Dynamic effects for structures caused by moving vehicles

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Abstract

In tests of bridge behaviour under influence of moving load, a quasi-harmonic vibration task should be solved if we want to take into consideration also the mass of the moving body. Use of finite element method requires analysis of systems of several degrees of freedom and this calls for appropriate numerical techniques. Both the effect of the external damping proportional to the velocity should be computed and the procedure must be able to consider also the structural damping. During numerical tests, computations were carried out on a reality-like structure. The results underlines the importance of considering the internal friction of the structure.

1. Introduction

Before the appearance of computers the test of bridges under the effect of a moving load could be analysed only by essential simplifications. In these tests, the bridge was simulated by a beam of constant stiffness and tested as continuum. Inbanathan and Wieland [1] modelled the vehicle as a single moving mass, and the effect of damping was neglected both for structure and vehicle. Later, as an advance, the vehicle was simulated not only as a moving mass but as a dynamical system of several degrees of freedom - like Green and Cebon [2] did it for road vehicles.

Application of the finite element method requires test of dynamical systems of several degrees of freedom and this calls for appropriate numerical methods. For computing matrix differential equations of constant coefficient for given starting conditions, several methods have been developed, among others the Wilson-O method described by Bathe and Wilson [3]. In their original forms these methods are for matrix differential equations with time-
dependent coefficients rather computation-intensive, and fit only for solving very simple tasks.

The known methods - reported e.g. by Olsson [4] - are able to compute the effect of the external damping proportional to the velocity, however, are not fit to take the structural damping into consideration. Györgyi [5] suggested a method for computing the structural damping as an equivalent external damping.

The technique published in this paper allows the simultaneous analysis of both the mass and stiffness matrix changing with time and the external damping proportional to velocity and the frequency-independent structural damping.

2 Mechanical models of the task

2.1 Dynamical equation for a two-axle road vehicle

The vehicle model will be arranged after Green and Cebon [2] as depicted in Fig. 1. The vehicle is supported by the structure in points a and b, the vertical displacements of these points are identical with vertical displacements of a given point of the structure.

![Figure 1: Mechanical model of a road vehicle](image)

We assume that points a and b do not displace and write the dynamical equations of the vehicle vibrating around the equilibrium state.

\[ M_v \ddot{u}_v + C_v \dot{u}_v + K_v u_v = 0. \]  

(1)

Here \( M_v \) is the mass matrix of the vehicle, \( C_v \) the damping matrix corresponding to hydraulic dampers, and \( K_v \) the stiffness matrix of the vehicle. In this case, the displacement vector \( u_v \) includes only the displacement components of the mass points, the rows and columns corresponding to supports can be deleted from the matrices. We obtain the following dynamical equations for the structure:

\[ M_B \ddot{u}_B + K_B u_B = r_i(t). \]  

(2)

In the matrix equation the whole weight of the vehicle in the nodes of elements containing points a and b will be given by the help of the elementary force
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vectors \( \mathbf{r}_{ef} = \mathbf{N}_t^T \mathbf{r}_0 \) as load in vector \( \mathbf{r}_1 \) (the elements of the matrix \( \mathbf{N}_t \) are the displacement functions describing the relation between the displacement of the element's internal points and the displacement of the element's different nodes, \( \mathbf{r}_0 \) is the force vector in the points a and b). When arranging the complete dynamical system, the displacements of the structure come in the first block of the displacement vector, and in the second one the displacement vector of the vehicle. Analysing the whole system, points a and b in the vehicle model will displace, and this displacement will be identical to the displacement of the given point of the finite element corresponding to the vehicle state. Accordingly, both matrix differential equations placed next to each other will join and the matrices and vectors will have the following structure:

\[
\begin{bmatrix}
\mathbf{M}_B & 0 \\
\mathbf{M}_v & \end{bmatrix}
\begin{bmatrix}
\mathbf{C}_1(t) & \mathbf{K}_1(t) \\
\mathbf{C}_v & \mathbf{K}_v \\
\end{bmatrix}
\begin{bmatrix}
\mathbf{u}_B \\
\mathbf{u}_v \\
\end{bmatrix}
\begin{bmatrix}
\mathbf{r}_1(t) \\
0 \\
\end{bmatrix}
\]

