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Modelling and computer analysing the stick-slip motion in a mass-spring system with friction

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Abstract

The paper presents a physical and mathematical model of mass-spring system with friction that enables to clarify and to analyse the stick-slip motion. The essential novelty in this model is inclusion of tangent contact flexibility and damping, also of optional characteristics on static, kinematic and dynamic friction. The characteristics take into consideration the increase of the static friction force (coefficient) with increasing time of repose (stick mode duration), also the relationship of the motional friction force from the sliding velocity and acceleration for the slip mode. The model has been developed basing upon the results obtained from autors experimental study. Presented are computer simulation program and examples. The result obtained match those obtained from the experimental study.

1 Introduction

Stick-slip vibration are self-excited oscillations induced by dry or mixed friction. They occur in numerous tribomechnical systems, both at high values of the friction factor (brakes, clutches) and low values of the factor (bearings, slideways), particularly often when starting and positioning various machinery units also where the relative velocity of two sliding surfaces is low.

The stick-slip phenomenon frequently occurs, as an undesirable and harmful effect, in various tribomechanical systems. It constitutes a real (current) and important (from both scientific and practical points of view) nonlinear problem of the contact dynamics.

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Thus far there is no unified, generally accepted, theory for this phenomenon. The existing theories are strongly simplified. These simplifications concern, above all, and from the necessity to avoid the difficulties of mathematical nature the model of friction and contact processes that proceed in a movable frictional joint. The advancement in the knowledge of physical phenomena occuring in a frictional contact and the development in numerical methods of solving complex nonlinear differential equations have enabled the scientists to assume more complex models, both physical and mathematical, and to investigate this phenomenon using the computer simulation method.

2 Conventional models and stick-slip motion theories

For clarifying the machanisms of occurrence also of mathematically defining stick-slip vibration adopted are generally in literature (Derjagin, Push, Tolstoi [2], Oden, Martins [5]) tribomechnical models with single degree of freedom as in Fig. 1. The models are equivalent and represent in some degree of simplification, wide class of dissimilar mechanical systems with friction. For instance the model as in Fig. 1b is adopted universally in research on stability of machine-tools feed drive units.

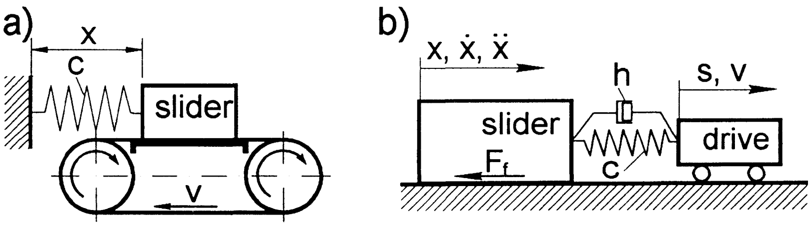

Figure 1: Tribomechanical models of two (equivalent) sliding systems which may have stick-slip oscillations.

As a main cause of stick-slip vibration (Fig. 2) regarded is (Kragielski, Gitis[1], Vejc [6]) a positive difference of value of static and kinetic friction force and/or decreasing value of sliding friction force while slipping velocity is going up. The vibration may have various course, various amplitude and frequency. It depends upon characteristics of (static and motional) friction, slipping velocity also upon parameter values of dynamic system (mass m, stiffness c, damping h, Figs 1a,b).

Characteristics about stick-slip motion is that it is feasible to distinguish the two successive modes viz stick mode and slip mode. Duration and courses of the modes depend largely upon adopted characteristics of static and kinetic friction. In the stick-slip vibration theories prevailing up to date adopted are most frequently friction characteristics as in Fig. 3 (Kragielski, Gitis [1]).

