Contact analysis for the modelling of anchors in concrete structures

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Abstract

A specific finite element program is designed for the analysis of the behaviour of anchors in concrete structures under dynamic loading. Since the anchors can include an expanding tubular casing, large strains and large rotations are available including the contact with friction of three bodies: anchor, tubular casing and concrete.

1 Introduction

The most of concrete structures need anchorage. In order to assess the safety of this type of structure, it is necessary to predict the mechanical response of the anchors and to evaluate their pull-out failure loads. Therefore, a specific finite element program, PLAST2, based on a dynamic explicit method and including large deformation, non-linear material behaviour and contact with friction was developed.

A smeared crack approach with non-linear plasticity and fracture mechanics is used for the description of concrete.

The contact problem, in this case, is computationally difficult because three deformable bodies must be taken into account: anchor, tubular casing and concrete. Two methods are implemented to introduce contact with friction: an incremental penalty formulation and a forward increment Lagrange Multipliers algorithm. Comparisons between this two methods were performed and showed that the second, which handles node to side contact, is more intrinsic than the first which requires penalty parameters to be adjusted.
Finite element simulations of pull-out tests for different types of anchorage were carried out and the results were compared with the experimental ones. Two friction coefficients are defined between tubular casing and concrete and between tubular casing and anchor bolt. The simulations carried out allowed to investigate the influence of this two friction coefficients on the load-displacement curve and on the crack propagation.

The aim of this modelling is to be able to predict the pull-out failure load of an steel anchorage in concrete in order to reduce experimental tests which are difficult to achieve and uneconomic.

2 The finite element code PLAST2

The program system developed includes a pre-processor, a solution program and an efficient post-processor. The finite element analysis can be interactively controlled and runs in several levels of real time graphics.

2.1 Formulation problem

The employed finite element method is based on a Lagrangian-mesh Cauchy-stress formulation in conjunction with time integration scheme. The equations of motions are developed via the principle of virtual work leading to the following system of differential equations of the second-order at time step $t$:

$$[M]\{\ddot{u}\}_t + [C]\{\dot{u}\}_t + \{F_{\text{int}}\}_t = \{F_{\text{ext}}\}_t$$

where $M$ and $C$ are respectively the mass and damping matrices, $F_{\text{int}}$ and $F_{\text{ext}}$ the nodal vector of internal and external forces respectively, $\dot{u}$ the velocity vector and $\ddot{u}$ the acceleration vector.

The time integration of the differential system (1) is performed by the central-difference method with $\Delta t$, a fixed time integration step.

If the mass and the damping matrices assume a diagonal form, displacements and velocities can be updated without necessity of equation solving.

A four node quadrilateral element, with 2x2 Gauss quadrature rule, but allowing large plastic strains and avoiding shear locking is implemented. The strains are evaluated using the configuration halfway between time steps $t$ and $t+\Delta t$ and this choice delivers an excellent approximation of the logarithmic strains. Upon transformation time-dependent co-rotating system, the numerical integration of the rate elasto-plastic equations are performed applying the radial return process.
2.2 Concrete behaviour modelling

The constitutive model of concrete is based on a smeared crack approach, similar to the one developed by Owen, Figueiras & Damjanic [5], with non-linear plasticity and fracture mechanics.

The behaviour of concrete is described by a four-branches diagram: 2nd degree parabola in compression loading, linear loading in tension and linear softening in both tension and compression (Figure 1).

![Concrete behaviour modelling in tension (a) and in compression (b)](image)

The cracking, or tensile fracture, is governed by a maximum tensile stress criterion (tension cut-off). Cracks are assumed to form in planes perpendicular to the direction of maximum principal tensile stress as soon as this stress reached the specified concrete tensile strength $f_t$. To avoid further complexities, only two orthogonal cracks are assumed to form, for an integration point, in planes perpendicular to the structural plane. After cracking has occurred, the concrete is rendered anisotropic by reducing to zero the Young's modulus and the Poisson ratio in the plane perpendicular to the cracked plane and a reduced shear modulus is employed.

