Transactions on Engineering Sciences vol 7, © 1995 WIT Press, www.witpress.com, ISSN 1743-3533

Design of three-screw positive displacement rotary pumps

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Abstract

Positive displacement rotary pumps, such as internal and external gear pumps, usually present an variable instantaneous flow rate. This characteristic produces undesirable noise and vibrations. On the other hand, three-screw pumps are usually considered as having a constant instantaneous flow rate, even if they work in a different way, whereas the working fluid motion is mainly axial. The three-screw pump operations will be analysed in this paper, starting from the study of its kinematics and then describing the geometry of the screw profile and its generation. The instantaneous flow rate generation will be explained and a simple phenomenological interpretation will be given. Then a screw profile optimisation will be proposed in order to improve the volumetric efficiency, providing the analytical tools to find the characteristic parameter values that allow us to obtain the optimisation.

Nomenclature

- h - worm depth;
- n - rotating speed;
- k - ratio between the worm depth h and pitch radius r;
- p - axial pitch;
- r - pitch radius;
- α_c - epicycloid parameter;
- α_{ct} - α_c value on the tip circle;
- α_t - epitrochoid parameter;
- α_{tt} - α_t value on the tip circle;
- β_c - angle subtended by the epicycloid;

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r_e - tip radius of the worm screw;
r_i - root radius of the idler rotors;
v - velocity;
Q_i - instantaneous flow rate;
S - reflux section;
T - period;
V - volume;
β_t - angle subtended by the epitrochoid;
γ - transverse worm thickness at the root;
σ - fluid axial section;
ω - angular speed.

1 Introduction

Screw pumps usually present a highly smooth running along with a high flow rate regularity. Literature describes various kinds of screw pumps, among which three families can be considered:

- single screw pumps; these pumps are a modern edition of Archimede's pump (Olivari [8]);
- two-screw pumps; the two screws have the same profile (Burenin & Gaevik [5]);
- Three-screw pumps.

In the present paper we will consider only the last kind of pump. Figure 1 shows a three-screw pump with its case.

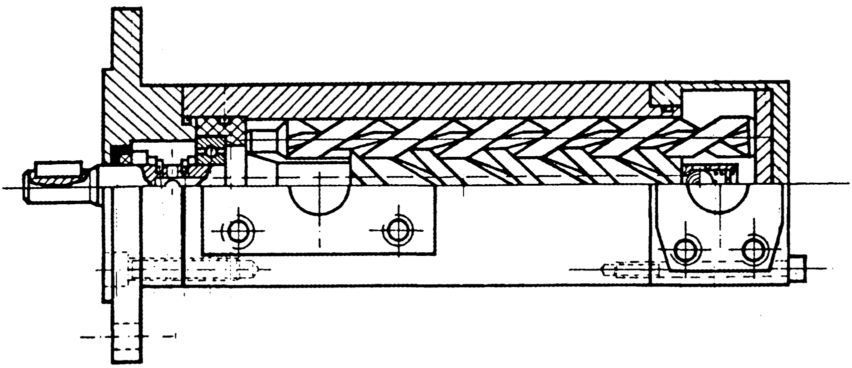

Figure 1: Three-screw pump section (courtesy of SEIM s.p.a.).

The central *power rotor* is a screw with two worms, and presents a diameter greater than that of the two lateral *idler rotors*. The root circle of the *power rotor* is equal to the tip diameter of the two *idler rotors*.

Suction and discharge are usually in the radial direction as shown in figure 1 but in some cases they are used in the axial direction.

Usually the length of the *power rotor* is equal to two pitches. It is longer on the two lateral idler rotors due to the necessity of priming the fluid at the suction. Due to the geometry of the rotors, they are subject to an axial component which acts on the bearings. For this reason two different solutions are adopted, the first one uses a pressure balancing through the rotors (see figure 1) that achieves a hydrostatic balance, the second uses worms with two opposite helicoidal directions.

The main advantages to using screw pumps are (Bazzocchi [2], Brennan [4]):

- construction simplicity and working reliability;
- quietness at high speeds due to the particular profile which pumps the fluid mainly axially without creating turbulent motion and radial loads on the rotors;
- continuous and uniform flux;
- auto priming for elevated heights.

