Mathematical modelling of porosity of plane and 3D woven structures

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Abstract

A mathematical model of 3D woven fabric internal structure computes a spatial placement of all yarns within a fabric repeat for the given 3D weave structure (a special coding algorithm is employed) and warp and weft yarns geometrical and mechanical parameters. The principle of minimum energy of a fibrous assembly leads to the prediction of yarn crimp, forces in contacts warp–weft and yarn compression within a fabric. This allows to calculate the porosity (or the fibrous content) for an arbitrary cross-section of a fabric and to produce an image of the cross-section which gives an estimation for filtration abilities of the fabric. The model is implemented in the software package CETKA.

1 Introduction

The software package CETKA (“net” in Russian) (Lomov[1,2,3]) allows to simulate the internal structure of multilayered woven fabrics, computing any desired geometrical property of the fabric. In the cited papers, how-
ever, 3D fabrics were not discussed. The latest version of the package – CETKA 3.1 – uses the method of coding of 3D fabric structure proposed by the authors (Lomov and Gusakov[4,5]) to cover this important class of woven reinforcement (to say nothing of other technical and apparel applications). This analysis is quite different from other works in this direction (Chen et al.[6], Okumura et al.[7], Zhao et al.[8]) as these works aim primarily on technological problems of the 3D woven reinforcements or mechanical properties of composite with a resin in it. In the present paper the essentials of CETKA woven fabric model will be given and then the package possibilities to simulate fabric porosity will be explored.

2 Essentials of CETKA woven fabric model

We assume that the following 3D fabric parameters are known (subscripts below designate: \(i\) – number of warp zone \((i=1...N_o)\), \(z\) – warp number in a zone \((z=1...N_{wa,io})\), \(j\) – number of weft row \((j=1...N_y)\), \(l\) – number of weft layer \((l=1...L)\), \(k\) – number of yarn intersection along a warp \((k=1...K_{il})\):

1. Weave structure – with the help of codes described in Lomov and Gusakov[4,5] (Figure 1)
2. Fabric warp and weft density \(P_{wa}, P_{we}\) and yarn spacing \(p_{wa}, p_{we}\).
3. Weft layers displacements in the warp direction, due to fabric movement in the loom \(\Delta_i\), (see Lomov[1]).
4. Warp and weft parameters: linear densities (we do not suppose all yarns identical!); yarns dimensions in free state; the law of yarns compression or some evaluation of yarns compressed dimensions in the fabric; yarn bending rigidities \(B_{wa,0}, B_{we,0}\). The law of yarns compression is used in a form \(d_{1w_w} = d_{1w_w}^{0}\eta(Q), \ d_{2w_w} = d_{2w_w}^{0}\eta(Q), \ d_{1w_e} = d_{1w_e}^{0}\eta(Q), \ d_{2w_e} = d_{2w_e}^{0}\eta(Q)\), where \(Q\) is a lateral force per unit length, \(d_1\) and \(d_2\) are two yarn dimensions in the direction of force \(Q\) and normal to it (flattening), superscript 0 designates uncompressed state (Harwood et al.[9]).

Giving these parameters, the CETKA package computes warp and weft crimp heights and the spatial placement of warp and weft yarns. This is done by the decomposition of the complex bent yarn shape into elementary bent intervals between yarn’s intersection. Let us consider warp bent intervals. Weft yarns intersected by the warp at ends of the bent interval will be called supporting yarns. The numbers of supporting yarns for any bent element are determined by the weave coding. Warp bent intervals can be classified into three categories (Figure 2a): horizontal (1), vertical (2) and
Figure 1: 3D weave. Intersection codes for the blackened warp: 
(1,1),(1,2),(1,3),(1,4),(1,5),(2,4),(2,3),(2,2),(2,1),(2,0)

Figure 2: Elementary bent intervals: a) weft bending directions; b) bent element shape: 1 – contact with the supporting contour on ellipse arc, 2 – spline approximation; c) a scheme for side deflection force
"quarter" (3). Weft bent intervals can correspond to the bending in two directions (see arrows on Figure 2a); these two types of bending are described independently. Considering a horizontal bent interval of a warp (Figure 2b; other types of bent intervals are treated similarly) we introduce supporting contours around cross-sections of supporting weft yarns at the direction of a the half of warp diameter from them. Bent line \( z(x) \) is approximate as a combination of ellipse arcs (in contact regions) and polynomial of \( 5^{th} \) order (in between); contact coordinates and polynomial coefficients are determined from the condition of minimum bending energy. Now the following characteristic functions can be defined and numerically tabulated:

\[
\begin{align*}
\text{bending energy } \quad w &= \frac{1}{2} B \int_0^\nu \frac{(z'')^2}{\left(1 + (z')^2\right)^{5/2}} \, dx = \frac{B}{p} F(h/p) \quad \text{an its derivative} \\
\frac{\partial w}{\partial h} &= B \cdot G(h/p) / p^2; \quad \text{transversal force } \quad Q \approx 2w/h = \\
&= \frac{2B}{p^2} f(h/p); \quad \text{element length } \quad l = \int_0^\nu \left(1 + (z')^2\right)^{1/2} \, dx = p \left(1 + \alpha(h/p)\right) \\
\end{align*}
\]