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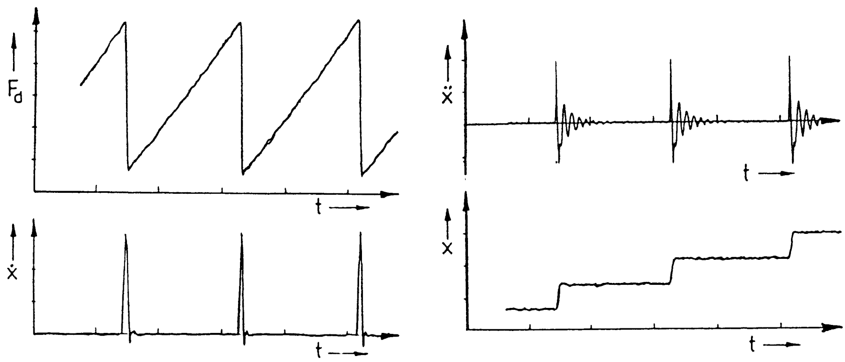

Figure 2: Examples of courses of drive force F_d, also of acceleration $\ddot{x}$, velocity $\dot{x}$, displacement x during stick-slip motion.

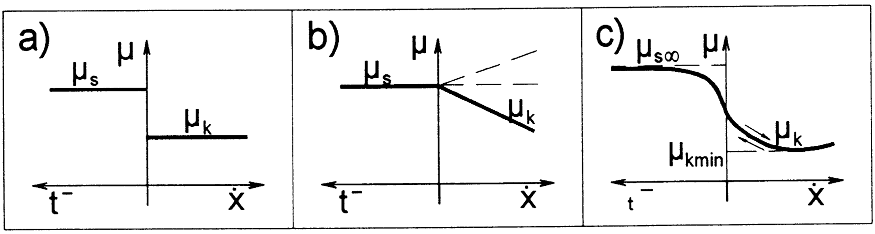

Figure 3: Conventional characteristics of friction adopted in up-to-date stick-slip theories.

Determined for the said friction characteristics courses and parameters of stick-slip vibrations do differ essentially among themselves also from the results of the experimental study (Grudzinski, Warda [4]). The experimental study in particular does not confirm assumptions adopted universally so far, concerning:

- strokewise variation of friction force when passing from slip mode to stick mode and vice versa (Figs 4a,b); friction force variaes more or less violently but not strokewise (Fig. 4c),
- univical relationship of motional friction force versus sliding velocity determined by kinetic friction characteristics (Figs 4g,h); in slip mode the friction force depends not only upon velocity but also upon acceleration; it describes some loop (Fig. 4i),
- occurrence of absolute deadlock of slider in stick mode (Figs 4d,e); due to contact flexiblility of mating surfaces occur in stick mode so called initial displacements which correspond to stick balance condition, also transient processes while slider goes through slip mode to stick mode (Fig. 4f).

The above facts stated on the basis of the experimental study, were instrumental in modyfing the model of the tribomechanical system under consideration.

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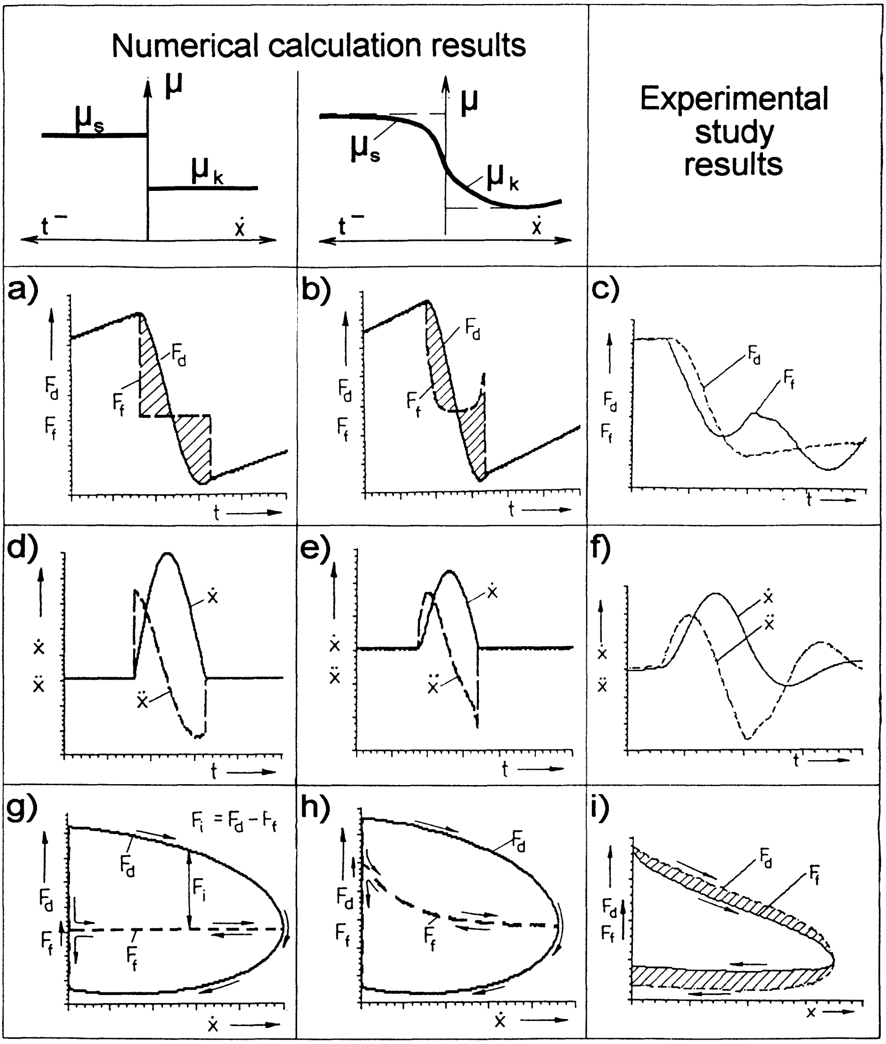

