Sequential linear power flow solution for AC electric railway power supply systems

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Abstract

Characterisation of AC electric railway power supply systems requires a power flow algorithm as a basic tool to determine bus voltages, line currents or power flows through feeder lines. The execution of the algorithm needs efficient search methods. Among these, the Gauss-Seidel and the Newton-Raphson methods have been successfully developed to obtain network solutions over several decades. Over the previous decade, some emerging power-electronic compensation technologies have affected practical power system behaviours, including AC railways. This advancement in technology has introduced new complications to the existing AC railway power systems. Thus, the optimisation techniques need refinement and more efficient algorithms need to be developed. This should enable more accurate optimum solution and require considerably less calculation time.

This paper describes a novel AC railway power flow method, which uses a current load model instead of a classical power load model. Therefore, nodal analysis together with a simple search algorithm is sufficient to solve such a problem. Also, this simplified algorithm considerably reduces the overall calculation time. This new method is named Sequential Linear Power Flow and is explained in depth in this paper. For benchmarking, the well-known Newton-Raphson power flow algorithm has been used to perform identical calculations. The test systems were the modified standard IEEE 24-bus, 57-bus and a ten-train AC railway system, respectively. The computing results indicate that the proposed method requires significantly less execution time than that required by the Newton-Raphson method while the same accuracy is attained.
1 Introduction

The main function of an AC railway power supply system is to deliver electric energy to the electric locomotives which are connected to the system adequately, sufficiently and economically. To analyse reliability, stability and security of power supplies needs basic tools such as power flow algorithms for characterising voltages, currents and power flows through the feeder lines. As long as AC railway power systems can be described by conventional power supply analysis [1], classical power flow methods are applicable to obtain their solutions. The Gauss-Seidel and the Newton-Raphson methods are two well-known iterative techniques used for numerical solutions of power flow problems. For several decades, the Newton-Raphson method has been successfully developed and broadly accepted as the most powerful algorithm. Before describing the newly developed method, both classical power flow techniques are briefly reviewed for comparative purposes.

Suppose there is an N-bus power system, one of which is treated as a reference bus, whose voltage magnitude and angle are both specified. The bus labelled N will be selected as the reference bus throughout this paper. According to the Gauss-Seidel algorithm [2], bus voltages are updated at each iteration, repeatedly. The following is the expression to update bus voltages for the Gauss-Seidel method.

\[
V_i^{(k+1)} = \frac{1}{Y_{ii}} \left( \frac{P_i - jQ_i}{V_i^{(k)}} - \sum_{m=1}^{i-1} Y_{im} V_m^{(k+1)} - \sum_{n=i+1}^{N} Y_{in} V_n^{(k)} \right)
\]

where the superscript \(k\) denotes the \(k\)-th iteration

\(V_i\) denotes the voltage vector at the \(i\)-th bus

\(P_i\) and \(Q_i\) are real and reactive power consumption at the \(i\)-th bus, respectively

\(Y_{im}\) is the \(i\)-th row and \(m\)-th column element of the system bus admittance matrix

* denotes the complex conjugate

For the Newton-Raphson method [2], Taylor Series expansion is used to approximate the power flow equations and decompose them into real and reactive power flow equations separately but there is still interaction between them. In the decomposed equations, all variables are real. Equations 2 and 3 represent real and reactive power mismatch equations after decomposition, respectively, and equation 4 shows the compact matrix form of the well-known Newton-Raphson power flow.

\[
\Delta P_i = P_{i,\text{sch}} - \sum_{k=1, k \neq i}^{N} |Y_{ki} V_k V_i| \cos(\theta_{ki} + \delta_k - \delta_i) = 0
\]

\[
\Delta Q_i = Q_{i,\text{sch}} + \sum_{k=1, k \neq i}^{N} |Y_{ki} V_k V_i| \sin(\theta_{ki} + \delta_k - \delta_i) = 0
\]
\[
\begin{bmatrix}
\Delta P \\
\Delta Q
\end{bmatrix}
= 
\begin{bmatrix}
J_1 & J_2 \\
J_3 & J_4
\end{bmatrix}
\begin{bmatrix}
\Delta \delta \\
\Delta V
\end{bmatrix}
\]

(4)

where \( P_{\text{sch}} \) and \( Q_{\text{sch}} \) denote scheduled real and reactive power at the i-th bus, respectively.

\( \Delta P_i \) and \( \Delta Q_i \) denote real and reactive power mismatches at the i-th bus, respectively.

