The measurement of in-plane permeability for sheared preforms
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Abstract

Computer simulations of composites manufacturing processes such as liquid moulding require accurate permeability data. Increasingly sophisticated models have been proposed for the prediction of the permeability of fibrous reinforcements; however these analytical tools are not yet able to account for the actual structure of the preforms. Parameters such as the textile structure or the level of imposed deformation in compaction and shear are known to modify the principal permeability values, as well as the orientation of the in-plane permeability tensor. If such parameters are to be considered within the simulations, their effects must be known with a level of accuracy that allows them to be separated from the natural variability observed with permeability data. In this paper, a method for the analysis of permeability measurements obtained from a planar apparatus instrumented along three axes is described. The principal permeabilities, as well as the values for the anisotropy ratio and orientation of the in-plane permeability tensor, are obtained without making any assumption relating the warp and weft directions of the materials to the orientation of the tensor. A metal cavity is used, allowing tests to be performed at high fibre volume fractions. Using this method, values can be obtained in a fraction of the time required for an equivalent work to be done using rectilinear measurements. Typical experimental data is presented to illustrate the potential of the method, and simplifying hypothesis used in other published methods are commented.
1 Introduction

The importance of the shear deformation that results from the draping of textile reinforcements on moulding tools has been demonstrated; therefore, studies of the effect of in-plane shear on permeability were undertaken. Working with balanced woven fabrics, Smit observed that the elliptic flow front tilts toward the bisector of the shear angle $\phi$ as defined in fig (1). Hammami et al. found that the front is not oriented toward the bisector of the shear angle $\phi$ but tilts progressively as the angle rises. Smith, Rudd & Long discussed the different effects associated to the rise of the fibre volume fraction and the reorientation of the fibres during shearing. Finally Lai & Young observed that as woven material are sheared, the flow front tilts toward the bisector of the shear angle $\phi$ without actually reaching it.

The principal flow directions of sheared reinforcements are not known prior to testing; this forces the use of a transparent cavity coupled with image analysis to position the flow front. However transparent cavities of sufficient dimensions are prone to deflection, especially for injection through preforms of high fibre volume fraction. Chick et al. proposed a method for the measurement of permeability in a metal tool, which can be properly reinforced. In this paper the method is adapted to the case of unknown flow orientation.

A number of algorithms for the calculation of in-plane permeability have been published. For the isotropic medium, Adams et al. showed the pressure $P$ to be distributed as:

$$\frac{P - P_f}{P_o - P_f} = 1 - \frac{\ln \left( \frac{R}{R_o} \right)}{\ln \left( \frac{R_f}{R_o} \right)}$$

where the indexes $o$ and $f$ represent the port and the front and $R$ is the radius. For the anisotropic medium an approximate solution is obtained through linear scaling of the physical domain:

$$X'_1 = X_1 \cdot \alpha^{1/4}$$

$$X'_2 = X_2 \cdot \alpha^{-1/4}$$

where $\alpha$ is the degree of anisotropy of the material and $X_i$ are positions along the principal flow directions 1 and 2, fig (1); $\alpha$ is expressed as:
\( \alpha = \frac{K_{22}}{K_{11}} \) \hspace{1cm} (4)

where \( K_{22} \) and \( K_{11} \) are the principal in-plane permeabilities as defined by Darcy's law. The elliptical front is defined by semi-axes of length \( A \) and \( B \) along directions 1 and 2, fig (1); its aspect ratio \( \beta \) is given as:

\[ \beta = \frac{A}{B} \] \hspace{1cm} (5)

Knowledge of the anisotropy of the material \( \alpha \) allows to express the aspect ratio \( \beta \) of the front as a function of \( A \); the ellipses defined by eqn (6) are isobars. The quantities \( A, B \) and \( R_o \) are illustrated in fig (1).

\[ \beta = \frac{A \cdot \alpha^{1/4}}{\sqrt{(A \cdot \alpha^{1/4})^2 - R_o^2 \cdot (\alpha^{1/2} - \alpha^{-1/2})}} \cdot \alpha^{-1/2} \] \hspace{1cm} (6)

Figure 1. Geometry of the flow front.

Chan & Hwang\(^8\) presented a non-iterative analysis for planar tests performed at constant pressure; in this method \( \beta \) is a constant.