Figure 4: Numerical calculation results for the two adopted friction characteristics compared with experimental study results.

3 Modification of physical and mathematical model

Divergent result of simulation study for adopted friction models also results obtained from laboratory research formed the basis for developing a modyfied physical and mathematical model of machine-tool feed-drive unit. Its simplified diagram is shown in Fig. 5.

The modification involved:

– incorporation into the model and into algorithm of tangent contact stifness also contact damping of the sliding connection,

– incorporation into calculation procedures of algoritm which take into consideration continuous variation of friction force value while slider goes through stick mode to slip mode in lieu of strokewise variation of friction force employed up to now. Additionally assumed was that friction force depends not only upon sliding velocity $\dot{x}$ but also upon acceleration $\ddot{x}$ of slider. It has a different plot for acceleration and deceleration in slip mode.

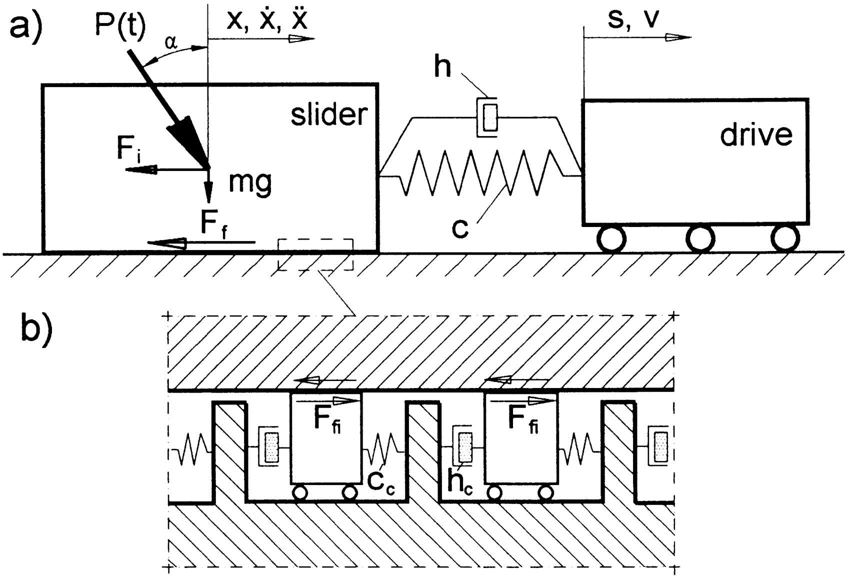

Figure 5: Modified physical model of the tribomechanical system.

To the algorithm and calculation program attached were modules that permit to implement any course of kinematic constraint velocity of drive also to implement periodically variable force external constrains acting upon slider, and kinematic constraints acting upon drive.

