Carrying capacity of railway networks: interaction of line and node models

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Abstract

Effective and widely tested models for the direct evaluation of the residual carrying capacity of the railway networks and the definition of the most effective actions for the full exploitation of this capacity are not yet available. The proposed approach is based on synthetic, probabilistic and combinatorial models for links and nodes, interacting each other, sensible to the performances of the signalling systems and not depending upon the timetable structure. Traditionally the link models require the knowledge of the train succession and the node models are based on the hypothesis of independence by the surrounding links. In the paper the limits of this independence are analysed and the effects of the interaction between the network elements (links and nodes) are formalised. The resulting model allows to evaluate the carrying capacity of each single node of a railway network taking into account the effects of the surrounding links and nodes. Of course the combination of these effects influences the carrying capacity of the network itself, which can be evaluated. The effectiveness of the model has been successfully tested on the urban railway network in Rome and the results of the pilot applications have been validated by means of experimental measures.

1 Introduction

The evaluation of the carrying capacity of railway networks with many nodes and short links is a typical problem to be faced in urban areas where the same infrastructures are used for urban, suburban and regional services. The
frequencies of these services are usually fairly high, constant for large periods during the day (basic interval timetables) and variable during seasons and years according to the demand needs. In these situations the most common problems to be faced are the evaluation of the residual carrying capacity of the network and the definition of the most effective actions for the full exploitation of this capacity.

2 Case study framework

A typical situation of a complex network commonly used for urban, suburban and regional services is the node of Rome, represented in figure 1: six main lines link the metropolitan area with other regions in the directions North, East and South, further secondary lines are dedicated to regional traffic. On a quite large part of these lines run both long distance trains (Eurocity, Intercity) every hour and short distance trains (regional, urban) with shortest time intervals (15-60 minutes). The signalling system is normally based on standard (1350 m) sections suitable for fairly high speed but strongly penalising urban trains, which are characterised by lower average speed and frequent stops (in some cases the stops are within the section without a proper signalling system). Particularly near the city centre the stations are so close each other that the circulation in one of them is conditioned by the others and the line linking these stations must be considered as a part of the station themselves (e.g. stations Tiburtina, Tuscolana and Ostiense within 5 km of line).

3 Methodological framework

The main general problems arising from the analysis of the case study are the definitions of: i) a station influenced area, from a train circulation point of view; ii) the effect of the succession of fast and slow running and stopping trains.

In fact, in order to avoid conflicts, some typical situations (figure 2) are:

1) entering movement (top of figure 2): when the slow train (S) runs after the fast one (F) it may enter the station only after that F is beyond the exit signal; when F runs after S it is not conditioned only by reaching the distant signal after the slow stopping train S released the entering route;

2) exiting movement (middle of figure 2): when S runs after F it may start after the stop when F released the line section in advance; when F runs after S it is not conditioned only by reaching the exit signal after the slow stopping train S released two line sections in advance;
movement between two close stations (bottom of figure 3): when F runs after S it is not conditioned only by reaching the exit signal after the slow stopping train S released the entering route of the following station.

Therefore the carrying capacity in the stations depends on the train succession and on the lengths of the line sections. Moreover the availability of the entering and exiting routes depends on the station topology and the routes compatibility.
4 Station carrying capacity

The train circulation in the stations is characterised by the possible compatibility of the different routes (e.g. two parallel routes without common track circuits may be contemporarily run, two converging routes may be run in different time periods). The carrying capacity is investigated by means of synthetic models based on the possible combinations of compatible routes [1] [2] [5].

Figure 2: Typical cases of trains succession

The proposed methodology allows the determination for the single station of the mean parameters: i) number of routes suitable to contemporarily run; ii) mean occupation time; iii) mean waiting time.

The mean number of routes suitable to be run is related to the percentage of compatible routes and to the distribution of the train movements \( \alpha \) among the various routes:
where $a_i$ is the summation at numerator is extended to all the routes, that at denominator is extended to all the couples of incompatible routes, $N_T$ is the number of train running in the station during the reference time ($T$), $n_i$ is the number of train on the generic route $i$. The number of movements ($N$) may be different from $N_T$ in fact each train generates a movement every used route.

The mean occupation time is:

$$n = \frac{\left(\sum_{i,j} \alpha_{ij}\right)^2}{\sum_{i,j} \alpha_i \alpha_j}$$

where $\alpha_{ij} = \frac{n_i}{N_T}$; $N = \sum_i n_i = N_T \sum_i \alpha_i$

the summation at numerator is extended to all the routes, that at denominator is extended to all the couples of incompatible routes, $N_T$ is the number of train running in the station during the reference time ($T$), $n_i$ is the number of train on the generic route $i$. The number of movements ($N$) may be different from $N_T$ in fact each train generates a movement every used route.

The mean occupation time is:

$$t = \frac{\sum_{i,j} \alpha_i \alpha_j t_{ij}}{\sum_{i,j} \alpha_i \alpha_j}$$

where $t_{ij}$ is the locking time of the route $i$ on the route $j$, the summations are both extended to all the incompatible routes and the time are calculated taking into account the close line sections (see figure 2).