In matrices \( \mathbf{C}_1(t) \) and \( \mathbf{K}_1(t) \) place of elements will change with time according to the vehicle's movement. These matrices can be obtained by compilation from the elementary matrices \( \mathbf{C}_{et}^a, \mathbf{C}_{et}^b \) and \( \mathbf{K}_{et}^a, \mathbf{K}_{et}^b \), respectively, belonging to the individual nodes. For computation of the elementary matrices, matrix \( \mathbf{N}_t \) already described above can be used, its elements are, of course, different in points a and b. For point a, e.g.:

\[
\mathbf{K}_{et}^a = k^a
\begin{bmatrix}
\mathbf{N}_t^T \mathbf{N}_t & -\mathbf{N}_t^T \\
-\mathbf{N}_t & \\
\end{bmatrix}
\]

Here \( k^a \) and \( c^a \) mean the stiffness of the spring joining in point a, and damping coefficient of the damper, respectively. Thus, the matrix differential equation of the task demonstrated in Fig. 1. is as follows:

\[
\mathbf{M} \ddot{\mathbf{u}} + (\mathbf{C}_C + \mathbf{C}_1(t))\dot{\mathbf{u}} + (\mathbf{K}_C + \mathbf{K}_1(t))\mathbf{u} = \mathbf{r}_1(t) \tag{3}
\]

In this relationship, subscript C designates the matrices with constant elements.
We note that in this model the mass matrix does not change with time. Writing the equation by the help of summed up matrices, we obtain

\[ M \ddot{u} + C(t)\dot{u} + K(t)u = r(t). \]  

(4)

2.2 Analysis of the effect of multi-axle railway vehicles

In this task, there are mass points of the vehicle also in contact points of the structure. The effect of these mass points will be expressed in the mass matrix of the structure as a time-dependent component. Consequently, the dynamical equation system of the coupled system is the following:

\[ (M_C + M_1(t))\ddot{u} + (C_C + C_1(t))\dot{u} + (K_C + K_1(t))u = r_1(t). \]  

(5)

\( M_C \) is the time-independent mass matrix shown in 2.1:

\[
\begin{bmatrix}
M_B \\
M_V
\end{bmatrix}
\]

\( M_1(t) \) matrix is calculated \( M_{ef} = N_f^T M_0 N_f \), where \( M_0 \) is the diagonal matrix in a size corresponding to the number of components of the internal point displacement, the value of elements belonging to displacements is \( m \), while the others are zero. If we write the equation by summed up matrices as shown above, we obtain

\[ M(t)\ddot{u} + C(t)\dot{u} + K(t)u = r(t). \]  

(6)

The last one out of the two models introduced is the general model. In this case, all the interacting matrices and the load vector are time-dependent.
3 Solution of the matrix differential equation by direct integration

Inbanathan and Wieland [1] and Olsson [5] successfully applied the Newmark procedure for computing displacements in simpler tasks, while Györgyi [6] showed that it was expedient to use the Wilson-\(\theta\) procedure for solution. Wilson [3] assumes linear acceleration change between moments \(t\) and \(t + \theta \Delta t\). Assuming linear change of \(r(t)\) for the given time period, we get

\[ r_{t+\theta\Delta t} = r_t + \theta (r_{t+\Delta t} - r_t) \]

and at the moment \(t + \theta \Delta t\) displacements can be obtained from the following relationship:

\[
\begin{align*}
\left( K + \frac{6}{(\theta\Delta t)^2} M + \frac{3}{\theta\Delta t} C \right) u_{t+\theta\Delta t} &= r_{t+\theta\Delta t} + M \left( \frac{6}{(\theta\Delta t)^2} u_t + \frac{6}{\theta\Delta t} \dot{u}_t + 2 \ddot{u}_t \right) + \\
&\quad + C \left( \frac{3}{\theta\Delta t} u_t + 2 \dot{u}_t + \frac{\theta\Delta t}{2} \ddot{u}_t \right). \\
\end{align*}
\tag{7}
\]

Afterwards, displacements, velocities and accelerations belonging to the moment \(t + \Delta t\) can be calculated.