The equation for the modified model (Fig. 5) is like this

$$m\cdot\ddot{x}-h(v-\dot{x})-c(s_0+vt-x)+(\kappa-1)F_f(t^-,\dot{x},\ddot{x})+\kappa F_c(c_c,h_c)=0 \quad (1)$$

Individual members of the equation defined as below:

$m\cdot\ddot{x}$ - inertia force,

$h\cdot(v-\dot{x})$ - damping force,

$c\cdot(s_0+vt-x)$ - elasticity force,

$F_f(\dot{x},\ddot{x},t^-)$ - friction force dependent in general upon rest time t^-, sliding velocity $\dot{x}$ and sliding acceleration $\ddot{x}$.

The last member defines contact tangential force in sliding connection

$$F_c=F_{ce}+F_{cd} \quad (2)$$

where: $F_{ce}=c_c\cdot\delta_c$ - contact tangential elasticity force,

$F_{cd}=h_c\cdot\dot{x}$ - contact damping force,

δ_c - elastic contact tangential deformation,
c_c - contact tangential stiffness factor,
h_c - contact damping factor in tangential direction.

Parameter κ introduced in the differential equation (1) utilises the values as below

$\kappa = 1$ - for slip mode,
$\kappa = 0$ - for stick mode.

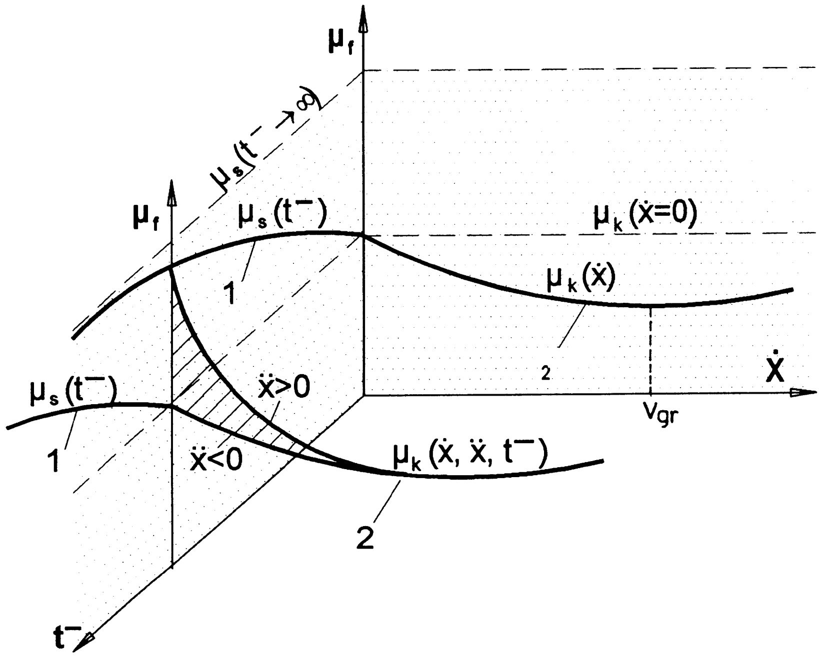

Figure 6: Dynamic and static friction characteristics assumed in the modified model

Introduction of the member defining contact tangential force to computations, takes place when slider changes from slip mode over to stick mode. In this very moment (i.e. when $\dot{x} = 0$), due to elastic contact deformation, transient process begins during which the slider vibrates slightly around momentary position of rest balance. The vibration fades away completely if stick mode is sufficiently long. The equation (1) is solved by use of suitable SYMTAR program (Grudzinski, Zaplata [3]).

4 Sample computation results

Numerical computations were made with contact stiffness excluded and then included. Adopted were the data as follows:

$m = 350kg$ $c = 0,6kN/mm$ $h = 4Ns/mm$ $\mu_s(t^- \to \infty) = 0.24$

$\mu_s(t^- \to 0) = 0.12$ $\mu_k(\dot{x} \to 0) = 0.12$ $v_{gr} = 35$ $\mu_k(v_{gr}) = 0.05$

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The outcome of the computations are given in Fig. 7 and Fig. 8.

