An analysis of contact stresses and displacements in a plane interference joint loaded by a concentrated force

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Abstract

The paper deals with the problem of modelling a plane interference joint (a ring with a wheel) by means of the finite element method (FEM), and determining stresses which are produced in this joint by the assembly (interference) itself and by locally acting working loads. An ADINA FE system is used for determining the contact stresses and the stiction forces that occur within the contact between the joined elements. An analysis was made on the possibility of the occurrence of local slips and the loss of contact of the joined surfaces in the zone of action of the concentrated force. A simple, Coulomb friction model was assumed and, on that basis, the condition of permissible loads, at which local slips of the joined surfaces would not occur, was formulated.

1 Introduction

The interference joints (obtained through shrink, expansion or force fit) offer many advantages. Realisation of these joints consists in applying definite assembly stresses and strains to them so as to clamp one element on the other. The friction forces that are produced by this grip enable these joints to carry considerable external loads of various types. Most frequently, these loads are of local character (Fig.1). Therefore, the analysis of the load capacity, the strength and the stiffness of the interference joints should generally take into account the following two states of stress and strain:

- assembly state
- operational state.
Determination of the assembly stresses and strains for axial symmetric elements within the elastic range poses no problem and is widely known (as so-called Lamé problem). The assembly stresses and strains can also be easily determined using the finite element method. The difficulties in solving the problem of the interference joints consist in determining stresses produced by external loads which occur during operation of these joints (Fig.1). These are particularly great when there are some discontinuities in stresses and displacements in the contact zone.

![Diagram](image)

**Figure 1.** Deformations caused by the action of local working loads on: (a) toothed ring, (b) rolling bearing race.

Within this scope, the methods of calculations of interference joints, proposed elsewhere [1-3], contain many simplifications and imperfections. This work presents a solution to this problem obtained using the finite element method.

### 2 Proposed model and method for problem solving

The subject of discussion in this work is a plane model of an interference joint, composed of a wheel and a ring with a modelled single tooth loaded by a concentrated force $P$ whose components are $P_z$ and $P_y$ (Fig.2a).

The purpose of the work is: 1) to determine the stresses $\sigma_\rho$, $\sigma_\theta$, and $\tau_{\rho\theta}$ in the joined elements, caused by the assumed geometrical interference $\Delta d$ and by the concentrated force $P$ that acts on the modelled tooth, and 2) to determine the contact stresses and find a value of the design safety factor which would prevent local slips from occurring. The presented model was solved making use of the finite element method and with the aid of the ADINA system. A FE-model of the interference joint analysed is presented in Fig.3.

This model is constructed with the use of six and eight-node 2-D SOLID elements (according to [4]). The division into finite elements in the ADINA system is realised automatically for predefined size of these elements. As it can be seen in Fig.3, the finite element network is finer at the surface of contact.
between the joined elements. This was done in order to obtain more exact results of computations in this area. The contact between the wheel and the ring is binodal in this model. The ADINA system allows for adoption of only the simplest model of contact, a Coulomb friction one. The elastic properties of the ring and wheel materials are defined by the parameters $E_i, v_i$ and $E_{ij}, v_{ij}$ respectively (Fig. 2b). The analysed model is pivotally supported at four points around the inside edge of the wheel so as to enable radial displacements to occur.

In order to determine the assembly stresses, caused by predefined geometrical interference $\Delta d$ (treated as elastic interference), the wheel was first cooled down by $\Delta T$ Kelvin's so as to enable (theoretically) the elements to be mounted without applying any force. After fitting the ring and heating the wheel up to the ring temperature a stress will arise in the elements joined. The value of this stress corresponds to the assumed interference. The value of $\Delta T$, by which the wheel should be first cooled and next heated in order to achieve the predefined geometrical interference $\Delta d$, results from the following relationship:

$$\Delta d = \lambda \cdot d \cdot \Delta T$$  \hspace{1cm} (1)

where $\lambda$ is the coefficient of thermal expansion.
3 Numerical calculations