The fluids treated by this kind of pump can also have a high viscosity but they must not have particles that might produce an abrasive action. Some common applications for these pumps are: the petrochemical industry with high viscosity oils, liquid-gaseous mixtures, syrups and molasses, oleodynamic devices for elevators, power units, etc.

2 Rotor geometry

Before proceeding with our description of the instantaneous flow-rate generation we have to introduce some considerations about the geometry of the rotors. Besides it is necessary to identify the parameters that allow a complete characterisation of the screws themselves. The shape of the rotors can be easily studied on a section perpendicular to the rotor axis, where the worms are complete (see figure 2).

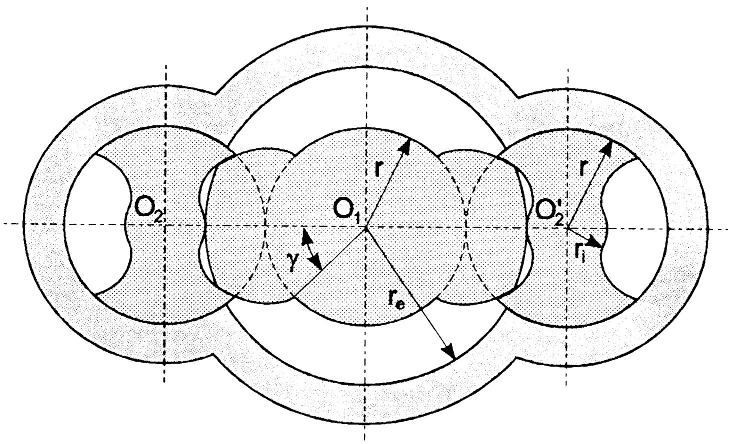

Figure 2: Normal section of a three-screw pump.

On the section of the central rotor we may find the two worms with their root r and tip r_e radiuses. On the two idler rotors, on the contrary, we observe the two vanes between the tip circle of radius r and an internal web with radius $r_i = 2\,r - r_e$

The pitch circles of the motion are the three circumferences which all have the same radius r. In order to trace the rotor profiles we may consider the relative motion of the rotors. First we will consider the motion of one of the two idler rotors in respect to the power rotor, and then the inverse motion.

2.1 Central screw tracing

Referring to figure 3 the point P, rigidly joined to one of the idler rotors -the extremity of the vane profile - during its relative motion, traces the flank of the central rotor tooth profile (PC arc). By choosing a reference system similar to that of figure 3, the parametric equation of the curve traced, as a function of the rotation angle α_c of the idler rotor, can be obtained by simple trigonometric considerations and results as:

$$\begin{cases} X = 2r\cos\alpha_c - r\cos 2\alpha_c \\ Y = 2r\sin\alpha_c - r\sin 2\alpha_c \end{cases} \tag{1}$$

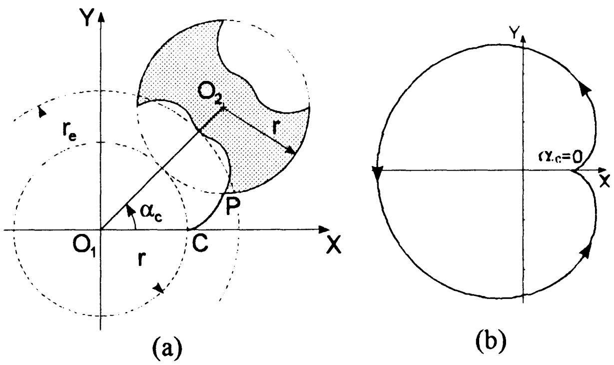

Figure 3: (a) Central rotor flank tracing. (b) Epicycloid.