For constant \(M, C, K\), the coefficient matrix should be decomposed only once after having taken the time step \(\Delta t\), while the right-side vector is time-dependent. We have seen that in the tasks analysed, at least one matrix was time-dependent, thus, the coefficient matrix will be time-dependent, and the time-consuming decomposition should be carried out at every time step. Inbanathan and Wieland [1] (who tested the model for a simply supported structure and one single mass point) took this way. If we want prevent time-dependence of matrix \(A\), time-dependent components in equation (5) should be taken to the right-hand side of the equation. Further on, neglecting subscript \(C\) in the constant element matrices, we can write the dynamical task in the following form

\[ M\ddot{u} + C\dot{u} + Ku = \ddot{r}, \]

where

\[ \ddot{r} = r - M_1\ddot{u} - C_1\dot{u} - K_1u. \]

In this case, displacements can be obtained from the relation

\[
\begin{align*}
\left( K + \frac{6}{(\theta\Delta t)^2} M + \frac{3}{\theta\Delta t} C \right) u_{t+\theta\Delta t} &= \ddot{r}_{t+\theta\Delta t} + M \left( \frac{6}{(\theta\Delta t)^2} u_t + \frac{6}{\theta\Delta t} \dot{u}_t + 2 \ddot{u}_t \right) + \\
&\quad + C \left( \frac{3}{\theta\Delta t} u_t + 2 \dot{u}_t + \frac{\theta\Delta t}{2} \ddot{u}_t \right), \\
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\tag{8}
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where

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In this case, displacements can be obtained from the relation

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\begin{align*}
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&\quad + C \left( \frac{3}{\theta\Delta t} u_t + 2 \dot{u}_t + \frac{\theta\Delta t}{2} \ddot{u}_t \right), \\
\end{align*}
\tag{8}
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Inbanathan and Wieland [1] and Olsson [5] successfully applied the Newmark procedure for computing displacements in simpler tasks, while Györgyi [6] showed that it was expedient to use the Wilson-\(\theta\) procedure for solution. Wilson [3] assumes linear acceleration change between moments \(t\) and \(t + \theta \Delta t\). Assuming linear change of \(r(t)\) for the given time period, we get

\[ r_{t+\theta \Delta t} = r_t + \theta \left( r_{t+\Delta t} - r_t \right) \]

and at the moment \(t + \theta \Delta t\) displacements can be obtained from the following relationship:

\[
\begin{align*}
\left( K + \frac{6}{(\theta \Delta t)^2} M + \frac{3}{\theta \Delta t} C \right) u_{t+\theta \Delta t} = & \ r_{t+\theta \Delta t} + M \left( \frac{6}{(\theta \Delta t)^2} u_t + \frac{6}{\theta \Delta t} \dot{u}_t + 2 \ddot{u}_t \right) + \\
& + C \left( \frac{3}{\theta \Delta t} u_t + 2 \dot{u}_t + \frac{\theta \Delta t}{2} \ddot{u}_t \right).
\end{align*}
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Afterwards, displacements, velocities and accelerations belonging to the moment \(t + \Delta t\) can be calculated.