\( \theta_{ki} \) denotes the angle of the complex vector \( Y_{ki} \)

\( \delta_i \) denotes the angle of the complex vector \( V_i \)

\( J_1, J_2, J_3 \) and \( J_4 \) denote Jacobian sub-matrices.

The Gauss-Seidel method proceeds by updating each unknown bus voltage separately from the first to the last and thus converges linearly, which needs much more execution time than the Newton-Raphson method which converges quadratically [3]. Although the Newton-Raphson method converges rapidly, usually within 3-5 iterations, it is still very complicated, requires substantial memory to store its variables and substantial computation because of complexity of its formulae.

In practice, AC railway power supply systems are special and there are some key differences compared to industrial power systems [4]. Most obviously the feed arrangement is single-phase and only has one feed (reference bus) as shown in figure 1. The loads are electric locomotives, can move along the railtracks and locomotives can be added into the system or be removed from the system at any instance, therefore the total number of buses, \( N \), varies with time. Furthermore, with traction controllers such as shown in figure 2, the modelling of the load is more variable and impedance or current load models are more appropriate to describe the locomotive characteristics than the power load model [5]. These factors lead to a need to re-analyse the power flow problem carefully for the possibility of developing a new power flow technique that is simpler, but still converges rapidly and exploits the special properties of the railway case.

![Figure 1: Single-phase AC railway feeding systems](image)
2 Sequential linear power flow method

Nodal analysis is one of the well-known methods for solving the unknown voltage at each node in a linear circuit. No iteration is needed to obtain bus voltages when all loads are in the form of currents [6]. Since loads in utility power systems are normally modelled as power loads, nodal analysis has not been able to be used to solve such power flow problems. In contrast, in AC railways where current load models are often acceptable, it is possible to derive power flow equations by applying nodal analysis.

As shown in figure 3, the net outgoing current flow from the k-th bus is obtained by applying KCL [6] as follows.

$$\sum_{i=1, i \neq k}^{N} y_{ki} (V_k - V_i) = I_{k,sch}$$

(5)

where $y_{ki}$ denotes the admittance connected between the k-th bus and the i-th bus

$I_{k,sch}$ denotes the scheduled current at the k-th bus
Since the primitive admittance $y_{ki}$ does not usually appear in any formulae used in network solvers and the N-th bus is the reference bus, equation 5 can be rewritten as equation 6 with the elements of the bus admittance matrix and the voltage at the N-th bus as given variables, as follows.

$$\sum_{i=1}^{N-1} Y_{ki} V_i = I_{k,\text{sch}} - Y_{kN} V_N$$  \hspace{1cm} (6)

Similarly, with N-1 unknown bus voltages, the matrix form of the node equations for this system is generated as

$$\begin{bmatrix} Y_{11} & Y_{12} & \cdots & Y_{1,N-1} \\ Y_{21} & Y_{22} & \cdots & Y_{2,N-1} \\ \vdots & \vdots & \ddots & \vdots \\ Y_{N-1,1} & Y_{N-1,2} & \cdots & Y_{N-1,N-1} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_{N-1} \end{bmatrix} = \begin{bmatrix} I_{1,\text{sch}} - Y_{1N} V_N \\ I_{2,\text{sch}} - Y_{2N} V_N \\ \vdots \\ I_{N-1,\text{sch}} - Y_{N-1,N} V_N \end{bmatrix}$$  \hspace{1cm} (7)

or in the short form of the matrix equation as

$$[Y_{\text{NODE}}][V] = [I]$$  \hspace{1cm} (8)

where $[Y_{\text{NODE}}]$ is a matrix derived from $[Y_{\text{BUS}}]$ by eliminating the N-th row and the N-th column.

Interestingly, equation 7 is linear when all loads are written as current models, not power models as industrial power systems. To extend the equation above to power load models, one possible method is to transform all the power loads into current loads. From the power-current relationship of a load, whose voltage is previously known somehow, the $k$-th element of the current matrix can simply be computed by equation 9.

$$I_k = I_{k,\text{sch}} - Y_{kN} V_N = \left(\frac{S_{k,\text{sch}}}{V_k}\right) - Y_{kN} V_N$$  \hspace{1cm} (9)

where $S_{k,\text{sch}}$ denotes the scheduled complex power of the $k$-th bus.