The non-linear fracture mechanics is introduced by means of the crack band theory [1]. In this theory, the maximum tensile strain, $\varepsilon_m$, is adjusted for each integration point according to the following formula:

$$\varepsilon_m = \frac{2.G_f}{\alpha. f_t. l_c}$$  \hspace{1cm} (2)

where $G_f$ is the fracture energy, which represents the energy dissipated per unit area and is assumed to be a material property independent to the spatial position of the point ($G_f$ constant). $l_c$ is called 'characteristic length' or 'crack band width' and is taken equal to $\sqrt{S}$ ($S$, surface associated to a Gauss point) for a two-dimensional problem or to $\sqrt[3]{V}$ ($V$, volume associated to a Gauss point) for a three-dimensional problem. Therefore, making the constitutive law depend on mesh size via the parameter $l_c$, objective results with regard to the finite element mesh are obtained.
2.3 Contact algorithms between bodies

Explicit algorithm are the most efficient for resolving a problem involving several contacting bodies. The finite element code PLAST2 allows the contact between body-rigid or body-body surfaces [3]. In the first part, the results obtained using the penalty method are presented and compared with the forward increment Lagrange multiplier method [4]. With the first method, the contact control requires the adjustment of the two penalty coefficients $k_n$ and $k_t$ for each new design of contacting bodies [2]. The Lagrange multiplier method in two dimensional and with friction is based on the numerical contact algorithm developed by CARPENTER and al. [4]. This method compatible with explicit time integration operators is briefly presented here. First, pairs of target and contactor surfaces are defined and a friction coefficient $\mu$ is associated to each pair. Then, the contact constraint, applied at every step of the simulation consists in preventing the contactor nodes from penetrating the target domain.

The equations system of the forward increment Lagrange multiplier method is :

Find $\{u\}_{t+\Delta t}$ such that :

$$
\begin{align*}
\begin{bmatrix} [M] & [C] & [F_{int}] \\
[C] & [G]_{t+\Delta t} & \{\lambda\}_t \\
[F_{int}] & [G]_{t+\Delta t} & [X_t + u_{t+\Delta t} + u_t]
\end{bmatrix}
\begin{bmatrix} \ddot{u} \\
\dot{u} \\
\lambda_t
\end{bmatrix}
+ \begin{bmatrix} \ddot{F}_{app} \\
\dot{F}_{app} \\
F_{app}
\end{bmatrix}
= 0
\end{align*}
$$

with $\lambda_t$ is the contact forces vector acted on the nodes of the contactor surface and $G_{t+\Delta t}$ is the global assembled matrix of the constraint matrices.

The basic idea is first to let the couple of target and connector surfaces penetrate in each other within a single time step (vector $\lambda_t$ equal to zero). Then, the identification of all the surface contactor nodes that have penetrated the target surface is performed, and an additional incremental nodal displacement associated with the calculated contact forces is introduced into the system such that the non-penetration conditions are strictly enforced. During these iterations, the surface contact force conditions in the normal ($\lambda_n < 0$) and tangent ($|\lambda_t| \leq \mu |\lambda_n|$) directions are taken into account. The coupled forward increment Lagrange multiplier equation solving strategy is an efficient Gauss-Seidel algorithm which prevent the matrix $G_{t+\Delta t}$ to be assembled. The advantages of this method are the precision, the stability and the very good and fast convergence at every time step.

3 Numerical results

Two types of anchorage were numerically modelled in order to cover the range of a particular industrial set. The first anchor (named A1) have a diameter and a length smaller than the second one (named A2).
3.1 Preliminary study

In this part, the analysis was performed with only one body for the modelling of the anchor and the tubular casing in order to compare the different contact algorithms and to predict the friction coefficient between concrete and anchorage, \( \mu_{c/c} \).

3.1.1 Finite element mesh and boundary conditions

A two-dimensional axisymmetric analysis was conducted for a steel anchorage (\( E=200000 \text{MPa}, \nu=0.3 \)) embedded in concrete (\( f_c=45 \text{MPa}, f_p=4.5 \text{MPa}, E=27000 \text{MPa}, \nu=0.2, G_p=0.1 \text{N/mm}) \). Four node quadrilateral elements were used for both concrete and anchor modelling. The mesh and the boundary conditions are defined in Figure 2.

![Mesh and boundary conditions (anchor A2)](image)

Table 1: Pull-out failure loads \( F \) of the two types of anchor in concrete for different values of the penalty coefficients

<table>
<thead>
<tr>
<th>( k_n=k_i )</th>
<th>( 5.10^4 )</th>
<th>( 1.10^5 )</th>
<th>( 2.10^5 )</th>
<th>experimental</th>
</tr>
</thead>
<tbody>
<tr>
<td>F (N) anchor A1</td>
<td>23110</td>
<td>27160</td>
<td>23620</td>
<td>24800</td>
</tr>
<tr>
<td>F (N) anchor A2</td>
<td>19910</td>
<td>74400</td>
<td>51120</td>
<td>70000</td>
</tr>
</tbody>
</table>
3.1.2 Contact control using penalty method
The friction coefficient between anchor and concrete, $\mu_{c/c}$, has been fixed to 0.5. Three simulations with different penalty coefficients were performed for both types of anchor. Results are given in Table 1.