Numerical calculations were made with the use of ADINA program. The following data were assumed (Fig. 2):

\[ d_1 = 0.02 \text{ m} \]
\[ d = 0.18 \text{ m} \]
\[ d_2 = 0.20 \text{ m} \]
\[ \delta = 0.01 \text{ m} \]
\[ h = 0.0175 \text{ m} \]
\[ E_i = E_{ii} = 2.1 \cdot 10^5 \text{ MPa} \]
\[ \nu_i = \nu_{ii} = 0.27 \]
\[ \lambda = 12.5 \cdot 10^{-6} \text{ 1/K} \]
\[ \Delta d = 16 \cdot 10^{-6} \text{ m} \]

Figure 4 shows the results of the computer calculations (according to ADINA program) that illustrate the assembly stress patterns (for both normal and tangential stresses) on the surface of contact between the ring and the wheel. The numbers 1 and 2 denote the assembly stress patterns for a smooth, axial symmetric ring – without the tooth. For this case, the pattern of normal contact stresses \( \sigma_p \) is uniform and their value, determined by means of FEM, agrees with the result of analytical calculations done according to Lame’s formulae. The tangential stresses here are equal to zero.

The presence of the single tooth on ring results in that the pattern of normal contact stresses \( \sigma_p \) is non-uniform in the close neighbourhood of this tooth (Curve 4 in Fig. 4). This tooth also causes the assembly shear stresses \( \tau_{p\theta} \) which occur in the contact surface (Curve 3 in Fig. 4). Figure 5 presents the radial displacements of the contact surfaces of the ring and the wheel after assembling.
Figure 4. Normal and tangential stresses in the contact surface between ring and wheel, caused by the assembly: 1 and 2 – for the ring without tooth, 3 and 4 – for the ring with a single tooth.

Figure 5. Radial displacement of contact surface of ring and wheel, caused by the assembly: 1- for the ring without tooth, 2 – for the ring with a single tooth.
Figure 6 presents the patterns of normal and tangential contact stresses which are due to the assembly and the action of an external force $P$ whose components are $P_r=500\ N$ and $P_z=115.4\ N$. The external force that acts on the tooth causes essential changes to the primary (assembly) pattern of the contact stresses. These changes are of local nature. The results of the calculations indicate (Fig.6) that the torque, produced by the force $P_r$ acting at the radius $\rho=0.5d+h$, is transmitted from the ring to the wheel, owing to friction forces, only by a small portion of the contact surface ($\theta \approx \pm 40^\circ$) within which the shear stresses occur. The methods which are commonly used for calculating the interference joints usually assume a simplification that the torque is transmitted by friction forces which are uniformly distributed around the entire surface of contact between the elements joined. The external forces $P_r$ and $P_z$ that act on the tooth produce local strains of the ring and have an essential effect on the patterns and values of the normal and tangential contact stresses only in the vicinity of places where they act. They can cause local slips on the joined surfaces (if $|\tau_{p\theta}|>\mu|\sigma_p|$) or, else, local loss of contact of the ring with the wheel (if $\sigma_p=0$). In Fig.6 one can see a great local decrease in the value of the stress $\sigma_p$. In this case, however, neither local slips nor the loss of contact of the joined surfaces take place since each point of the joint satisfies the conditions $|\tau_{p\theta}|<\mu|\sigma_p|$ and $\sigma_p<0$. The distributions of radial displacements of the contact surfaces of the ring and the wheel are shown, for the discussed example, in Fig.7. The radial
displacements of both the surfaces are identical. This means that they remain in contact with each other. Also, the local slips of the contacting surfaces do not occur in the discussed example.

Figure 7. Radial displacement of the contact surfaces of the ring and the wheel, caused by the assembly and by the forces $P_y=500 \, N$ and $P_z=115.4 \, N$.

Figure 8. Normal and tangential contact stresses caused by the assembly and by the forces $P_y=1300 \, N$ and $P_z=300 \, N$ that act on the tooth.

Figure 8 shows the patterns of normal and tangential contact stresses caused by the assembly and the action of a force $P$ whose components are $P_y=1300 \, N$
and $P_z=300\ N$. The peripheral force $P_y$, whose value here is much greater than previously, caused a local loss of contact between the ring and the wheel, and local slips of the contacting surfaces. In the place where the contact is lost, there is a positive allowance. The values of the normal and tangential contact stresses in this place are equal to zero (Fig.8). The distributions of radial displacement of the joined surfaces are shown, for this example, in Fig.9. Curve 1 in this Figure illustrates displacements caused by the assembly while Curves 2 and 3 illustrate the displacements of the contact surfaces of the ring and wheel respectively, occurring after applying the external forces $P_y$ and $P_z$.