Using these functions, crimp heights of warp and weft yarns are found from a system of algebraic equations. To build this system, the minimum energy principle for warp and weft simultaneous bending is expressed as:

\[
\sum_{i=1}^{N_w} \sum_{z=1}^{K_{iz}} \frac{\partial W_{Wz}}{\partial h_{iz}} + \sum_{j=1}^{L} \sum_{i=1}^{N_w} K_{ij}^{Wz} \frac{\partial W_{Wz}}{\partial h_{ij}} = 0, \quad i' = 1 \ldots I, j' = 1 \ldots N_{we}
\]

Here elementary bending energies \( W_{Wz} \) and \( W_{We} \) are related to crimp heights via characteristic functions; warp and weft crimp heights \( h \) are related via

\[
H_{Wz} = \Delta Z + (d_{Wz} + d_{We}) - \left(h_{1Wz} + h_{2We}\right) / 2
\]

where yarns numbers subscripts are omitted, \( \Delta Z \) is a vertical displacement between weft layers for supporting weft yarns designated by subscripts 1 and 2, \( d \)’s are computed using the characteristic function for \( Q \).

Using the characteristic function \( G \) and eqn. (1),

\[
-\frac{1}{2} \sum_{i=1}^{N_w} \sum_{z=1}^{K_{iz}} \sum_{k=1}^{K_{iz}} \left( B_{izk}^o \right) G\left( \frac{h_{izk}^o}{p_{izk}^o} \right) + \sum_{j=1}^{L} \sum_{i=1}^{N_w} K_{ij}^{Wz} \sum_{k=1}^{K_{ij}} B_{ijk}^v G\left( \frac{h_{ijk}^v}{p_{ijk}^v} \right) = 0,
\]
The sum for warp includes only bent elements of warp yarn \( i \) which has contact with weft yarn \( (j \cdot l') \); \( \frac{1}{2} \) before this sum appears because each weft crimp height \( h_{ij} \) enters the energy expression for the warp twice (see eqn. (1)). Eqns. (1,2) constitute a system of \( I \cdot L \cdot J \) equations for \( I \cdot L \cdot J \) unknown crimp heights \( h_{a} \) of warp and weft yarns. From these algebraic equations \( h_{a} \) are determined with the iterations method.

Computed crimp heights allow to evaluate side crimp of warp and weft – bending in the direction parallel to the fabric surface. Computing the side force \( l_{b} \) with the scheme Figure 2c: \( l_{b}=\eta Q\tan \theta /2 \), where \( \theta =\arctg(z'(p/2)) \) is computed with the polynomial approximation of \( z(x) \), and \( \eta \) is the compression coefficient of the yarn (see eqn. (1)), the energy balance between the work done by the force \( l_{b} \) on the displacement \( b/2 \) (side crimp height), bending energy increase and friction work (coefficient of friction \( \mu \)) provides the equation for \( b \).

Warp and weft yarns bend in two directions and \textit{three} crimp heights are determined for each weft yarn: vertical crimp, horizontal crimp in vertical woven layers (see Figure 1) and side deflection. Then warp crimp will be determined by eqn. (2). Yarn middle line displacement is computed as a vector sum of three components of crimp. Now all spatial placements of ends of warp and weft elementary bent intervals are determined; the shape of yarn on an elementary interval is determined by polynomial approximation. Yarns positions in the space are therefore fully determined, allowing to compute any desired geometrical property of the fabric.

3 Modelling a 3D fabric porosity

3.1 Plane fabrics: non-rectangular pores in twills and non-uniform pore section

Side deflection of yarns in twill weaves leads to the non-rectangular pores (Figure 3). The porosity computed without side deflection of yarns is quite close to the results for the deflected configuration (1.5% and 1.4% respectively for the fabric shown on Figure 3); the same holds for the mean pore dimension, evaluated with the “hydraulic diameter” \( d_{h} = 4S_{pores}/L \), where \( L \) is total pores perimeter (0.08 and 0.07 mm). But the minimum and maximum pore size is affected by yarn deflection considerably (0.05 and 0.12 mm compared to 0.08 mm without a side deflection).

Another factor affecting permeability of a fabric is the porosity distribution in the direction normal to fabric surface: the fibrous content has a
Figure 3: Porosity of twill 2/1. Warp and weft: 20 yarns per cm, polyester 111 tex, \( \eta = 0.8 \), \( \eta' = 1.1 \). a) Through porosity. One yarn is marked; black rectangles - pores for undetected yarns; b) porosity vs. a section \( z \)-position.

Figure 4: Porosity of two-layered polyester fabric. Warp: multifilament 310 tex, 18.6 yarns per cm, \( \eta = 0.7 \); weft: monofilament 272 tex (0.5 mm), 14.0 yarns per cm: a) cross-section along warp and weft; b) pores in oblique direction; c) porosity and hydraulic diameter of pores vs. direction; d) cross-section by plane inclined at \( \phi = 35 \) degrees (parallel to the arrow).
maximum in the middle of the fabric. Figure 3a shows that this dependence is very strong: the channel with parameters corresponding to eqn. (1) goes only through 1/3 of fabric thickness; neglecting this fact can lead to the gross error in permeability estimations.

