A simplified passenger flow model using Coloured Petri Nets

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Abstract

A model for the representation of passenger flow using Coloured Petri Nets is proposed. In this model, the locations and the walking routes are modelled as a network of places and transitions, and passengers as tokens moving around in this network. A token is assigned a “colour”, which can be used to show information that is carried around by the token, such as destination, the number of passengers that the token represents, and any other data that may be required for evaluating the passenger flows. Passenger flow rate fluctuation and capacity limitation can be included in the model. The model can easily be integrated with train movement models, which can also be expressed using Petri nets. This model yields a simpler representation of passenger flow than those based on fluid-flow modelling, while still being able to handle the reality of the terminal adequately, and is potentially suitable for applications that require accelerated time simulations.

1 Introduction

Managing connections between public transport services at a major transport interchange, such as a railway terminus, can be a very complex task especially when services are disrupted. Management of the interconnections between services, especially in disrupted conditions, requires a dynamic model of the services themselves, at least within some defined neighbourhood of the interchange, and the flow of passengers within the interchange. To construct some kind of decision support system aimed at the optimisation of the interchange, there must be a simplified model to allow accelerated time simulations with which to test different management strategies. In attempting to construct this dynamic model, the authors have so far concentrated on passenger flow modelling with a view to devising a model which is suitable for extensions to include the local transport services themselves.
In present operational practice, however, only the average walking time for passengers to move from one service to the other is considered if, indeed, any such consideration is made at all. Given the origin and destination, the time value is fixed regardless of the prevailing conditions, which can be quite unrealistic in some cases. On the other hand, there do exist several models to calculate passenger flows which can reflect overcrowding, for example, but these require complex calculations and are not obviously suitable for such embedded simulation applications[1].

As a compromise, the authors are now working on the development of a more simplified model, using Coloured Petri Nets[2]. Using this technique, a simplified model suitable for use within embedded simulations can be obtained. It can handle complex terminus layouts, and can represent the fluctuations in passenger flow rates and/or capacity limitations. In addition, it can be easily linked to transport network models of the various transport modes serving the terminus.

2 The basic idea: using Coloured Petri Nets (CPNs)

The basic approach in this new model is to use Coloured Petri Nets (CPNs) to describe the movements of passengers in and around the terminus.

A CPN can be considered as one of the modified classes of Petri nets (PTNs, short for Place-Transition Nets).

A PTN is made up of “places”, “transitions”, “arcs” and “tokens”. Each arc is directed, and links either from a place to a transition (called an “input arc”) or from a transition to a place (called an “output arc”). Each arc is assigned a non-negative integer number, which is called the “arc inscription”. A place may contain one or more tokens. A transition will “fire” when all the places that are connected to the transition by input arcs (input places) have tokens equal to or more than the input arc inscription number. When a transition fires, tokens equal to the input arc inscription number will be removed from the input places, and tokens equal to the output arc inscription number will be added to the output places.

PTNs can be used to describe, for example, passengers’ movements along a corridor, by corresponding PTN places, tokens and the series connection of an arc, a transition and another arc with certain locations in the corridor, passengers and walking routes, respectively. Firing of the transitions, which will “move” tokens from one place to the other, will occur according to the movement of passengers along the corridor. Using “timed” variants of PTNs, the time required for a token, i.e. a passenger, to move from one location to the other can also be taken into consideration.

For example, in Figure 1, the firing of transition P will remove a token from location A and add token to location B (equivalent to moving a token from A to B). Then, the next transition Q can be fired, moving this token further to location C.

In the realistic terminus, however, there are many passengers heading for individual destinations. The walking route of a passenger will inevitably converge and diverge with those of other passengers with different destinations.
To describe this situation, it is more convenient to have the destination information attached to the tokens: This will enable the control of token routes according to their destinations.

![Diagram of PTN-based corridor model](image)

Figure 1. PTN-based corridor model, in which passengers move from location A to C.

![Diagram of Coloured Petri Net (CPN) based model](image)

Figure 2. Coloured Petri Net (CPN) based model: diverging passenger routes.

