Study on measurement of rail longitudinal force on slab track by rail lifting

Y. Sato
Nippon Kikai Hosen KK. (JR Central Group), Japan

Abstract

The measurement of rail longitudinal force in CWR (Continuously Welded Rail) is important so as to avoid the rail failure in winter and the track bucking in summer. The effectiveness of rail lifting method for it in winter was demonstrated on the ballasted track experimentally and theoretically. As the application of it to the slab track is conceived in Japan, theoretical analyses are proceeded for it. As a first approach, the elastically supported rail with single stiffness outsides of a released section for rail lifting at the center of it is analyzed. The ruff characteristics of the measurement are given with it. However, as the responses of rail fastening devices are quite different for the amount and direction of force in the section of elastically supported rail, the second model with two kinds of stiffness for this section is analyzed. As astonishment, the remarkable decrease of force to the first rail fastening device from the boundary of the released section is given. It is sure that such a decrease in displacement is brought by the large stiffness in the outer section of rail elastic support. With these facts depending on more plausible analysis, all the difficulties for the measurement are excluded. The linearity of lifting force to the rail tension is guaranteed.

1 Introduction

The measurement of rail longitudinal force in tension in winter in CWR (Continuously Welded Rail) is important so as to know the rail neutral temperature and to avoid rail failure in winter and track bucking in summer. The effectiveness of measuring it by rail lifting in the released section of rail from sleepers is demonstrated experimentally in South America [1] and UK [2] and
theoretically by the present author [3]. This measurement is named simply “Rail lifting method”.

As the application of it is conceived for the slab track in Japan, the theoretical analysis is proceeded. here

2 First approach

2.1 Model

The model for analysis is set as shown in Figure 1 for a half space as the first approach. In it the released section of rail is assumed to be a beam with weight and to be connected to a semi-infinite long beam on elastic foundation. The latter is not necessary to consider its weight by putting it at the settled position under its weight. The beams are in tension as shown in the figure and lifted at the center of the released section.

![Figure 1: Model of massive beam connected to elastically supported beam](image)

2.2 Equations

The differential equations for the model are given as follows;

\[ 0 \leq x < L \quad EI \frac{d^4 y}{dx^4} - T \frac{d^2 y}{dx^2} - \rho A = 0 \quad (1) \]

\[ L \leq x \quad EI \frac{d^4 y}{dx^4} - T \frac{d^2 y}{dx^2} + ky = 0 \quad (2) \]

where \( x \) is the distance from the position of lifting, \( L \), a half length of released section, \( EI \), vertical rigidity of rail, \( y \), deflection, \( T \), rail tension, \( \rho A \), rail weight per length and \( k \), rail supporting stiffness.

The General solution is given as follows;

\[ 0 \leq x < L \quad y = A_{11} e^{\frac{T}{\sqrt{EI}}} + A_{12} e^{-\frac{T}{\sqrt{EI}}} - \frac{\rho A}{2T} x^2 + A_{13} x + A_{14} \quad (3) \]
They are solved for following boundary conditions.

At $x=0$

\[ y'=0, \quad -EIy'''=P/2 \quad \text{(1)} \]

At $x=L$

\[ y_{L1}=y_{L1}y', y_{L1}''=y_{L1}''', y_{L1}'''=y_{L1}''''=L_1 \quad \text{(2)} \]

### 3 Analyzed results 1

#### 3.1 Conditions

As an actual condition, 60kg rail fastened on slab with rail fastening devices of type 8 shown in Figure 2 and pulled with the tension of 40tf is released from the slab for the section of 20m and lifted by 7cm at the center of it. Following three cases for rail supporting stiffness are analyzed.

1. $k=272\text{kgf/cm}^2=17\text{tf/cm}^2/62.5\text{cm}$: Stiffness of two ends of leaf spring. Rail is parted from rail pad by rail lifting force which is larger than rail pad pushing down force.
2. $1232\text{kgf/cm}^2=77\text{tf/cm}^2/62.5\text{cm}$: Stiffness by two ends of leaf spring and rail pad considering excess rail pad pushing down force
3. $969\text{kgf/cm}^2=60\text{tf/cm}^2/62.5\text{cm}$: Stiffness by the inner end of leaf spring and rail pad. Normal condition

The end of rail released section is to be at the center of a slab.