The system represents an epicycloid whose cusp is on the positive semi-axis (see figure 3 b). It is useful identifying the value α_{ct} of the parameter that corresponds to the intersection with the tip radius r_e for tracing the epicycloid arc on the flank of the worm:

$$X^2 + Y^2 = r_e^2$$

$$\left(2r\cos\alpha_c - r\cos 2\alpha_c\right)^2 + \left(2r\sin\alpha_c - r\sin 2\alpha_c\right)^2 = r_e^2$$

$$\cos\alpha_c = \frac{5r^2 - r_e^2}{4r^2} \tag{2}$$

$$\alpha_{ct} = \arccos\frac{5r^2 - r_e^2}{4r^2}$$

2.2 Idler rotor tracing

In order to trace the idler rotor, we will consider the point P of figure 4(a) that represents the edge of the worm, with height h, of the central screw. The following positions of point P trace the profile of the vane flank of the idler rotor (see arc PC in figure 4 (a)).

The curve parametric equation, which refers to the co-ordinate axes shown in figure 4 (a), is an epitrochoid, and can be obtained by simple trigonometric considerations.

$$\begin{cases} X = 2r\cos\alpha_t - r_e\cos 2\alpha_t \\ Y = 2r\sin\alpha_t - r_e\sin 2\alpha_t \end{cases} \tag{3}$$

The double point of this epitrochoid is on the positive semi-axis. The value α_{tt} of the parameter in correspondence with the intersection with the pitch radius, which is useful for tracing the arc of the epitrochoid on the flank of the worm is:

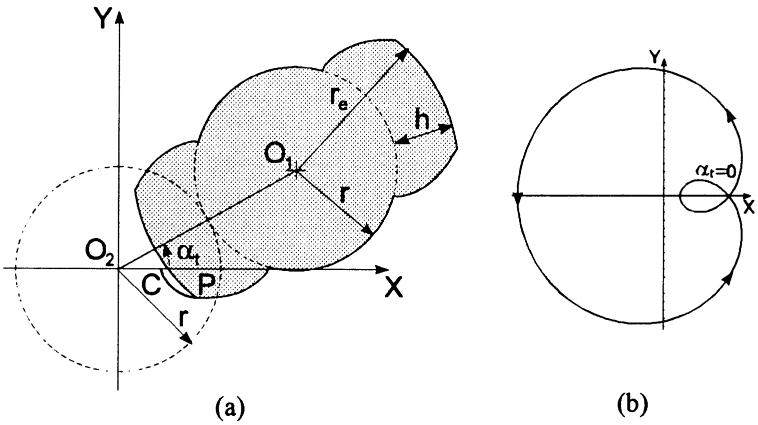

Figure 4: (a) Idler rotor vane tracing. (b) Epitrochoid.

$$X^2 + Y^2 = r^2$$

$$(2r\cos\alpha_t - r_e\cos 2\alpha_t)^2 + (2r\sin\alpha_t - r_e\sin 2\alpha_t)^2 = r^2$$

$$\cos\alpha_t = \frac{3r^2 + r_e^2}{4rr_e} \tag{4}$$

$$\alpha_{tt} = \arccos\frac{3r^2 + r_e^2}{4rr_e}$$

The α_{tt} value is useful for tracing the arc of epitrochoid on the flank of the worm. Finally, in order to trace the flanks of the profiles, we have to determine the angle subtended under the epicycloidal arcs of the central worm and of the epitrochoid of the two lateral idler rotors toward the centre of the relative rotor.

For the central screw, using the α_{ct} value given by eqn (2) we have:

$$\beta_c = \arctan \frac{2r \sin\alpha_{ct} - r \sin 2\alpha_{ct}}{2r \cos\alpha_{ct} - r \cos 2\alpha_{ct}} \tag{5}$$

Similarly considering the α_{tt} value given by eqn (4):

$$\beta_t = \arctan \frac{2r \sin\alpha_{tt} - r_e \sin 2\alpha_{tt}}{2r \cos\alpha_{tt} - r_e \cos 2\alpha_{tt}} \tag{6}$$

It is possible to demonstrate that eqns (5) and (6) are equal in module.

2.3 Design parameters

Let us now introduce some adimensional parameters, function of r. The first is the coefficient k, which represents the ratio between the height h of the driving screw worm (corresponding to the radial dimension of the vane of the idler rotors) and the pitch radius r. So, for the external radius, we obtain $r_e = (1+k)\, r$. The parameter k ranging is from 0 to 1 even if the values next to the extremes are not of practical interest.

