC network, its critical circuit (black links) and associated component (white links), and delay sensitivity of IC 1600 Amf-Dvd (coloured nodes)

Also the recovery matrix is computed. All trains on the critical circuit have a feedback recovery time of 24:00 min. As an example, Figure 1 also visualizes the delay sensitivity of the IC1600 train from Amersfoort to Duivendrecht (the coloured nodes). The preceding stops of the IC1600 train are most critical, where a delay at Apeldoorn has zero recovery time, whereas the robustness is gradually increased over the preceding stations to 4 min recovery time at Enschede. The

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preceding stops of the feeder IC500 train are next critical, with 4 min recovery time after departure at Zwolle, increasing to 8 min in Groningen and 9 min in Leeuwarden (the trains from Leeuwarden and Groningen are coupled in Zwolle). Moreover, the IC1600 train may get delayed by the IC21600 from Amsterdam CS (11 min), the IC12500 from Rotterdam (13 minutes), and the IC500 from The Hague CS (19 min) and the intermediate stops with decreasing recovery time. This clearly shows the interactions in the most critical network component.

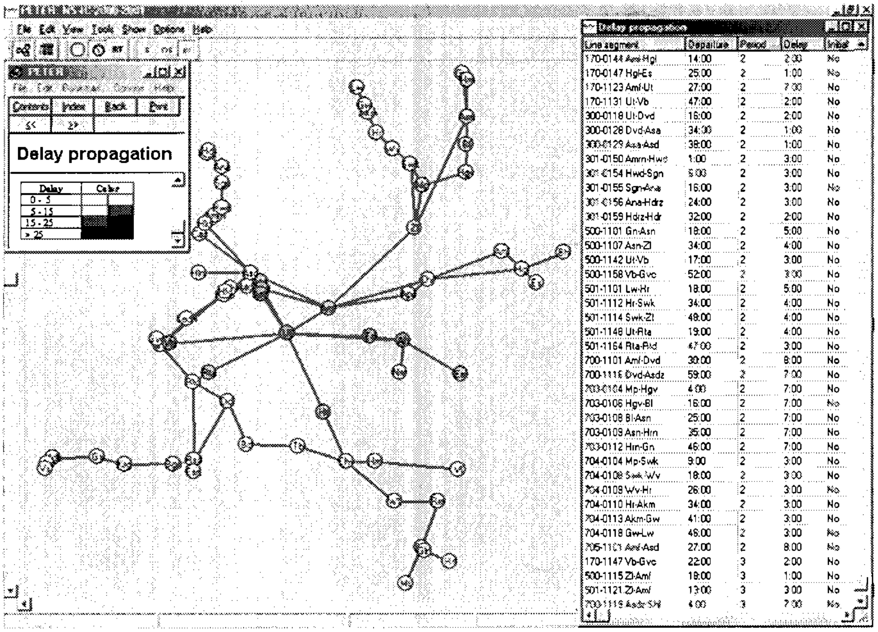

Figure 2: Delay propagation over the IC network

More detailed insight in the interconnectedness of the IC train services is obtained by studying the delay propagation of initial delays. As an example, we consider a scenario in which during one hour each of the 20 departing IC trains from Utrecht CS has a 10 minute departure delay. Figure 2 shows the resulting delay propagation. In total 43 stations are reached, including the terminal stations Groningen, Leeuwarden, and Nijmegen, where the delays propagate over new train circulations unless reserve train units are dispatched. The delays also reach 5 other terminal stations, however at these stations enough buffer time is available to guarantee stable train circulations, i.e., new circulations can be initiated in accordance to the process plan. In the other directions the delays have been eliminated by the recovery times before reaching the terminal stations. Apart from the 10 initial trains, 13 additional trains get delayed, with an average departure delay of 5:14 min, including trains starting a new circulation. In total 103 departure delays are computed, including the (reduced) delays at successive stops of a train line. Eventually, all delays are eliminated after 3 hours.

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6 Conclusions

Network timetables have an intrinsic structure that is conveniently modelled by max-plus algebra. Efficient analysis methods are applied to this max-plus model including spectral analysis, shortest path methods, and matrix computations. The stability analysis yields network performance indicators (minimum cycle time, throughput, stability margin), recovery times (delay impact vector, delay sensitivity vector, feedback recovery times), and delay propagation results.

The max-plus modelling and analysis techniques are implemented in the software tool PETER. PETER has a user-friendly man-machine interface and all computations are in real-time. PETER is also compatible with DONS – the timetable design tool of the Dutch Railways. All analysis methods in PETER are hence available to quickly evaluate timetable structures generated by DONS, which is a major contribution to Dutch computer-aided timetable design.

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