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Theoretical analyses of measurement of stress-free temperature in continuous welded rail

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Abstract

The actual value of the neutral temperature in CWR (Continuous Welded Rail) is of high concern for track people so as to avoid rail failure in winter and track buckling in summer. Recently the measurement of the rail tension by releasing the rail from sleepers in the section of a certain tens meters and lifting the rail at the center of it has been successfully tried. It is analyzed theoretically with four mechanical models. Analyzing them, it is made clear that the calculation with the model of the massive beam connected with heavier beams at ends shows excellent coincidence with the practice. With use of the model, the numericals for another practice, necessary items to be checked and the optimum conditions in the measurement on Japanese Tokaido Shinkansen are calculated. Through such investigation, not only the characteristics of the measurement is made clear, but also it is confirmed that this measurement is very reliable and with high precision because just the mechanical characteristics of rail and the weight of track skeleton are concerned. Further, the application of it on curve is checked.

1 Introduction

The longitudinal force in continuous welded rail is varied with temperature. The actual value of the neutral temperature in it is of high concern for track people so as to avoid rail failure in winter and track buckling in summer. It had been very difficult to measure it at any time wanted. Recently the measurement of the rail tension by releasing the rail from sleepers in the section of 20–30m and lifting the rail at the center of it has been successfully tried. Measuring the temperature of rail then, the neutral temperature is estimated from the tension.

It is here analyzed theoretically with four mechanical models. Through such analyses, not only the characteristics of the measurement are made clear, but also it is confirmed that this measurement is very reliable and with high precision because just the mechanical characteristics of rail and the weight of track skeleton are concerned. Further, the application of it on curve is checked.

2 Measurement of rail longitudinal force by lifting rail

In the Interactive Conference on "Cost Effective and Safety Aspects of Railway Track" held by UIC-ERRI at Paris on December 8/9 in 1998, a paper titled "Determining the Stress-free Rail Temperature of continuous Welded Rails (CWR)" was presented by Mr. van Tonder from South Africa (van Tonder[1])and it was introduced that the same measurements were practiced in UK and USA.. The work in UK is introduced in WCRR held by RTRI in Japan on October 19/21, 1999 (Lemmon [2])

The main concern on the paper by Mr. Tonder is as follows;

"The neutral temperature of CWR were measured by cutting the rail formerly. It needed fairly long work time and weakened the rail strength. With the introduction of the new measurement 70% of the work for stress releasing which had been practiced in ballast cleaning normally was excluded. The measurement on R25000 was performed with a trial machine and that on R5000, with the practical one. The rail is released in the section of 20m and the longitudinal force in rail is calculated from the force for lifting the rail at the center of the section by 70mm. The neutral temperature is calculated with the force and the temperature measured at the same time. The work is executed by 4 workers just when the rail is under tension."

The relation of lifting force to tension in rail is given in Figure 1. It shows that the lifting force is 1255kgf at the zero tension, 1750kgf at the 40tf tension and linearly proportional to the longitudinal tension.

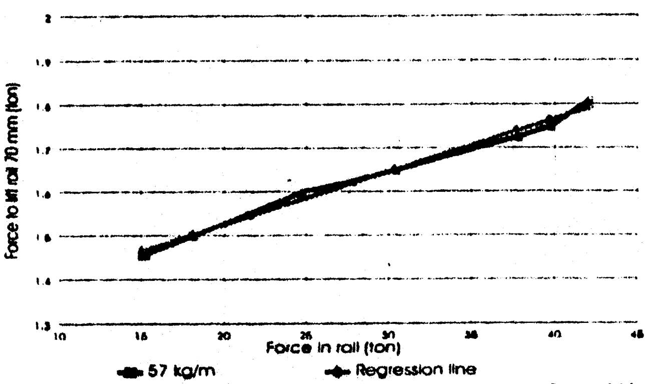

Figure 1: Relation of lifting force to rail tension (van Tonder [1])

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Additional knowledge given in the discussion was as follows;
"The measurement on sharp curves are down to the radius of 500m under 800m" from UK
"In US, if the temperature is to be higher than 4℃, the rail could be pushed back even if it is deformed by tension"
"In UK the equipment named as VERSE (VErtical Rail Stiffness Equipment) was made in 1994. So far 500 measurements have been made. The rail is released in the section of 30m. The measurement is feasible on the curve of R800 with the precision of ±1. 3℃"

3 Mathematical analysis with models

3.1 Massive string

To know the fundamentals of rail lifting under the tension, it is analyzed with the model of massive string as in Figure 2.