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\[ M \ddot{u} + C \dot{u} + Ku = \ddot{r}, \]

where

\[ \ddot{r} = r - M_{11} \ddot{u} - C_1 \dot{u} - K_1 u. \]

In this case, displacements can be obtained from the relation

\[
\begin{align*}
\left( K + \frac{6}{(\theta \Delta t)^2} M + \frac{3}{\theta \Delta t} C \right) u_{t+\theta \Delta t} = & \ r_{t+\theta \Delta t} + M \left( \frac{6}{(\theta \Delta t)^2} u_t + \frac{6}{\theta \Delta t} \ddot{u}_t + 2 \dddot{u}_t \right) + \\
& + C \left( \frac{3}{\theta \Delta t} u_t + 2 \dddot{u}_t + \frac{\theta \Delta t}{2} \ddot{u}_t \right),
\end{align*}
\]
where
\[ \ddot{r}_{t+\theta\Delta t} = r_{t+\theta\Delta t} - M_{l+\theta\Delta t} \left( \frac{6}{(\theta\Delta t)^2} (u_{t+\theta\Delta t} - u_t) - \frac{6}{\theta\Delta t} u_t - 2\ddot{u}_t \right) - C_{l+\theta\Delta t} \left( \frac{3}{\theta\Delta t} (u_{t+\theta\Delta t} - u_t) - 2\ddot{u}_t - \frac{\theta\Delta t}{2} \dddot{u}_t \right) - K_{l+\theta\Delta t} u_{t+\theta\Delta t} . \] (9)

It can be seen that on the right-hand side of the equation, vector \( u_{t+\theta\Delta t} \) appears and its computation calls for an iteration procedure. Györgyi [6] experienced that convergence of the method could only be assured if the supplementary mass was not too large as compared to the structure's mass.

4 Use of the modal analysis

Györgyi [5] showed that the differential equation of motion was proportional to internal damping (when the damping constant of all the structural elements is the same):

\[ \ddot{v} + \omega_u \dot{v} + \omega_u^2 v = \frac{4\gamma}{4 + \gamma^2}, \quad u = \frac{4 - \gamma^2}{4 + \gamma^2}, \quad \omega_u = \frac{\omega_r}{\sqrt{1 + \frac{\gamma^2}{4}}} , \quad \gamma = \frac{\vartheta}{\pi} . \] (10)

Here, \( \omega_r \) is the logarithmic decrement of damping, \( \omega_r \) can be computed from the \( r \)-th eigenvalue of the eigenvalue problem \( K v = \omega^2 M v \) belonging to the undamped case, while \( V \) is the matrix containing \( M \)-normed eigenvectors. Apparently, in the case of internal damping the direct integration must be preceded by the solution of an eigenvalue problem.

Out of the tasks analysed, the one in 2.2 is the most general. Let us rewrite the relationship (5) in the following form:

\[ M_C \ddot{u} + K_C u = -C_C \dot{u} + r_1(t) - M_1(t) \ddot{u} - C_1(t) \dot{u} - K_1(t) u \] (11)

The matrices on the right-hand side are hyperdiagonals. Accordingly, also the eigenvalue problem \( K_C v = \omega^2 M_C v \) splits in two parts. If we generate some smallest eigenvectors both for the structure and the vehicle (\( m_b \) for the structure and \( m_v \) for the vehicle).

Collect the eigenvectors in matrix \( V \), also matrix \( V \) becomes a hyperdiagonal (its blocks will be quadratic only in that case when all the eigenvectors both for the structure and the vehicle were computed).
If we seek the solution again in the form of \( u = Vx \), the task becomes again diagnosable. Nothing will hamper the consideration of the proportional internal damping - different for the bridge structure and the vehicle.

\[
V = \begin{bmatrix}
V_B & 0 \\
0 & V_V \\
\end{bmatrix}
\]