3.1.3 Contact control using the method of Lagrange Multipliers
Since the method of Lagrange Multipliers does not require the determination of coefficients as the penalty ones, the influence of the friction coefficient between anchor and concrete, $\mu_{c/c}$, was studied. Results obtained from several tests on both type of anchorage are given in Table 2.

<table>
<thead>
<tr>
<th>$\mu_{c/c}$</th>
<th>0</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.35</th>
<th>0.4</th>
<th>experimental</th>
</tr>
</thead>
<tbody>
<tr>
<td>F (N)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>anchor A1</td>
<td>13590</td>
<td>17960</td>
<td>21500</td>
<td>21640</td>
<td>22500</td>
<td>23350</td>
<td>24800</td>
</tr>
<tr>
<td>F (N)</td>
<td>35400</td>
<td>43700</td>
<td>52410</td>
<td>61670</td>
<td>67420</td>
<td>82630</td>
<td>70000</td>
</tr>
</tbody>
</table>

Table 2: Pull-out failure loads $F$ of the two types of anchor in concrete for different values of the friction coefficient, $\mu_{c/c}$

![Diagram](image_url)  
Figure 3: Crack propagation during a pull-out test (anchor A2)
The best numerical results were obtained for a friction coefficient $\mu_{oc}$ in the range of 0.35 to 0.4. Figure 3 shows an example of crack propagation during a pull-out test. The shape of the crack is similar to the one observed experimentally (cone with an angle at the top equal to $-120^\circ$).

3.1.4 Discussion
The penalty coefficients $k_n$ and $k_t$, which must be fixed by the user, have an high influence on the results, in particular for the anchor A2. The forward increment Lagrange Multipliers method is more efficient and intrinsic than the penalty one and give results in good agreement with experiment.

3.2 Modelling of pull-out tests of anchorage including tubular casing

The contact algorithm used in this section is based on the Lagrange Multipliers method. The mesh is the same than in the previous part but the anchorage is divided in two deformable bodies: anchor bolt and tubular casing. Hence, an other friction coefficient $\mu_{at}$ between anchor and tubular casing is introduced. The friction coefficient $\mu_{oc}$ between tubular casing and concrete is taken equal to 0.4 according to the results obtained in §3.1.3. The boundary conditions are also the same except for the tubular casing which is not constraint. The concrete and steel characteristics are unchanged.

The friction coefficient $\mu_{at}$ have an high influence on the load-displacement curve but not in the same way as for the two types of anchorage (Figures 4 and 5).

![Figure 4: Pull-out load(F)-displacement curves for various values of $\mu_{at}$](image)

(anchor A1-experimental pull-out failure load = 24800 N)
According to the simulations, a friction coefficient of 0.2 between the anchor and the tubular casing is the best compromise for this kind of concrete/anchorage configuration. The same results have been obtained in the past from experiments conducted with different coatings for the anchor and tubular casing.

Figure 5: Pull-out load (F)-displacement curves for various values of $\mu_{a/t}$ (anchor A2-experimental pull-out failure load = 70000 N)

Figure 6: Crack propagation during a pull-out test (anchor A2)
Figure 6 shows the shape of the crack obtained in the case of the anchor A2 with a friction coefficient $\mu$ equal to 0.2, which is identical to the experimental one. The friction coefficients between the three bodies have a great influence on the results and cannot be neglected. Another simulations with different types of concrete and anchor have led to a better understanding of the compromise (concrete/anchor/friction) obtained through the experiments.

4 Conclusion

The dynamic and quasi-static load carrying capacity of anchors do not rely entirely on the tensile strength and toughness of the concrete. The material and particularly the contact friction behaviour are significant, mostly in the case of an anchoring steel element expanding a tubular casing in the depth of concrete specimen. Hence, it is necessary to model the contact with friction with an efficient algorithm such as the forward increment Lagrange Multipliers method. The various studies have shown the effectiveness of the code to determine pull-out failure loads of anchorage embedded in concrete structure.

Acknowledgements

The authors wish to thank the society SPIT for the financial support of this work.

Key words : FEM; concrete; cracking; contact; Lagrange Multipliers

References