High order sliding manifold control for vibration reduction in flexible structures

A. Cavallo, G. De Maria, C. Natale
Dipartimento di Ingegneria dell’Informazione, Seconda Università degli Studi di Napoli, Italy

Abstract

In this paper a high order sliding manifold control approach is adopted as a technique to effectively reduce the vibrations of a metal rectangular panel. A novel theoretical result for an output feedback control law is presented to show how the singular perturbation theory can be used to tackle the problem of active vibration control.

In this paper we address the problem of reducing vibration of a flexible structure subject to exogenous disturbances by using an output feedback control law. In a first phase the model of the panel is deduced, by experimentally identifying the parameters of the system. Next, we consider active DVA’s (Dynamic Vibration Absorbers) as actuators, and displacement sensors, whose models are integrated into a MATLAB model of the system to be controlled. Finally the control problem is considered, and the controller is designed, by using a high order sliding manifold approach. Simulation results are presented to show the effectiveness of the proposed control strategy.

1 Introduction

Vibration reduction in high-speed railway vehicles is a research area which has received considerable attention over recent years. A factor to be considered in improving the train ride comfort is the reduction of acoustic noise produced by the vibrating structures. The traditional approach to acoustic noise suppression uses passive techniques, such as sound-absorbing materials (silencers). However the attenuation of silencers is small when the acoustic wavelength is large compared
to silencer dimension, thus effectiveness of passive techniques is limited at low frequencies. A proposed solution to overcome this drawback is the use of active noise control, in which additional secondary sources (typically loudspeakers) are used to cancel noise from the primary source. Unfortunately, this approach necessarily (due to wave propagation physics) produces zones where the noise and the antinoise have the same phase, then the noise is amplified. A proposed solution to avoid this drawback is to focus on reducing the vibrations of the noise source itself (for instance the floor of the car), that is, to contrast the cause rather then the effect; this implies addressing directly the problem of controlling the vibration amplitude of flexible systems.

Control of flexible systems is a very challenging problem, mainly due to its infinite-dimensionality: in fact, since flexible systems are described by means of linear partial differential operators, the mathematical model of flexible structures is an infinite-dimensional one, obtained by replacing the partial differential equation (PDE) formulation with an infinite set of ordinary differential equations (ODE’s), each ODE corresponding to a resonance mode of the structure. In particular, for a bidimensional flexible structure, the elastic deformation $w(t, P)$ at each point $P$ of the structure evolves according to the following equation

$$\ddot{w}(t, P) + 2\delta \dot{w}(t, P) + Aw(t, P) = Bf(t, P)$$

(1)

where $A$ is a suitable differential operator depending on the geometry of the plate, $B$ is an operator depending on how the distributed forces $f(t, P)$ act on the structure, and $\delta$ is a small damping coefficient [1].

However, infinite-order are very hard to treat, thus the usual approach is simply to truncate the ODE’s to obtain a finite set of ordinary differential equations. However, this approximation can lead to undesired effects, even instability in the closed-loop system (“spillover” phenomenon, [2]), hence particular attention must be paid in the order reduction phase.

In this paper we propose a control law to reduce panel vibrations by using dynamic vibration absorbers (DVA, [3]) as actuators. Specifically, active DVAs are considered, with a linear actuator commanding the seismic mass displacement.

Moreover, the use of a colocated control strategy, allows us to employ an high-gain second order sliding manifold output feedback strategy [4]. This strategy guarantees several advantages: indeed, it is very robust with respect to unmodeled dynamics and uncertain parameters, is able to reduce the effect of external disturbances even in structures with high modal density, and requires only output feedback, thus increasing reliability and reducing cost. The main drawback our control approach is its need for minimum-phase plants to be controlled, thus the use of a colocated strategy is mandatory.