Figs 7a,d show values of drive force F_d and of friction force F_f as well as sliding velocity $\dot{x}$ and acceleration $\ddot{x}$ for several first stick-slip cycles upon prolonged slider rest. Figs 7b,e are fragments of Figs 7a,d in magnified scale of time. They illustrate plots of F_d, F_f, $\dot{x}$ and $\ddot{x}$ during slip mode of stick-slip motion. Fig. 7f portrays stick-slip motion under consideration, on the so called mode plane. Plots of the said motion tend to the limit cycle this indicating stable character of the vibration. Fig. 7c presents plots of drive force F_d versus sliding velocity, during slip mode, for several first stick-slip cycles. Here the friction force is not a univocal function of sliding velocity $\dot{x}$. It has different values during accelerated ($\ddot{x} > 0$) and decelerated ($\ddot{x} < 0$) motion travel which is compatible with the experiment.

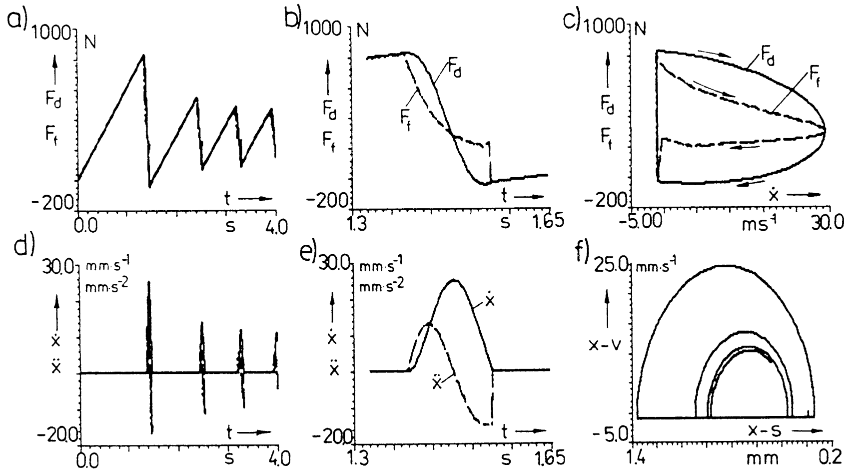

Figure 7: Stick-slip characteristics for the modified model of the tribomechanical system excluding the contact stiffness

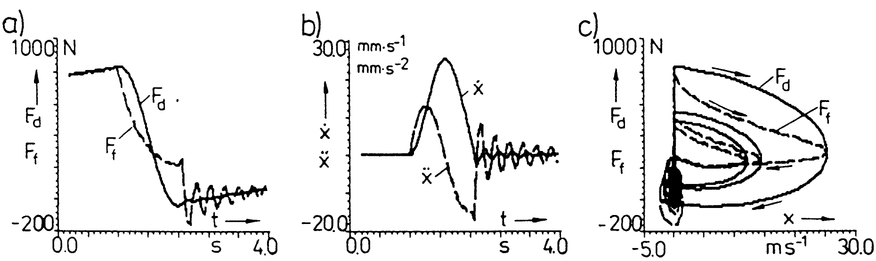

Figure 8: Stick-slip characteristics for the modified model of the tribomechanical system including the contact stiffness

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Fig. 8 gives the result of the numerical computations for the same data as before, including contact stiffness and damping: $c_c = 5kN/mm$ $h_c = 15Ns/mm$. The values of the parameters were obtained experimentally. Inclusion of contact stiffness and damping explains the arising of low vanishing oscillations while slider passes from slip mode to stick mode. The said oscillations constitutes solely certain transient process and do not alter essentially the character and parameter of stick-slip motion.

5 Resume

The developed tribomechanical model (Fig. 5) and equation (1) also dynamic friction characteristics (Fig. 6) gave in effect computation outcome more compatible with experimental results than the models and theories of stick-slip found in the literature so far. The result of the numerical computations obtained by use of SYMTAR, for given tribomechanical system, permit to define the extent of frictional self-excited vibration also permit to describe the vibration with precision.

Accuracy of simulation research depends upon the value of the magnitudes adopted in computation the magnitudes describing mass/spring qualityies also tribological attributes of the object under research.

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