#### 3.2 Displacements and forces

Results are shown in Figure 3. As the rail pad is parted from the rail with the displacement exceeding $0.017\text{mm}=1\text{tf}/60\text{tf}$ of rail pad deformation under fastening force, the Case (1), just two points touch of leaf spring ends, is considered to be a good approximation for the large displacement of first and second rail fastening devices from the boundary of the released section, though as for the displacement to the lower direction or small displacement in other rail fastening devices, the Case (3) could be a good approximation.

By multiplying the rail supporting stiffness to the rail displacement and...
integrating them for the length of 62.5 cm in both sides of rail fastening device the forces acting on each rail fastening device are calculated as shown in Table 1. It shows that the forces acting on the first rail fastening device are 3.8 to 5.8 tf. They are fairly large values, but it may be at the level of 3.8 tf because the case (1) could be the actual situation. The leaf spring could support it because the strength of it is 4.8 tf. However, the effects of Case (3) for the second or third rail fastening device and followings are not clear.

The resultant force to lift the slab of 5 m long works as shown in Figure 4. It is the difference of forces acting to every rail fastening devices in Table 1. It is 1.5 to 2 tf. The acting position of the slab lifting force is near the slab end in the counter side of fastened rail fastening devices as shown in Figure 4. The actual positions of the resultant force are shown in Table 2. As the track slab is...
Table 2. Position the resultant force

<table>
<thead>
<tr>
<th>Case</th>
<th>1 (272kgf/cm²)</th>
<th>2 (1232kgf/cm²)</th>
<th>3 (969kgf/cm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>L (cm)</td>
<td>342</td>
<td>512</td>
<td>493</td>
</tr>
</tbody>
</table>

Figure 5: Positions of connection of slab loaded to neighboring slabs

connected to neighboring slabs through the rail fastening devices at three places, the both ends of slab under counter rail and the end of the slab at fastened side under lifted rail in Figure 5, and the weight of slab itself is about 5tf, the slab worked by the resultant force could not be lifted.

3.3 Lifting force to rail tension

The lifting force acting at the center of released section is calculated for the rail tension as shown in Figure 6. That is, the lifting forces are in proportion to the rail tension and about 2.1tf at the rail tension of 40tf.

Figure 6: Lifting force to rail tension
4 Amelioration of model

4.1 Model

Through the analysis having done so far it is considered that the model with elastic support of single stiffness could not explain the real behavior of the measurement enough. Actually the first and maybe second rail fastening devices from the boundary of released rail section are pulled up and following ones are compressed or the deflection of them are very small. So, it is appropriate that the ameliorated model is set as the one with the rail fastening devices separated to two kinds, Case (1) of two ends of leaf spring and Case (3) of normal condition.

Such a model is shown in Figure 7.

![Figure 7: Model of massive beam connected with two kinds of elastically supported beams](image)

4.2 Equations

The differential equation for it are given as follows;

\[
\begin{align*}
0 & \leq x < L_1 & E_I \frac{d^4 y}{dx^4} - T \frac{d^2 y}{dx^2} - \rho A &= 0 \quad (6) \\
L_1 & \leq x < L_2 & E_I \frac{d^4 y}{dx^4} - T \frac{d^2 y}{dx^2} + k_1 y &= 0 \quad (7) \\
L_2 & \leq x & E_I \frac{d^4 y}{dx^4} - T \frac{d^2 y}{dx^2} + k_2 y &= 0 \quad (8)
\end{align*}
\]

where \( k_1 \) is the upward rail supporting stiffness of first and maybe second rail fastening devices and \( k_2 \), that for others.

The general solution is given as follows;

\[
y = A_1 e^{\frac{T}{2E_I} x} + A_2 e^{-\frac{T}{2E_I} x} - \frac{\rho A}{2T} x^2 + A_3 x + A_4
\]  

(9)
where $A_{ii}$ is coefficient, $S = \sqrt{\frac{k}{EI}}$, $\theta = \frac{1}{2} \tan^{-1} \left( \sqrt{\frac{4EIk}{T^2}} \right)$, $L_1$, the distance from the lifting force to the end of released section and $L_2$, the boundary between Case (1) and Case (3).