Weitzenböck et al.\(^9\) mentioned the possibility of deriving the orientation of the front from the time readings of three thermistors placed between the textile layers. Because of the perturbations\(^10,11\) to the flow associated to this technique, no analysis was proposed. In a further publication\(^12\) the authors described an iterative method for the calculation of the principal in-plane permeabilities using time values obtained from transducers laid along three arbitrary axis. Given the nature of the transducers, the authors use a single time value from
each location, determined by a change in the voltage read. In contrary, as the technique developed by Chick et al. uses pressure transducers, the time values obtained by the authors are derived from interpolation of all the pressure values read over time for each transducer, making them more robust. Furthermore the technique is less intrusive, the transducers are localised more precisely and the pressure values obtained at different points over time can be used to determine the shape of the pressure distribution as well as the shape of the front.

2 Analysis

The in-plane flow of a fluid through an anisotropic porous medium is defined by the vectorial field of fluid velocity, the scalar field of fluid pressure and the permeability tensor. The local pressure gradient and superficial velocity vector are not parallel except along the principal axes of the material. Also, most authors assume that the flow front can be described as an ellipse; Adams et al. justified this assumption. The principal permeabilities can be calculated along the principal directions of the material from the pressure gradient, the superficial velocity of the fluid and the one-dimensional form of Darcy’s law,

\[ \nu = -\frac{K_{ii} \cdot \partial P}{\mu \partial x_i} \]  \hspace{1cm} (7)

where \( \nu \) is the superficial velocity of the fluid and \( \mu \) is the viscosity. Therefore the problem consists in defining the shape of the pressure distribution and calculating the flow rates in the principal directions.

The knowledge of the injection flow rate \( Q \), combined with two time readings \( t_f \) and \( t_g \) from transducers \#f and \#g, do not suffice to obtain the orientation angle \( \theta \) and aspect ratio \( \beta \) of the front. As shown in fig (2), \( R_f \) and \( R_g \) are the radial positions of transducers \#f and \#g; the couples \((X_f, Y_f)\) and \((X_g, Y_g)\) are their positions along the principal axes 1 and 2. If \( A_f, B_f, A_g \) and \( B_g \) are the semi-axes of the front as it touches transducers \#f and \#g at times \( t_f \) and \( t_g \), the equations defining the fronts at these times are:

\[ \frac{X_i^2}{A_i^2} + \frac{Y_i^2}{B_i^2} = 1 \hspace{1cm} i = f, g \]  \hspace{1cm} (8)
where $X_f$, $Y_f$, $X_g$ and $Y_g$, as $\theta$, are unknown. Consideration of the area of the front at times $t_f$ and $t_g$ leads to the expression:

$$
\frac{Q \cdot t_i \cdot \beta_i}{\pi \cdot h \cdot \varepsilon} = R_i^2 \left[ (\cos \theta)^2 + \beta_i^2 (\sin \theta)^2 \right], \quad i = f, g
$$

(9)

where $h$ is the cavity height and $\varepsilon$ is the porosity. If an average value of $\beta$ is assumed between $t_f$ and $t_g$ these equations can be combined as:

$$
R_g^2 \beta^2 - \left( \frac{Q \cdot t_f \cdot R_g^2}{\pi \cdot h \cdot \varepsilon} + \frac{Q \cdot t_g}{\pi \cdot h \cdot \varepsilon} \right) \beta + R_g^2 = 0
$$

(10)

which has two possible solutions for $\beta$; the solutions are not separated by an angle of ($\pi/2$) but correspond to two possible physical cases.

If the transducers $#f$ and $#g$ are laid along two non-orthogonal lines, the flow rate $Q$ and the times $t_f$ and $t_g$ suffice to determine $\beta$ and $\theta$; however that solution is highly very sensitive to fluctuations of the flow rate and is unpractical as a cavity should be equipped with two orthogonal lines to allow the use of the standard algorithms.

A third line of transducers allows to discriminate between the two cases resulting from eqn (10). However the times readings $t_f$, $t_g$ and $t_h$ from three transducers $#f$, $#g$ and $#h$ suffice to determine $\beta$ and $\theta$ without knowledge of the injection rate $Q$. The positions of transducers $#f$, $#g$ and $#h$ along the principal axis $X$ and $Y$ are functions of their positions along the axes $x$ and $y$ of the cavity:

$$
X_i = \sqrt{(x_i^2 + y_i^2)} \cdot \cos \left[ \tan^{-1} \left( \frac{y_i}{x_i} \right) - \theta \right], \quad i = f, g, h
$$

(11)

$$
Y_i = \sqrt{(x_i^2 + y_i^2)} \cdot \sin \left[ \tan^{-1} \left( \frac{y_i}{x_i} \right) - \theta \right], \quad i = f, g, h
$$