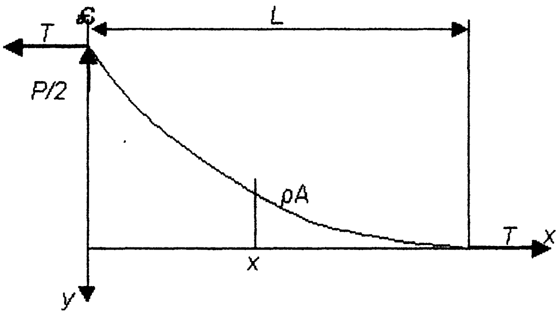

Figure 2: Model of massive string

The fundamental equation is given as

$$T\frac{d^2y}{dx^2} + \rho A = 0 . \qquad (1)$$

where T is rail tension, y, deflection, x, position and ρA, rail weight per length.
It is solved for following boundary conditions

$$\left.\begin{aligned} &\text{at } x=0 && y' = \frac{P}{2T}, && (1) \\ &\text{at } x=L && y=0. && (2) \end{aligned}\right\} (2)$$

where P is lifting force.
As a result, the rail lifting forces are 570kgf for zero tension and 1125kgf for the tension of 40tf. They are too small compared with the above results.

3.2 Fixed massive beam

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Secondly the rail lifting is analyzed for the fixed beam with mass considering the effect of beam typically as in Figure 3.

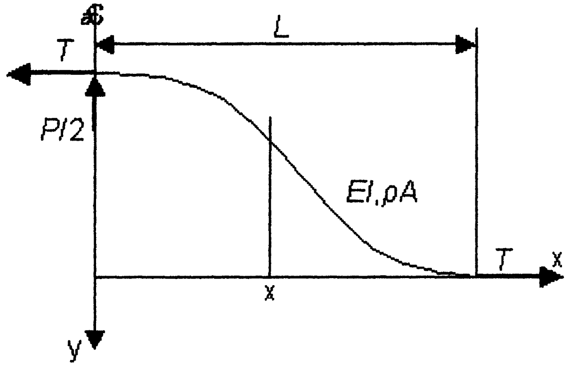

Figure 3: Model of fixed massive beam

The fundamental equation is given as

$$EI\frac{d^4y}{dx^4} - T\frac{d^2y}{dx^2} - \rho A = 0\,, \tag{3}$$

where *EI* is vertical rigidity of rail.

It is solved for following boundary conditions

$$\left.\begin{array}{llll} \text{at } x=0 & y'=0, & (1) \\ & -EIy'''=P/2, & (2) \\ \text{at } x=l & y=0, & (3) \\ & y'=0. & (4) \end{array}\right\}(4)$$

As a result, the rail lifting forces are 1488kgf for zero tension and 2154kgf for the tension of 40tf. They are too large compared with the above data.

3.3 Massive beams connected with elastically supported beams

Thirdly the rail lifting is analyzed for the massive beam connected with elastically supported beams at both ends as in Figure 4.