2 Modeling

In this section we deduce the model of a fuselage skin panel of a Boeing 717 aircraft. The panel, depicted in Fig. 1, has three bulkheads and two orthogonal
stiffeners with curvature radius of approximatively 1.5 m. This implies that the panel cannot be modeled with a set of PDE’s by simply resorting to the classical approximation of a rectangular plate. Although the structure of the mathematical model is still in the form (1), the operators $A$ and $B$ are considerably more difficult to obtain. Therefore, a different modeling strategy has to be pursued, e.g. FEM, assumed mode approaches [5], [6] or experimental modal analysis [7]. In detail, a modal analysis approach is aimed at solving an eigenvalue problem of the type

$$A\varphi(P) = \lambda\varphi(P)$$

(2)

where $P$ is the generic point of the structure belonging to a suitable domain describing the geometry of the structure. The solution of this problem is an infinite set of eigenvalues $\lambda_n$ and eigenfunctions $\varphi_n(P)$, and thus the solution $w(t, P)$ of the differential equation (1) can be expressed as the infinite superposition

$$w(t, P) = \sum_{n=1}^{\infty} \frac{1}{\lambda_n} w_n(t) \varphi_n(P)$$

(3)
with each \( w_n(t) \) satisfying the ODE

\[
\ddot{w}_n(t) + 2\delta \lambda_n \dot{w}_n(t) + \lambda_n w_n(t) = \lambda_n (B f(t), \varphi_n(P)), \quad n = 1, \ldots, \infty
\]  

By truncating the infinite expansion (3) we obtain a finite set of \( N \) ordinary differential equations. When a closed form of the operator \( A \) is not available, it is possible to estimate eigenvalues and eigenfunctions by means of experimental measurements [7].

This technique is here applied with a hybrid MDOF (Multi Degree Of Freedom) method, consisting of two successive steps: first, the eigenvalues (natural frequencies and damping coefficients) are estimated via a circle-fitting algorithm, then a LMS Least Mean Square estimation method is used to determine the eigenfunctions \( \varphi_n(P_i) \) at some grid points \( P_i, i = 1, \ldots, L \) suitably chosen on the structure. The numbers of selected modes \( N \) and grid points \( L \) are determined based on the frequency range of interest.

After a preliminary experimental analysis phase, it has been found that the frequency range where most of the elastic energy is stored is the interval \([80, 180]\) Hz. Based on the maximum frequency of interest, on the size of the panel \((1350 \times 865 \text{ mm})\) and on the propagation velocity of the elastic wave in the material (Aluminium), a rectangular grid of \(11 \times 15\) \((L = 165)\) points has been selected in order to assure a \(83 \times 83\) mm spatial resolution.

Two sets of measurements have been performed at two different locations of the grid. In particular, two accelerometers have been placed, one at the center of the panel (point A), the other at a stiffner (point B); the panel has been excited at the other points of the grid by using an instrumented hammer. Thus a set \( S_A \) and a set \( S_B \) of 165 acceleration/force frequency response functions \( h_k^{(A)}(\omega), h_k^{(B)}(\omega) \), respectively, \( k = 1, \ldots, 165 \), have been obtained. Based on the data in the two sets, the number \( N \) of significative modes to be taken into account in the model has been deduced. To this purpose, a preliminary mathematical of the data has been performed. In set \( S_A \), for each frequency \( \omega_0 \) a measure \( \mu^{(A)}(\omega_0) \) of the global contribution of data has been computed by using the 1-norm

\[
\mu^{(A)}(\omega_0) = \sum_k |h_k^{(A)}(\omega_0)|
\]

By applying the same procedure to the set \( S_B \), two functions of the frequency, \( \mu^{(A)}(\omega) \) and \( \mu^{(B)}(\omega) \), are thus obtained.

The functions \( \mu^{(A)}(\omega) \) and \( \mu^{(B)}(\omega) \) emphasize the contribution of the modes in the given frequency range. However, not every mode appears in both functions, hence the total number of modes to take into account into the model must result from the union of the modes detected in each set. In order to fairly compare the two function, it is convenient to normalise them, in the frequency range of interest, with respect to any weighted norm, e.g. the 1-norm

\[
\mu_1^{(A)} = \int_0^\infty \Pi_{80}^{180}(\omega) |\mu^{(A)}(\omega)| d\omega
\]
Thus, we obtain the two normalised functions $\tilde{\mu}^{(A)}(\omega) = \mu^{(A)}(\omega)/\mu^2_{\text{avg}}$ and $\tilde{\mu}^{(B)}(\omega) = \mu^{(B)}(\omega)/\mu^2_{\text{avg}}$, whose plots are depicted in Fig. 2 assuming a fixed frequency step size of 0.125 Hz.