They are solved for following boundary conditions.

At $x=0$, $y'=0, -EIy'''=P/2$ (1)

At $x=L_1$, $y_{-L_1}=y_{+L_1}$, $y'_{-L_1}=y'_{+L_1}$, $y''_{-L_1}=y''_{+L_1}$, $y'''_{-L_1}=y'''_{+L_1}$ (2) (12)

At $x=L_2$, $y_{-L_2}=y_{+L_2}$, $y'_{-L_2}=y'_{+L_2}$, $y''_{-L_2}=y''_{+L_2}$, $y'''_{-L_2}=y'''_{+L_2}$ (3)

5 Analyzed results 2

5.1 Conditions and calculations

These equations having ten unknown variables are not directly solved in reasonable time length by computer. So, the equations are combined so as to decrease the number of unknown variables. Finally they are decreased to 4 unknowns. Solving these for the same condition as in the first approach, that is, 40tf for the rail tension and 70mm for rail lift in he rail released section of 20m, the rail displacement is given as shown in Figure 8 and forces acting on each rail fastening device are given in Table 3.

5.2 Discussions

![Figure 8: Deformation in elastically supported section of two stiffness](image)
Table 3. Forces acting on rail fastening devices (kgf)

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1427</td>
<td>131</td>
<td>-624</td>
<td>-420</td>
<td>514</td>
</tr>
</tbody>
</table>

As an astonishment, the force to the first rail fastening device from the boundary of released rail section is dramatically decreased from those in Figure 2. The actual value of displacement is 0.15mm. In the first approach, the author estimated that the Case (1) would explain the matter, but the forces acting to the rail fastening devices are much smaller than those in Table 1. It depends on the effect of the stiffness $k_2$ which constrains the rail displacement in the section of stiffness $k_1$ to a little smaller than those in Case (3) and Case (2) in the first approach.

As the forces acting downward at the boundary is just 1.4tf, there is no risk for the breakage of spring which has the strength of 4.8tf. Also, the force lifting the slab of 0.5tf is much smaller than the weight of slab, 5tf, without any risk on the lift of slab. The position of resultant force of 5.07m from the outer end of slab is nearly as same as in the first approach.

The rail lifting force acting at the center of the released section is calculated for the tension of rail as shown in Figure 9. In this model also the linearity of lifting force to tension is confirmed.

![Figure 9: Lifting force to rail tension for 2 stiffness model](image)

In closing the discussion the dramatic difference of results between the first approach and the ameliorated model suggests the necessity of seeking the more realistic models. Also, it is sure that, depending these facts, the related matters must be checked in the test on actual track.
6 Concluding Remarks

Depending on the needs on the measurement of rail tension for knowing the neutral temperature of CWR on slab track, the characteristics of measurement is pursued theoretically.

Followings are the results for them.

(1) For the first approach
- The rail deformation is calculated for the released rail connected to elastically supported rail with single stiffness
- Three cases of different stiffness of rail fastening device are dealt with under the rail lifting force.
- In the result the Case (1), where just the leaf spring is working, seems plausible because the force acting to the first rail fastening device is considered to be the most important.
- The force acting to the first rail fastening device in case (1) is 3.8tf. It is under the allowable strength of 4.8tf.
- The role of Case (3) for second or third rail fastening device and followings is not clear.
- The lifting force of slab is 1.5tf to 2.2tf. They are much smaller than the weight of slab, 5tf. The place of resultant force is at the end of slab of released side. As for the stability of slab, there will be no problem because the slab is fixed through rails connected to slab at three places.
- The linearity of lifting force to the tension in rail is demonstrated.

(2) With the ameliorated model
- To clarify the question on the difference of stiffness of rail fastening depending on the direction and the value of forces acting on them, an ameliorated model with two kinds of stiffness for rail support is analyzed.
- As astonishment, the force acting on the first rail fastening device is dramatically decreased to 1.5tf.
- This phenomenon is understood with the downward constraining of the rail on stiffness of Case (1) by the stiffness of Case (3).
- With these, the strength of leaf spring and the stability of slab are guaranteed.
- The linearity of the rail lifting force to the rail tension is demonstrated.

(3) It is essential to use the model which is as realistic as possible.

References