The second parameter is the semi-amplitude γ of the worm on the root (see figure 2). This angle determines its circumferential amplitude; as far as that parameter is concerned, we must observe that it usually assumes the value 45°.

3 Instantaneous flow rate generation

In order to calculate the instantaneous flow rate generation, first of all it is necessary to understand the mechanism that produces it. Considering an axial cross section, the conjugated theoretical profiles of the rotors present five contact points as shown in figure 5.

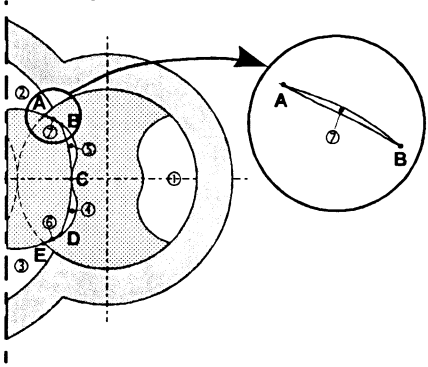

Figure 5: Contact points between the rotors on a cross section.

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The contact points A, B, C, D and E represent the ideal sealing points between the screws on the section plane. The zones occupied by the fluid, indicated by the numbers ① ÷ ⑦, turn out to be separated on the various sections both by these seals and by the seals existing through the rotors and the case. The sections ① ÷ ⑦ have helicoidal development that produces closed volumes with a length of one step.

This has been demonstrated numerically by determining how the cross section area of each of those closed volumes changes during the rotation and in particular observing how it reaches zero in correspondence with each axial step.

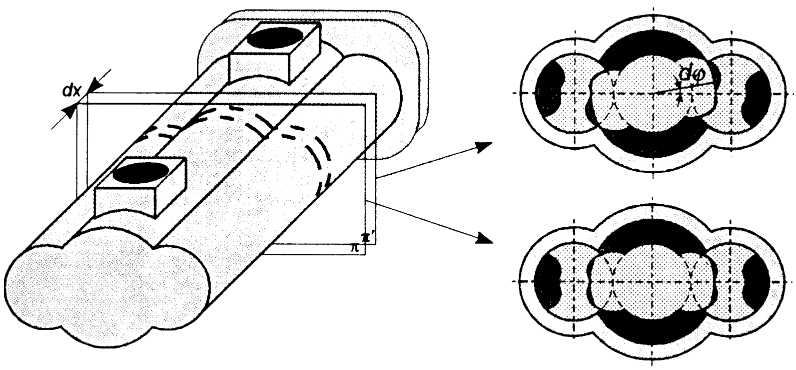

Figure 6: Subsequent angular positions of the rotors in cross section.

So, to avoid the reflux it is necessary to use the worm length of at least one axial step. The screws are commonly made with a length of two steps.

The instantaneous flow rate will be given by the following formula:

$$Q_i = \frac{dV}{dt} = \sigma(k,\gamma)\frac{dx}{dt} \tag{7}$$

where $\sigma(k, \gamma)$ identifies the surface occupied by the fluid, and dx the elementary axial translation due to the screw rotation.

For a constant angular speed of the rotors, the axial velocity of the fluid is constant and equal to

$$v = \frac{dx}{dt} = \frac{p}{T} = p\frac{\omega}{2\pi} \tag{8}$$

where p is the axial pitch.

Since the cross section $\sigma(k, \gamma)$ remains constant during the rotation of the rotors, the instantaneous flow rate is constant and, for a unitary pitch radius, given by

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$$Q_i = \sigma(k, \gamma) \cdot p \cdot \frac{\omega}{2\pi} \tag{9}$$

The actual volumetric flow rate can be obtained by multiplying the formula (9) by r^2.