The fundamental equations are given as

$$\left.\begin{array}{lll} \text{in } 0 \leqq x < L_1 & EI\dfrac{d^4y}{dx^4} - T\dfrac{d^2y}{dx^2} - \rho A = 0\,, & (1) \\ \text{in } L_1 \leqq x & EI\dfrac{d^4y}{dx^4} + Ky = 0\,. & (2) \end{array}\right\}(5)$$

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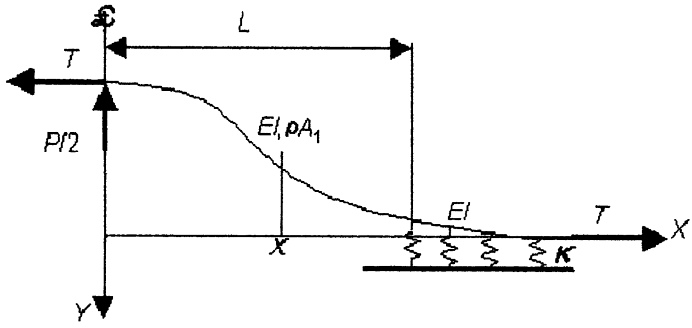

Figure 4: Model of massive beam connected with elastically supported beams

They are solved for following boundary conditions

at x=0	y'=0,	(1)
	$-EIy'''=P/2$,	(2)
at x=L	$y_{-L}=y_{+L}$.	(3)
	$y'_{-L}=y'_{+L}$,	(4)
	$y''_{-L}=y''_{+L}$	(5)
	$y'''_{-L}=y'''_{+L}$.	(6)

(1)–(6) → (6)

As a result, the rail lifting forces for the beam connected with the beam supported with elastic spring is 1340kgf for zero tension and 1959kgf for the tension of 40tf for the spring coefficient of 500kgf/cm^2. They were also too large compared with the above data. They are also checked without prominent difference for the spring coefficient of 125kgf/cm^2 putting it smaller.

3.4 Massive beam connected with heavier beams

Finally the rail lifting is analyzed for the massive beam connected with heavier beams at both ends characteristically as in Figure 5.

The fundamental equation is given as

in $0 \leqq x < L_1$ $$EI\frac{d^4y}{dx^4} - T\frac{d^2y}{dx^2} - \rho A_1 = 0\,, \qquad (1)$$

in $L_1 \leqq x$ $$EI\frac{d^4y}{dx^4} - T\frac{d^2y}{dx^2} - \rho A_2 = 0\,. \qquad (2)$$

(1)–(2) → (7)

They are solved for following boundary conditions

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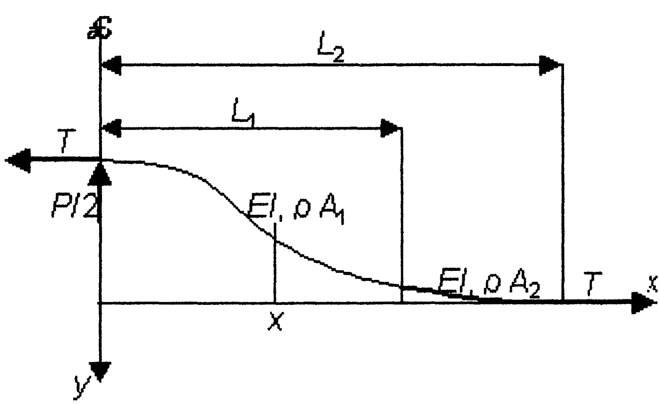

Figure 5: Model of massive beam connected with heavier beams

at x=0	$y'=0,$	(1)	
	$-EIy'''=P/2,$	(2)	
at $x=L_1$	$y_{-L1}=y_{+L1},$	(3)	
	$y'_{-L1}=y'_{+L1},$	(4)	(8)
	$y''_{-L1}=y''_{+L1},$	(5)	
	$y'''_{-L1}=y'''_{+L1},$	(6)	
at $x=L_2$	$y_{-L2}=0,$	(7)	
	$y'_{-L2}=0.$	(8)	

The length of L_2 is given for

At x=L_2 $\qquad y''=0 \qquad$ (9)

As a result, the rail lifting forces for the massive beam connected with heavier beams at both ends are1250kgf for zero tension with the raised length of 2.77m for the heavier beam and 1735kgf for the tension of 40tf with the raised length of 3.43m. They were in very good coincidence with the above data. It means that the phenomena can well be explained with this model. Further, as this model does not include uncertain factors just using the weight of rail and sleepers, it is very reliable

With this model the lifting force to the rail tension is calculated as shown in Figure 6 with good coincidence with Figure 1. The rise at the end of released section is 5.4mm for the tension of 40tf.