Applying the Gauss iterative method, with given initial bus voltages, $[V]$ is updated iteration-by-iteration, thus the bus voltage vector at the $k+1$ iteration can simply be obtained as equation 10.

$$[V]^{[k+1]} = [Y_{\text{NODE}}]^{-1} [I]^{[k]}$$  \hspace{1cm} (10)

As can be seen, unlike the Jacobian matrix, $[Y_{\text{NODE}}]^{-1}$ is completely known at the beginning and can be used repeatedly without re-calculation throughout the process. In addition, such a matrix is an (N-1)-by-(N-1) matrix whereas the Jacobian matrix is a 2(N-1)-by-2(N-1) matrix, which considerably reduces the memory required by the power flow programme.

This method transforms the non-linear matrix equation into a linear matrix equation at each iteration, so that the developed method is named the Sequential Linear Power Flow Method (SLPFM). In summary, the SLPFM is described by the flow chart in figure 4.
3 Current load modelling of an electric locomotive

The power flow problem is generally concerned with obtaining system solutions, with the problem formulation being generated from all elements connected in the system. Therefore, each element has to be modelled as simply as possible to decrease problem complexity, but still adequately represent the original. Just as a load in an industrial power system is modelled by a power model, a locomotive in an AC railway can simply be represented in the same way. Nevertheless, because of certain differences, it is sometimes specified in other forms, such as current, impedance or some more complicated model as shown in figure 5, dependent on the type of electric locomotive [7].

![Figure 5: Locomotive models for power flow analysis](image-url)
The SLPFM employs the current model in figure 5-c to characterise the locomotives. Thus, the locomotive current model must be calculated from its dependence on the original load model. It can simply be obtained case-by-case as follows.

- The impedance or admittance model
  \[ I_t = \frac{V_t}{Z_t} \text{ or } Y_t V_t \]  

- The power model
  \[ I_t = \begin{bmatrix} S_t \\ V_t \end{bmatrix} \]  

- The linear model by Thevenin’s equivalent circuit
  \[ I_t = \frac{V_t - E_t}{Z_t} \]

Using the above equations, any locomotive model can easily be transformed into a current model and the transformation needs only simple numerical calculations. This leads to the SLPFM being capable of solving precisely complicated railway systems where there exist several locomotives modelled by various different models.

4 Simulation results

By using the latest version of MATPOWER [8] (a well-known Newton-Raphson power flow programme for the MATLAB environment developed by the School of Electrical Engineering, Cornell University, USA in December 1997) to solve the modified (to have only one reference bus) standard IEEE 24-bus, 57-bus and a ten-train railway test systems as shown in figures 6, 7 and 8 respectively, the effectiveness of the developed method compared to the standard Newton-Raphson power flow has been tested.
Figure 6: The modified standard IEEE 24-bus test system

Figure 7: The modified standard IEEE 57-bus test system
Figure 8: The ten-train AC railway test system with all power load models

These tests were performed on a Pentium 133 MHz, 48 MB RAM, with MATLAB 5.3 student version. Of the power flow solutions for three cases, only the simulated result from the ten-train railway test system has been selected to present as an example in this paper and are shown in table 1. Table 2 is a table summary that illustrates the effectiveness for each of the three cases by giving the required iterations and the calculation times. It should be noted that both power flow methods used $1\times10^{-8}$ per-unit as the terminating condition; maximum power mismatch in per-unit for the Newton-Rapson method and maximum voltage error in per-unit for the SLPFM.

Table 1. Results from the ten-train railway test system

<table>
<thead>
<tr>
<th>BUS</th>
<th>Voltage magnitude in per-unit</th>
<th>Voltage angle in degree</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Newton-Raphson</td>
<td>SLPFM</td>
</tr>
<tr>
<td>1</td>
<td>0.841</td>
<td>0.841</td>
</tr>
<tr>
<td>2</td>
<td>0.795</td>
<td>0.795</td>
</tr>
<tr>
<td>3</td>
<td>0.761</td>
<td>0.762</td>
</tr>
<tr>
<td>4</td>
<td>0.739</td>
<td>0.739</td>
</tr>
<tr>
<td>5</td>
<td>0.841</td>
<td>0.841</td>
</tr>
<tr>
<td>6</td>
<td>0.795</td>
<td>0.795</td>
</tr>
<tr>
<td>7</td>
<td>0.761</td>
<td>0.762</td>
</tr>
</tbody>
</table>
5 Conclusions

The results confirm the effectiveness of the SLPFM to solve AC railway power flow problems with their special features. Even though the developed method requires more iterations to obtain the solution, because of spending less calculation time per iteration, the overall calculation time is only approximately 50% of that used by the standard Newton-Raphson method. Significantly, the developed method does not require the calculation of derivatives. Furthermore, the SLPFM uses current load models computed from various alternative load models, hence enabling the use of different types of load models characterising different types of locomotives in one problem.

6 References