From this figure, we deduce that the number of modes to be taken into account is $N = 22$. In order to estimate accurately the 22 natural frequencies, along with the associated damping coefficients, a SDOF circle fitting [7] method is applied, leading to the values in Table 1.

Finally, in view of (3) and (4), each frequency response function can be expressed as

$$h_k^{(A)}(\omega) = (j\omega)^2 \sum_{r=1}^{N} \left( \frac{A_{k,r}}{j\omega - \lambda_r} + \frac{A^*_{k,r}}{j\omega - \lambda^*_r} \right)$$

where $\lambda_r$ are the poles associated to the modes and $A_{k,r}$ the residual of the $r-$th mode at the $k-$th grid point, and the term $(j\omega)^2$ takes into account acceleration measurements. Obviously, in order to deduce the force/displacement transfer functions, it is sufficient to remove the term $(j\omega)^2$. 
To estimate the unknown residuals, we note that the functions $h_k^{(A)}(\omega)$ depend linearly on the residuals, therefore a least mean square estimation algorithm has been employed.

The result of the procedure is shown in Fig. 3, for the case of the identified transfer function at one of the grid points on the panel. The plot evidences the good accordance between experimental data (dotted line) and the identified mathematical model (solid line).

### 3 Control Design

We propose a control strategy based on the use of active DVA's as actuators and a sliding control strategy to drive the DVA's. The DVA schematic model is depicted in Fig. 4, where $x_m$ is the displacement of the seismic mass $M_a$, $y$ is the displacement of the panel point where the DVA is fixed and $x_a$ is the commanded displacement (control variable). The seismic mass is connected to the vibrating structure through an elastic fixture with stiffness $k_a$ and friction coefficient $\beta_a$. The spring elongation is controlled by a linear actuator, e.g. a solenoid or a magnetostrictive motor.

The force exerted by the DVA on the structure is given by

$$f(s) = -2\zeta\omega_0 \frac{s^2(s + \omega_0/2\zeta)}{s^2 + 2\zeta\omega_0 s + \omega_0^2} y(s) - \frac{s^2\omega_0^2}{s^2 + 2\zeta\omega_0 s + \omega_0^2} x_m(s)$$

(9)

where $\zeta = \beta_a/2\sqrt{M_a/k_a}$ and $\omega_0 = \sqrt{k_a/M_a}$.

From eqn (9) it is apparent that the DVA introduces a structural feedback even when the control action is null, i.e. $x_a = 0$ (passive DVA).
For the control action, we consider a colocated strategy, i.e. we measure the displacement of the structure at the point where the DVA is attached. Therefore, the transfer function between the control variable $x_a$ and the system output $y$ is minimum phase. This allows us to resort to a high order sliding manifold strategy with high-gain action [4]. The order of the sliding is the controlled plant pole-zero excess, at the most. In our case it is easy, in view of (8) and (9), to show that the pole-excess is 2, hence a second order sliding manifold, at the most, can be used.

In particular, we define a sliding manifold as follows

$$\Sigma = \{(y, t) \in \mathbb{R} \times \mathbb{R}_+ : \sigma(y, t) = 0\}, \quad (10)$$
where \( \sigma : \mathbb{R} \times \mathbb{R}^+ \to \mathbb{R}^r \) is given by

\[
\sigma(y, t) = -y + e^{wt} [y_0 + t(y_1 - wy_0)]
\]

being \( y_0 = y(0), y_1 = y'(0) \) and \( w \in \mathbb{R}_- \).