4 Other calculations

4.1 Studies on measurement in UK

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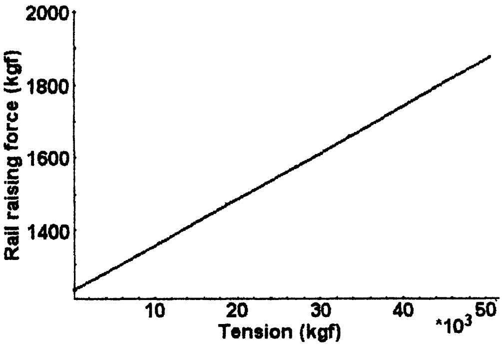

Figure 6: Rail lifting force to rail tension for South Africa

In UK the rail lifting force is measured at the center of released section of 30m. It is calculated for the rise of 70mm being same as that in South Africa assuming the rail of UIC60. The lifting forces are 1250kgf for zero rail tension with the raising of heavier beam section of just 12cm long and 1793kgf for the rail tension of 50tf with the raised length of 3.70m long. The rail rise at the end of released section is 2.8mm.

4.2 Proposal to the measurement in Japan

The lifting force to the rail tension on Tokaido Shinkansen is calculated on the track of 60kg rail and 3T sleepers for the rise of 3mm limiting the rail tension up to 55tf by refraining the measurement in very cold time. The necessary rail releasing section length for it is given as 24m and the rail lifting force to the tension, in Figure 7.

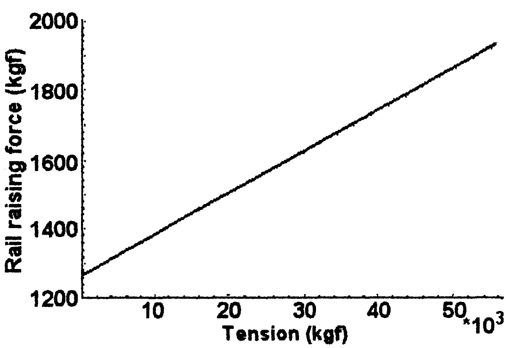

Figure 7: Rail lifting force to rail tension for Japan

5. Conditions for applying on curve

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5.1 Rail without longitudinal tension on curve

The application of rail lifting method for measuring its longitudinal tension on curve is analyzed for the lateral form with the model in Figure 8.

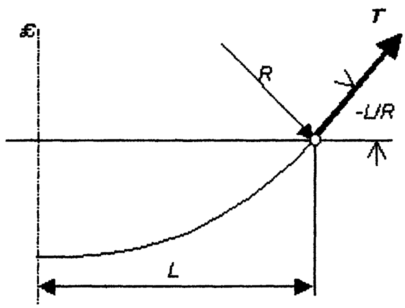

Figure 8: Model for rail form with tension in released section

With this model the rail is considered as the simple beam released from sleepers in a section and pulled with the slope of circular curve at the end of the section. In the model the rail is held as a circular form without tension, that is, the deviation from the chord is as same as the deflection of the beam at T=0tf and the curvature of the beam is in proportion to $1/R$. The actual value of the deviation for R=500m ($1/R$=0.00002cm-1) is 14cm at the center of released section as shown in Figure 9. Here, with the tension of T=55tf it decreases to 3cm. It means that the rail moves inner side by 11cm.