This is realised by the application of CPNs. In CPNs, a token is assigned a "colour", which can be used to make the token carry a variety of information identified with it as it moves around the network. The colour should be an element of a predefined "colour set". Although the network itself is still made up of places, transitions and arcs, the arc inscriptions will no longer be simple integer numbers; they will be expressed using the concept of "multi-sets", and can include fairly complex functions. This idea will enable the modelling, for
example, of the diverging passenger routes illustrated in Figure 2, in which the firing of transition P will move a token from location A to location B if the token has the colour “to_B”, and from A to C if “to_C”. It will therefore be easy to expand the idea to a realistic station layout such as in Figure 3.

Figure 3. Example CPN model superimposed on a station layout.

The colour set can have an infinite number of elements, and therefore the token can carry far more information than just its destination. For example, it can carry the number of passengers it represents, thus making the model free from the assumption that one token represents one passenger only. This is realised by defining the colour set as a pair of data, one showing the destination and the other the number of passengers. This idea is useful when modelling the stochastic behaviour of passengers, in which case the data will take non-integer values.

3 Implementing passenger flow rate fluctuation and capacity limitation

In the CPN model of Figure 3, the transitions can “hide” various rules or functions behind them. These rules or functions can also be expressed as CPNs in the form of “subnets”. For example, passenger flow fluctuation and capacity limitation can be implemented in this model.

3-1 Passenger flow rate fluctuation

The walking speed of a passenger can differ from that of others. Therefore, the time it takes to go from one location to another will fluctuate. There are many different methods of implementing this into the CPN based passenger flow model, but to maintain simplicity it has been done as follows.

Assume that the colour of the passenger token can be written as \((d, x)\), where \(d\) is the destination and \(x\) is a real number showing the number of passengers...
represented by this token. Also, \((d, x)@T\) denotes a token with colour \((d, x)\) which arrives at a location at time \(T\). Because the time is quantised, it can be assumed that \(T\) is an integer number.

Now consider a case in which a token should move from an input place to an output place through a transition representing a corridor. If passenger flow rate fluctuation is not considered, the time required to complete this movement is a fixed value, say \(T_c\). Then, if a token \((d, x)@T\) arrives at the input place and moves to the output place, the result will be a single token \((d, x)@(T + T_c)\) at the output place, without any change in \(x\). If the fluctuation exists, however, the arrival time at the output place will not be identical for all passengers, and will vary according to a specific probability distribution function (p.d.f.).

Let the p.d.f. be \(P(T_c), T_c = 0, 1, 2, \ldots\). Then, if a token \((d, x)@T\) at the input place moves to the output place, it will be divided into a number of tokens. The token arriving at the output place at time \(T + T_c\) will be:

\[
(d, xP(T_c))@(T + T_c)
\]  

(1)

Realistically, it can be assumed that the range of \(T_c\) for which \(P(T_c) > 0\) will be limited, e.g. \(T_s \leq T_c \leq (T_s + n)\). Therefore, it should be appropriate and convenient to assume that \(P(T_c)\) is a binomial distribution, i.e.:

\[
P(T_c) = f(T_c - T_s; n, p) \quad \text{for } T_s \leq T_c \leq (T_s + n)
\]  

(2)

\[
P(T_c) = 0 \quad \text{otherwise}
\]  

(3)

where

\[
f(t; n, p) = \binom{n}{t} p^t (1 - p)^{n-t}
\]  

(4)

Equations (2) through (4) mean that the token at the input place will be “divided” into \((n + 1)\) tokens at the output place after moving through the transition. Using this model, the fluctuation can be handled with good approximation by carefully selecting the parameters \(n\) and \(p\).

3-2 Capacity limitation

There are many features within a terminus where certain capacity limits exist. For example, the staircases are not capable of accepting too many passengers at a time. In a crowded situation, there must be a queue approaching the staircase waiting for the persons in front to go through.

To model this phenomenon, there should be a mechanism to calculate the total number of passengers in a place, which can be designed and implemented using the CPN itself. If the number is below the capacity limit, the token should be able to move freely. If the number exceeds the limit, then some of the passengers should be allowed to go but the rest should stay in the original place to form the queue there.
Suppose there is only one token, \((d_1, x_1)\), in the input place trying to go through a corridor with capacity limitation of \(C_c\) (persons per unit time). If \(x_1 < C_c\), then the token can move on to the output place without any constraint. Otherwise, the token should be split into two, namely \((d_1, C_c)\) and \((d_1, (x_1 - C_c))\), and the former can move on but the latter should stay in the input place.