Now we can introduce the feedback control law, \( x_a \in \mathbb{R} \), as the solution of the following ordinary differential equation

\[
\dot{x}_a + \varepsilon^2 d_2 \dot{x}_a + \varepsilon d_1 \dot{x}_a = n_2 \dot{\sigma} + n_1 \dot{\sigma} + n_0 \sigma,
\]

where the dot indicates the total derivative with respect to time, e.g. \( \dot{\sigma} = \frac{\partial \sigma}{\partial y} \frac{dy}{dt} + \frac{\partial \sigma}{\partial t} \), \( \varepsilon > 0 \) is a “small” scalar parameter and the coefficients \( d_i, i = 1, 2 \) and \( n_i, i = 0, 1, 2 \) are to be suitably chosen, as to satisfy the following conditions:

- the polynomials \( d(s) = s^2 + d_2 s + d_1 \), \( n(s) = n_2 s^2 + n_1 s + n_0 \) (13)
- the polynomials \( n(s) = n_2 s^2 + n_1 s + n_0 \) (14)

are Hurwitz, where \( h_2 \) is the second Markov’s parameter \( h_2 = CAB \), obtained from any space-state representation \( (A, B, C, D) \) of the system to control.

4 Simulation Results

In this section we apply the results of the previous paragraphs to reduce the vibrations of the panel. We consider the DVA and displacement measures at the point \( A \) defined in Section 2. Then, by following the steps outlined in the previous section, we start by choosing the first polynomial in (13) so as its roots are located in \( \{-2\pi 80, -2\pi 80, -2\pi 0.01\} \) and thus the value of \( n_2 \) is computed by dividing the last coefficient by \( h_2 \). Then, we choose the polynomial \( n(s) \) as

\[
n(s) = n_2 n'(s)
\]

with \( n'(s) \) is a monic polynomial whose roots are located in \( \{-2\pi 10, -2\pi 10\} \). In this way, the controller is completely defined once the value of the “small” parameter \( \varepsilon \) has been fixed

\[
C(s) = \frac{n_2 s^2 + n_1 s + n_0}{\varepsilon^3 s^3 + \varepsilon^2 d_2 s^2 + \varepsilon d_1 s}
\]

The smaller the \( \varepsilon \), the closer will the control be to the “equivalent control” and the disturbance will be better and better rejected. Notice that the input of the controller is simply the system output, since null initial conditions for both position...
and velocity have been assumed, hence in eqn (11) the second term is zero. Finally, the mechanical parameters of the DVA are set to the following values

\[ M_a = 0.1 \text{ kg}, \quad k_a = 4.2 \times 10^4 \text{ N/m}, \quad \beta_a = 38.8 \text{ Ns/m} \]

resulting in a natural frequency of about 100 Hz and a damping coefficient of 0.3, kept low to increase the absorbing effect around the natural frequency.

In order to show that the effect of the controller is to reduce the disturbance in a wide frequency band, we excite the panel with a chirp force signal with harmonic contents from 80 Hz to 180 Hz and an amplitude of 1 N. The results of the simulation are reported in Figs. 5, 6.

In Fig. 5 three output behaviours are compared: the open-loop output and two different closed-loop situations, with \( \epsilon = 10^{-2} \) and \( \epsilon = 10^{-3} \). Note that the smaller the \( \epsilon \), the smaller the output vibration.

In Fig. 6 the control action is depicted. In particular, the difference between the disturbance and the control variable is shown for the two previous values of \( \epsilon \); we note that decreasing \( \epsilon \) reduces the difference, hence a better rejection of the disturbance is achieved.
5 Conclusions

In this paper the vibration reduction for a bidimensional panel has been addressed. The model of the panel has been experimentally identified first, then a second order sliding manifold control strategy has been selected, using active DVA actuators and colocated displacement sensors. The identified panel model frequency response is in good accordance with experimental data. The effectiveness of the control law has been shown via simulations.

Acknowledgments

This work was financially supported by the Ministry of Scientific Research (MIUR) under the research project “Innovative Controls in High Speed Transport Systems”.

References