As the individual blocks of the stiffness and mass matrices will be only multiplied by the \( V \) matrix blocks belonging to them due to the structure of \( K_C \) and \( M_C \) and \( V \), the internal friction may be different for the blocks. The relationship for calculation of vector \( x \) is the following:

\[
\begin{align*}
\dot{\mathbf{x}} + \frac{6}{(\Delta t)^2} \mathbf{B}_{\dot{t}+\Delta t} + \frac{3}{\Delta t} \left( \mathbf{H}_C + \mathbf{H}_t + \Delta t \right) + \dot{\mathbf{P}}_{t+\Delta t} \mathbf{x}_{t+\Delta t} = \\
= \dot{f}_{t+\Delta t} + (\mathbf{B}_{t+\Delta t} + \mathbf{E}) \left( \frac{6}{(\Delta t)^2} \mathbf{x}_t + \frac{6}{\Delta t} \dot{\mathbf{x}}_t + 2\ddot{\mathbf{x}}_t \right) \\
+ \left( \mathbf{G} + \mathbf{H}_C + \mathbf{H}_t + \Delta t \right) \left( \frac{3}{\Delta t} \mathbf{x}_t + 2\dot{\mathbf{x}}_t + \frac{\Delta t}{2} \ddot{\mathbf{x}}_t \right).
\end{align*}
\]

(12)

Here vector \( x \) has the size of \( m_B + m_V \) and \( \mathbf{E} \) is a unit matrix. Matrices \( \mathbf{D} \) and \( \mathbf{G} \) are hyperdiagonal matrices, in their first element \( m_B \) with the characteristics of the structure, then those of the vehicle:

\[
\begin{align*}
\mathbf{D} &= \begin{bmatrix}
\omega^2_T & \cdots & \omega^2_T \\
\omega^2_S & \cdots & \omega^2_S \\
\gamma V \omega_{nu} & \cdots & \gamma V \omega_{nu} \\
\gamma V \omega_{su} & \cdots & \gamma V \omega_{su}
\end{bmatrix}, \\
\mathbf{G} &= \begin{bmatrix}
\gamma V \omega_{nu} & \cdots & \gamma V \omega_{nu} \\
\gamma V \omega_{su} & \cdots & \gamma V \omega_{su}
\end{bmatrix}.
\end{align*}
\]

Effect of masses in the supports of the vehicle is contained in \( \mathbf{B}(t) \). It is clear that only block \( i \) generated by the eigenvectors of the structure of the size \( m_B \times m_B \) is not zero.

\[
\hat{\mathbf{B}} = \begin{bmatrix}
& & & & i \\
& V_B^T M(t) V_B & 0 & i \\
& & & & j \\
& 0 & 0 & j
\end{bmatrix}
\]
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The damping system of the vehicle proportional to the velocity appears, on the one hand, in matrix \( \hat{H}_C \) containing constant elements, on the other hand, in matrix \( \hat{H}(t) \) expressing effect of dampers joined to vehicle supports. Only block \( jj \) of matrix \( \hat{H}_C \) differs from zero, and this block can be computed from the following relationship:

\[
\hat{H}_{C_{jj}} = V V^T C_C V.
\]

The block \( jj \) of matrix \( \hat{H}(t) \) is zero, but this can be computed from the relationship. Matrix \( \hat{P}(t) \) expressing the effect of the vehicle springs joined to vehicle supports is similar to matrix \( \hat{H}(t) \):

\[
\hat{P}(t) = V^T K_1(t) V.
\]

Also vector \( \hat{f}(t) \) can be partitioned in two parts. The \( i \)-th block is time-dependent, while \( j \)-th block will be zero:

\[
\hat{f}(t)_i = V_B^T r_i(t).
\]

The road vehicle demonstrated in 2.1 will be simpler than this general model highlighted. In that case, matrix \( \hat{B}(t) \) will be zero.

5 Numerical experience

5.1 Data of the structure and vehicle tested

During numerical tests, computations were carried out on a reality-like structure. Data of the bridge spanning 30 m are shown in Fig. 3. The internal damping factor \( \gamma \) was 0.1.