(12)

As a constant flow rate is imposed, eqn (13) is obtained assuming a constant value of $\beta$ on the interval covering $t_f$, $t_g$ and $t_h$:

$$
Area = z \cdot t_i = \pi \cdot A_i \cdot B_i = \pi \cdot \left( A_i^2 / \beta \right), \quad i = f, g, h
$$

(13)
where \( Area \) is the area of the impregnated domain and \( z \) is an unknown constant. The equation defining the front can be written as:

\[
z = \frac{\pi X_i^2}{t_i \beta} + \frac{\pi Y_i^2}{t_i \beta}, \quad i = f, g, h
\]  

(14)

The equality of \( z \) for different pairs of transducers gives:

\[
\left( \frac{X_g^2 t_f - X_f^2 t_g}{Y_f^2 t_k - Y_g^2 t_f} \right) - \left( \frac{X_h^2 t_f - X_f^2 t_h}{Y_f^2 t_k - Y_h^2 t_f} \right) = 0
\]  

(15)

which can be solved numerically for \( \theta \) along with eqns (11,12). Two solutions separated by \( \pi/2 \) are obtained, corresponding to the same ellipse with aspect ratios of \( \beta \) and \( \beta^{-1} \); therefore the solution is unique.

This solution allows, in theory, to obtain \( \beta \) and \( \theta \) from the time readings of three transducers located at different radii \( R \). However this is not always true in practice. Even if \( \beta \) is often assumed to tend rapidly toward \( \alpha^{-1/2} \), test results show that the hypothesis of a constant
\( \beta \) in eqn (13) does not always apply. However the pressure transducers allow the advance of the front to be monitored continuously by interpolating the elliptic\(^7\) equivalent of eqn (1). Hence positions \( x, y \), corresponding to the same time, and associated to a constant \( \beta \), can be used in eqns (11,12). As the front is localised without using the injection rate \( Q \), this value can be monitored; Ferland et al.,\(^{14}\) pointed out that the flow rate may fluctuate in real laboratory experiments.

Once the shape of the pressure distribution is known for each time step, the superficial velocities are calculated along the principal axes and the principal permeabilities are obtained in a regular manner, from eqn (7). If the cavity is equipped with two transducers laid along three lines and of one transducer located at the injection port, then this transducer is only used to determine the injection start time. In contrary to other methods the value of the injection pressure is not used in the calculation. This represents an advantage as it is not possible to determine precisely a radius for the boundary separating the fibre-free zone above the injection port and the fibrous preform.

### 3 Results

Pressure curves appear in fig (3) for 6 layers of the unbalanced stitch-bonded glass textile UC/600/100 (600 g/m\(^2\)) from Flemings Industrial Fabrics; the fibre volume fraction is 0.63, and the fluid, SAE 30 motor

![Figure 3. Evolution of the pressure with time, anisotropic material.](image-url)
oil with $\mu = 0.3$ Pa·s at 22 °C, is injected at $6.7 \times 10^{-7}$ m$^3$/s. The cavity is instrumented with a central transducer and 6 others laid along three lines (2 per line) oriented at 0°, 45° and 90° relative to axis x (warp direction). The figure shows the anisotropy of the material.

Curves obtained in identical conditions are presented in fig (4) for the balanced 600 g/m$^2$ plain weave glass material from Vetrotex. The isotropy can be seen from the times at which inner and outer transducers mounted on axes x and y are touched by the front.

The evolution of the orientation angle $\theta$ as a function of the

![Figure 4](image1.png)

Figure 4. Evolution of the pressure with time, isotropic material.

![Figure 5](image2.png)

Figure 5. Orientation angle $\theta$ as a function of the shear angle $\varphi$. 
shear angle $\phi$ is presented for the material UC/600/100 in fig (5). One series of points (black triangles) was obtained by using directly the time readings obtained from the transducers. These points were all obtained in identical conditions; it is seen that the interpolated time values must be used. The second series of points, represented as white squares, was obtained by varying the shear angle $\phi$. The observed trend presents similarities to the results obtained by others.\textsuperscript{2-5}

4 Conclusion

A method for the calculation of the orientation, degree of anisotropy and permeability of textile reinforcements laid in an arbitrary orientation in a planar cavity was presented. It was evidenced that the hypothesis of a constant aspect ratio $\beta$ for the elliptic front can not be regarded as a constant value under all circumstances. Supplementary experimental results produced using the method will be included in further publications.

5 References


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