Figures 10 and 11 present the relative radial displacement $\Delta u = u_r - u_w$ and the relative peripheral displacement $\Delta v = v_r - v_w$ of the contact surfaces of the ring and the wheel. From the calculations one can conclude that the local slips occur over a portion of contact which is much greater than of the occurrence of the loss of contact (Figs. 10 and 11).

Figure 9. Radial displacement of the contact surfaces of the ring and the wheel, caused by the assembly and by the forces $P_y=1300\ N$ and $P_z=300\ N$.

Figure 10. Relative radial displacement of the surface of contact between the wheel and the ring, caused by the assembly and by the forces $P_y=1300\ N$ and $P_z=300\ N$ that act on the tooth.
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Figure 11. Relative peripheral displacement of the surface of contact between the wheel and the ring, caused by the assembly and by the forces $P_Y=1300 \, N$ and $P_Z=300 \, N$ that act on the tooth.

Under real conditions of operation of the interference joints the local slips can displace around the wheel’s periphery, following the external force, and cause a slow drift of the entire ring. This may lead to the destruction of the joint and cause a failure.

In a well designed interference joint the load capacity condition should be satisfied, at an adequate safety factor, not only for the sake of excluding the global slip of the ring over the wheel but also for the sake of excluding the local slips, with working loads being properly taken into account. Selection “at a guess” of a high value of the safety factor for the sake of global slip, which has been often used till now in practice, is unreasonable and not always eliminates the danger of the occurrence of local slips. This issue will be illustrated based on the above presented examples.

For the interference joint, presented above, the maximum torque is equal to the moment of friction which is usually calculated from the formula $M_F = 0.5 \mu \pi d \delta$. For the data assumed earlier we obtain $M_F = 732.9 \, Nm$.

The torque produced by the force $P_Y$ is dependent upon the value of this force and is as follows:

- for $P_Y=500 \, N$ \hspace{1cm} $M_{01} = P_Y(0.5d+h) = 53.75 \, Nm$
- and for $P_Y=1300 \, N$ \hspace{1cm} $M_{02} = P_Y(0.5d+h) = 139.75 \, Nm$

The safety factors are calculated from the following formulae:

- with the global slip criterion being taken into account:

\[
X_{Gl.} = \frac{M_T}{M_O} \tag{2}
\]

- with the local slip criterion being taken into account:

\[
X_L = \frac{\tau_{\rho \theta}}{\mu |\sigma_\rho|} \tag{3}
\]
In the first example, the safety factors, calculated from the formulae (2) and (3), assume values of $X_{GL1}=13.64$, $X_{L1} \geq 1$. In this case there are no local slips. A low value of the coefficient $X_{L1}$ indicates that a small increase in the value of the force $P_Y$ will bring about the occurrence of such slips.

In the other example, at a fairly high value of the coefficient $X_{GL2}=5.24$, the coefficient $X_{L2}$ is $X_{L2} < 1$. In this case not only considerable local slips are found but also local loss of contact between the ring and the wheel takes place (Fig.8, 9 and 10).

4 Conclusions

In this work, the finite element system “ADINA” was used for modelling a plane interference joint of a ring with a wheel, and for determining stresses and displacements in this joint, caused by the assembling process and by the action of a concentrated external force.

A particular goal of numerical calculation was to determine normal and tangential contact stresses in the surface of contact between the ring and the wheel, and the relative displacement of the joined surfaces, caused by the assembly and the action of local external load. The obtained results of calculations are highly accurate, both in qualitative and quantitative terms.

The presented examples of the calculations of the interference joints indicate that there is a potential for an effective use of the finite element method in evaluating the load capacity of interference joints, with a due consideration given to the working loads of local nature. In particular, this method enables us to determine the value of the safety factor $X_L = \min \mu \sigma_f/\tau_{pl}$, i.e. one which excludes the possibility of the occurrence of local slips.

References