For a more general case, suppose there are more than one tokens, say \((d_1, x_1)\), \((d_2, x_2)\), \ldots, \((d_k, x_k)\) at the input place, and the sum of the number of passengers represented by these \(k\) tokens (\(\Sigma x_i\)) is \(X\). If \(X < C_c\), then all the tokens can move on. Otherwise, the token \((d_i, x_i)\) \((i = 1, 2, \ldots, k)\) will be divided into \((d_i, x_i C_c / X)\) and \((d_i, x_i (X - C_c) / X)\), and the former moves on and the latter stays at the input place.

### 3-3 Embedding these functions as subnets

The functions described in the previous sections can be embedded as a “subnet” hidden behind a transition, as shown in Figure 4.

![Figure 4. Subnets embedded in transitions.](image)

### 3-4 The output of the model

The resulting output of the model incorporating both functions described in sections 3-1 and 3-2 is shown in Figure 5.

![Figure 5. A series of locations connected by corridors.](image)
In Figure 5, there is no capacity limit at the corridor between locations P1 and P2, while between P2 and P3 there is a maximum capacity of 60 passengers per unit time. Both corridors have been given passenger flow rate fluctuations, with the parameters of $n = 6$ and $p = 0.5$. Passengers will “emerge” at P1 at times 6, 7 and 8 with the fixed rate of 200 passengers per unit time.

The calculation result using this model is shown in Figure 6. Because of the flow rate fluctuations, the peak value of the histogram of passengers arriving at each location decreases and the deviation increases as passengers proceed from P1 to P3. Because of the capacity limit of 60 passengers per unit time at corridor P2 – P3, people have to queue at location P2, and the number of passengers flowing into P3 per unit time is further limited. The maximum length of the queue at P2 reaches 250 passengers.

![Figure 6. Histogram of passengers arriving at locations P1 – P3 in Figure 5, together with the occurrence of the queue at P2.](image)

### 4 Extending the model to the neighbouring transport network

One of the major advantages of using CPNs in modelling passenger flow inside the terminus is that it can easily be extended to the neighbouring transport network.

It is possible to model the movements of trains, buses, etc. in the network using the CPN[3][4][5]. In this model, trains and/or buses should be represented by tokens moving in the network. These tokens can carry information on the number of passengers on board, and the destinations for which these passengers are heading.

It is easily realised by defining the colour set as a “list”. A list is a colour set that contains a number of elements which belong to another colour set. The list can be empty, or can contain any number of elements.
The colour of a passenger token defined in section 3 is \((d, x)\), where \(d\) shows the destination and \(x\) the number of passengers. Using this notation, the colour of a train / bus token can be written as \([\left( d_1, x_1 \right), \left( d_2, x_2 \right), \ldots, \left( d_k, x_k \right)\] ), where \(k\) is a non-negative integer showing the number of passenger tokens contained in the list.

When a train or a bus arrives at a platform in a terminus, the elements in the list that should get off at this terminus are “alighted” onto the platform, and only those that should be staying in the train / bus remain in the list. The “alighted” tokens will then move into the terminus model network for their final destination. Passengers arriving at the platform to ride the train / bus will be accepted into the list until the dwelling time of the train / bus runs out.

5 Conclusion

The model proposed in this paper is simple in its nature with the ability to handle real complexities with a good level of approximation when well-tuned parameters are chosen. It offers interesting promise as a method of representing passenger flows, including stochastic variation in passenger “performance” and the effects of building infrastructure capacity limits. The graphical input interface and use of standard software are considerable benefits for widespread use and ease of maintenance. However, the ability to model details such as stochastic variation and the user-friendly interface come at the cost of slower execution time. The authors are currently exploring this trade-off in some detail as the model development is part of a project to look at decision support for managing transport interchanges. In this context, it will need to be embedded in accelerated time on-line simulations where execution time is of the essence.

References