3.2 Multilayered fabrics: pores in oblique directions

Let us consider a fabric shown on Figure 4. The warp spacing in it is so dense, that there are no pores in the direction normal to fabric surface (and $\Pi = 0$ in eqn. (1)). But there are pores in the direction shown by the arrow on Figure 4b. Computation reveals the complex pore shapes in this direction. This data can be used for evaluation of permeability of the fabric. Let us consider the dependence $\Pi(\phi)$ and let $\phi_-$ and $\phi_+$ be points of its maximum (when the direction $\phi$ is inclined at positive and negative angle), and let $\Pi_-, d_-, \Pi_+, d_+$ be the respective values of porosity and hydraulic diameter. Then apply some semi-empirical law of aerodynamic resistance (Robertson[10]) to air flow through pores with parameters $(\Pi_-, d_-)$ and $(\Pi_+, d_+)$ to obtain velocity estimations $u_-$ and $u_+$. Then the velocity of the air flow through the fabric for pressure $\Delta p$ would be given by $u = u_- + u_+$.

For the fabric shown on Figure 4 this simplified model produces estimation $u = 1.46$ m/s and $u = 2.27$ m/s for $\Delta p = 50$ and 127 Pa respectively. Experiment gives for these $\Delta p$ values $1.28 \pm 0.04$ m/s and $2.84 \pm 0.15$ m/s.

This type of evaluation is very approximative. As can be seen from Figure 4d, the real structure of channels inside a fabric is quite far from the model of a plate with holes, implied by eqn. (5).

3.3 3D fabrics: parametric study of the porosity

We shall consider a multilayered fabric with a rather simple weave structure, but with a considerable (21) number of layers (Figure 5a) to study the dependence of fabric parameters on warp and weft crimp height and relative yarn diameters in the following intervals:

- the ratio of warp and weft crimp: straight warp, equal crimp, straight weft (in horizontal an vertical layers);
- diameters of three groups of yarns: warp yarns in vertical ($d_w$) and horizontal ($d_h$) layers and weft ($d_w$) yarns – the diameter of yarns in one group is 1/3...3 when other yarns diameters is 1.

Cross-sections of all yarns are circular and they do not compress in a fabric (this is an idealised case for the parametric study).
Figure 5: 3D fabric for parametric study: a) weave (section in warp plane), b) computed fabric structure: sections in weft planes
Table 1. Warp/weft crimp in horizontal (up) and vertical (down) woven layers, %. Equal yarn diameters

<table>
<thead>
<tr>
<th>Crimp in horizontal layers</th>
<th>Crimp in vertical layers</th>
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<tbody>
<tr>
<td></td>
<td>S.Wa.</td>
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<tr>
<td>S.Wa.</td>
<td>0/1958</td>
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<tr>
<td></td>
<td>57.29</td>
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<tr>
<td>Wa = We</td>
<td>14/2030</td>
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<tr>
<td></td>
<td>14/29</td>
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<tr>
<td>S.We.</td>
<td>57/2106</td>
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<td>0/29</td>
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Table 2. Fibrous volume content, %

<table>
<thead>
<tr>
<th>d_h</th>
<th>d_v</th>
<th>d_w</th>
<th>Crimp in horiz. layers</th>
<th>Crimp in vertical layers</th>
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<td></td>
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<td>S.Wa.</td>
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<td>S.Wa.</td>
<td>41</td>
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<td></td>
<td>Wa = We</td>
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<td></td>
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<td></td>
<td>S.We.</td>
<td>41</td>
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<td>51...15</td>
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<td></td>
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<td>44...59</td>
<td>47...51</td>
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Note. In tables 1–2: S.Wa – straight warp, S.We – straight weft, Wa=We – equal warp and weft crimp

Tables 1 and 2 give the insight in the porosity trends. The most dramatic influence on fabric fibrous contents is produced by warp diameters both in horizontal and vertical layers. By the variation of it the fibrous contents can be made as low as 15% (and porosity as high as 85%) and as high as 89% (porosity 11%). The weft diameter does not affect fabric fibrous content so strongly. With the filter applications of 3D fabric in mind, it is interesting to consider the dimensions of channels in a fabric.
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Figure 5b clearly shows that in spite of the dense fabric structure within vertical woven layers pores between them can be rather big, allowing to pass much bigger particles.

4 Conclusion

CETKA 3.1 software package proves to be an efficient tool in 3D fabric structure study, providing an insight in fabric internal structure. The parametric study shows the possibilities to control 3D fabric properties with the choice of yarns parameters, achieving the broad interval of variation of fabric porosity for a given weave structure. The porosity of 3D fabric variates strongly within the fabric volume.

References

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[2] Lomov S.V., Truevtzev N.N. A software package for the prediction of woven fabrics’ geometrical and mechanical properties, Fibres & Textiles in Eastern Europe, 3, N2, pp.49–52, 1995