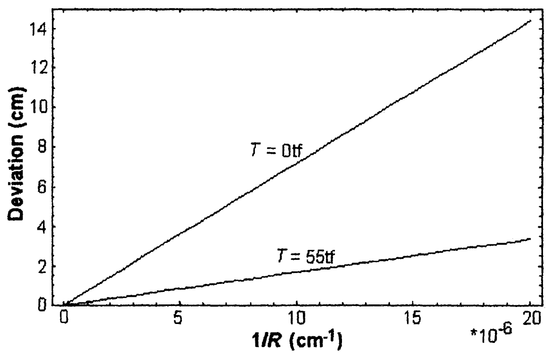

Figure 9: Deviation of rail from circular curve in released section

5.2 Rail with constraint on curve

In such a situation, as the line form could not be maintained, the rail is assumed to be held at the position of rise in circular form of curve. With this measure the

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rail is supported with the constraint *P* as shown in Figure 10.

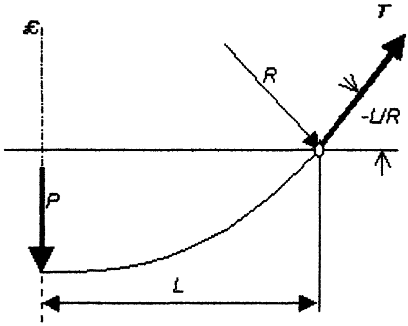

Figure 10: Model with constraint for holding line form

The constraint *P* is calculated as shown in Figure 11. In the figure the constraint for *R*=500m is 1300kgf and that for *R*=2500m (1/*R*=0.000004cm^{-1}) is 250kgf.

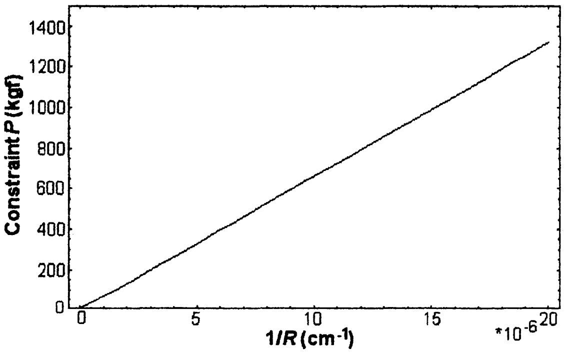

Figure 11: Constraint for holding circular curve

The constraint for *R*=5000m in the measurement in South Africa with practical machine is calculated as 150kgf. Considering the friction under lifting force of about 2.0tf, this is acceptable enough even with a simple equipment. On the other hand, the equipment in UK is considered being prepared enough to catch the constraint attaining 1300kgf on the curve of R500.

The reaction applied to the sleepers at the ends of released section is given as shown in Figure 12. It gives the reaction on the curve of R500 as 700kgf. As the sleepers are released just for a rail and fastened to another rail resisting to the movement enough, the reaction is steadily accepted without any movement.

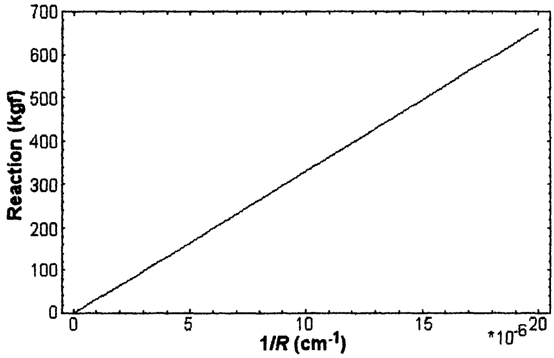

Figure 12: Reaction at ends of released section

6 Concluding remarks

Through the analyses solving linear simultaneous equations for 8 variables and hose including transcendent functions for 3 variables and checking the rail lifting and lateral movement of rail with the longitudinal tension, following conclusions are obtained.

(1) The model of the massive beam connected with heavier beams explains well the measurement in South Africa.
(2) As the model takes into account just the weight of a rail and sleepers, the measurement is very reliable.
(3) The conditions for the measurement on curve are given.
(4) The measurement in UK is explained well.
(5) The conditions for the measurement in Japan are given.

7 Acknowledgement

In the analysis on the measurement in South Africa, the data on their tracks such as rail, tie and rail fastening devices are sent from Mr. van Tonder of the Spoornet in South Africa. I appreciate it so much.

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