For the calculation of the mean waiting time the probability of the train arrivals on the routes is assumed to be constant, so that this time is:

$$r = \frac{N^2 \sum_{i,j} \alpha_i \alpha_j t_{ij}^2}{\left(\sum_i \alpha_i\right)^2} \cdot \frac{1}{2Tn}$$

where the summation at numerator is extended to the couples of incompatible routes and that at denominator to all the routes.

The value of $N$ defining the carrying capacity of the station (corresponding to the full utilisation of the station) derives from the solution of the following equation:

$$\frac{N}{n} \left(\sum_{i,j} \alpha_i \alpha_j t_{ij}^2\right) \cdot \frac{1}{2Tn} = \gamma T$$

where $\gamma$ is a prudential coefficient (typically 0.6-0.7).

This value depends on the topology of the station (compatibility among routes), on the occupation time (including the sections close to the station) and on the
distribution of the movements on the various routes. For the same topology the carrying capacity depend on the utilisation of the routes: obviously should the trains run only on compatible routes \( N \) will be greater than in the case when they should run only on incompatible routes. The dependence of \( N \) on the movements distribution requires an iterative process when the parameters \( \alpha \) are unknown. This is the case of a station far from the saturation condition in which is investigated the carrying capacity by increasing the number of the trains on defined routes: after the formulation of hypotheses of increased use of the routes the new values of \( \alpha \) and \( N \) may be calculated. If the carrying capacity is still not reached further increases may be considered. In figure 3 an example for the station of Roma Tiburtina: a solution has been found after four iterations.

![Figure 3: Iterative process for the evaluation of the station carrying capacity](image)

### 5 Line carrying capacity

The carrying capacity on a line may be evaluated by means of the models based on the calculation of the mean minimum time between two trains running on the line itself [3] [4] [5]. The general expression is:

\[
P = \frac{T}{\sum t_i}
\]

where \( T \) is the reference time and the single terms \( t_i \) take into account the performances of the signalling system and, particularly, the minimum time interval on the critical section between two stations, the probability of queues formation and the number of block sections within this section. In figure 4 an example of calculation of the carrying capacity with various lengths of the critical section (standard elementary sections and trains characteristics) is shown.
The carrying capacity is strongly lower than the theoretical value defined by the minimum safety distance.

The proposed model is developed starting from the quoted model by refining the calculation of the minimum time interval and the effects of the succession of the trains. In particular the minimum time interval take also into account the possible stops within a section without specific signalling systems (longer occupation times) and the successions including stopping and not stopping trains running with different speeds.

![Graph showing carrying capacity for various lengths (L_b) of the critical section and its theoretical limit](image)

**Figure 4:** Carrying capacity for various lengths (L_b) of the critical section and its theoretical limit

The new expression of the minimum time interval between two trains i and j imposed by the length of the section k becomes (figure 5):

$$ t_{ij} = S_i + \sum_{x=1}^{k} T_{i,x} + R_{k,j} - S_j - \sum_{x=1}^{k-1} T_{j,x} $$

where S it the time for approaching the line, T_{i,x} and T_{j,x} are the times for running on the generic section x, including the possible stop time, and R the time for releasing the section. The calculation is extended to all the sections of the line. The maximum value of \( t_{ij} \) represents the minimum time interval between the trains i and j. With a similar procedure may be obtained the expressions for the
oth: r cases of trains succession (different speeds and presence/absence of a stop).

The minimum time interval to be included in the general expression of the carrying capacity is the mean value of the various cases of trains successions:

\[ t = \sum_y \beta_y \cdot t_{ij,y} \]

where the coefficient \( \beta \) is a weight proportional to the frequency of the generic succession.

![Figure 5: Minimum time interval between trains running at different speeds](image)

6 Application procedure

The two proposed models may be used in an integrated way for the analysis of the complex networks characterised by the sharing of the infrastructure between different kinds of railway services. The integrated model allows to define the global carrying capacity of the network as a whole and of its single elements.
(stations and line sections), their rates of utilisation and the residual carrying capacity (figure 6).

The residual carrying capacity is available for a further traffic with use of station routes and succession of trains on the lines similar to the existing ones (α and β coefficients). The possibility to use this capacity for a defined service may be evaluated by means of an iterative procedure including: i) hypotheses of increased traffic; ii) carrying capacity verifications.

7 Conclusions

The methodology has been tested on the urban railway network in Rome.

Figure 6: Comparison between traffic and carrying capacity of the railway network elements

Extended comparisons with experimental data observed in periods with different traffic density have been carried out. The results of these tests confirm the effectiveness of the proposed approach in terms of validity of the results, flexibility and applicability to a wide set of topological and technological situations.

Nevertheless future developments of the research may be envisaged in the following directions:
• homogenisation of input data for stations and line sections;
• better definition of the various application fields, based on systematic and extended sensitivity analyses;
• evaluation of the possible application of the methodological principles for supporting the traffic management choices.

References