![Figure 3: Data of the structure tested](image)

On the bridge, a railway vehicle was moving. Vehicle velocities were taken between 0 and 50 m/s in the different cases. Its internal friction factor was chosen 0.05, its stiffness, mass and damping data from Fig. 2 are:

\[
\begin{align*}
& m_1 = 2.5 \text{ t}, & m_2 = 10 \text{ t}, & m_3 = 50 \text{ t}, & \theta_2 = 8 \text{ tm}^2, & \theta_3 = 1500 \text{ tm}^2, \\
& k_1 = 3000 \text{ kN/m}, & k_2 = 2100 \text{ kN/m}, & c_1 = 20 \text{ kNs/m}, & c_2 = 150 \text{ kNs/m}.
\end{align*}
\]
5.2 Eigenvectors of the structure needed for the modal analysis

The dynamical task was solved by modal analysis. For determination of the number of eigenvectors needed for properly correct solution, a numerical experiment was performed. Displacement of the central point of the structure was examined reckoning with increasing numbers of eigenvectors. Table 1 shows the percentages of excess displacements generated by the dynamical effect at different velocities. The data above reveal that for the structure consideration of already five eigenvectors provides the additional dynamical displacements with sufficient accuracy, nevertheless, we calculated with 7 structural eigenvectors.

Table 1. Additional displacement due to dynamical effect

<table>
<thead>
<tr>
<th>number of eigenvectors</th>
<th>10</th>
<th>20</th>
<th>35</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>velocities [m/s]</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>2.1</td>
<td>2.3</td>
<td>5.5</td>
<td>20.2</td>
</tr>
<tr>
<td>20</td>
<td>1.0</td>
<td>1.3</td>
<td>4.6</td>
<td>18.8</td>
</tr>
<tr>
<td>35</td>
<td>0.8</td>
<td>1.2</td>
<td>4.4</td>
<td>18.7</td>
</tr>
<tr>
<td>50</td>
<td>0.9</td>
<td>1.2</td>
<td>4.4</td>
<td>18.7</td>
</tr>
</tbody>
</table>

As the vehicle model had six degrees of freedom, in this case we had to solve a system of equations with 13 unknowns in the individual steps, what did not require considerable computation time.

5.3 Influence of damping on additional dynamical displacements

During tests, it was analysed how the change of the vehicle's damping system influences the dynamical excess displacements. The rows of Table 2 contains the additional displacements computed with the initial damping value in the percentage of the static displacement in the different damping case.

Table 2. Effect of the dampers of the moving vehicle and the structure's internal friction

<table>
<thead>
<tr>
<th>velocity of vehicle</th>
<th>damping</th>
<th>$v=20$ m/s</th>
<th>$v=50$ m/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma_B$, $\gamma_v$, c</td>
<td>1.2</td>
<td>18.7</td>
<td></td>
</tr>
<tr>
<td>$\gamma_B$, $\gamma_v$, 0.1c</td>
<td>0.6</td>
<td>16.2</td>
<td></td>
</tr>
<tr>
<td>$\gamma_B$, $\gamma_v$, 10c</td>
<td>1.3</td>
<td>19.2</td>
<td></td>
</tr>
<tr>
<td>$\gamma_B = 0$, $\gamma_v$, c</td>
<td>2.4</td>
<td>24.7</td>
<td></td>
</tr>
</tbody>
</table>
Values of the table relate to two velocities. It can be seen that the damping effect is much stronger with a higher vehicle velocity than at an average speed. Tests revealed that the internal friction of the vehicle in cases tested did not influence the movements of the structure.

6 Summary

The paper discusses the test of bridges under moving vehicle applying the finite element method, and shows a procedure able to compute the influence of the external damping proportional to velocity and, at the same time, is fit for considering structural damping, too. Application of modal analysis essentially reduced the computation time. The numerical results cited proved the efficiency of the algorithm for joint test of moving vehicle and the structure as well as the importance of taking the influence of the internal friction of the bridge structure into consideration. As per the numerical results, the possible most correct analysis of the interaction of the vehicle's damping structure and the bridge structure is necessary mainly in the case of high-speed vehicles.

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Keywords

effect of moving mass, structural damping, direct integration, finite elements

References