The value of $\sigma(k, \gamma)$, is given by the following integral

$$\sigma(k,\gamma) = -4\left(\int_{\Gamma_{AB}} y\,dx + \int_{\Gamma_{BC}} y\,dx + \int_{\Gamma_{CD}} y\,dx + \int_{\Gamma_{DE}} y\,dx + \int_{\Gamma_{EF}} y\,dx + \int_{\Gamma_{FG}} y\,dx + \int_{\Gamma_{GH}} y\,dx + \int_{\Gamma_{HA}} y\,dx\right) \tag{10}$$

The analytical result of eqn (10) is rather complicated and we show here only the diagram as reported in figure 7.

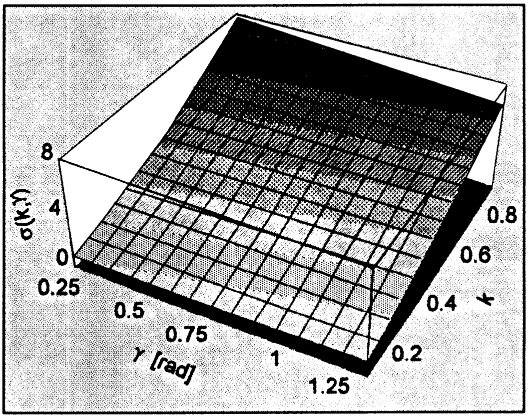

Figure 7: Diagram of the cross section of the area of the fluid $\sigma(k, \gamma)$.

We have found in literature some approximate formulas for the calculations of the instantaneous flow rate.

$$Q_i = \tfrac{3}{4}\pi \cdot p \cdot k(2+k) \cdot n \tag{11}$$

$$Q_i = \pi \cdot p \cdot k(2+k) \cdot n \tag{12}$$

The comparison of eqn (10) with the approximate formulas (11) (Bertani [3]) and (12) (V.V.A.A. [1]) shows that the first of these gives a value lower than the exact one while the second gives a higher value. In particular eqn (11) seems to give a better approximation and may be useful when only an approximate evaluation of the flow rate is needed and calculation with eqn (10) is not suitable.

4 Design optimisation

Actual screw profiles are rather different from the theoretical ones previously shown. Manufacturing errors and the presence of solid particles in the pumped fluid make it necessary to undercut the tips of the rotors and to blunt the edges of the rotor flanks, see also (Burenin & Gaevik[5], Okorokov et al. [7]).

Due to these profile variations the volumes between the screws are not yet closed and the fluid can reflux back. In order to limit this phenomenon, we arrived at a profile shape optimisation, reducing the maximum reflux section.

This minimisation has been made on the area shown in figure 8 whose surface is function only of k and γ.

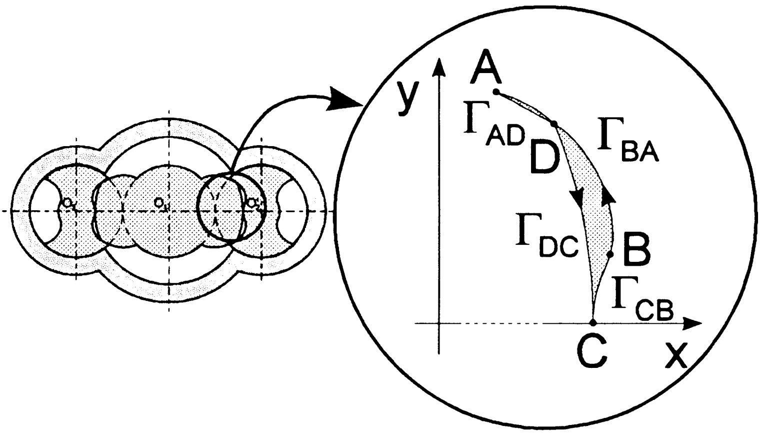

Figure 8: Reflux surface to be minimised.

The expression of the surface S is given by

$$S(k,\gamma) = \int_{\Gamma} x\,dy = \int_{\Gamma_{BA}} x\,dy + \int_{\Gamma_{CB}} x\,dy + \int_{\Gamma_{DC}} x\,dy + \int_{\Gamma_{DA}} x\,dy \tag{13}$$

Figure 9 shows the diagram of the function $S(k, \gamma)$ for k in the range from 0.4 to 0.8 and γ between 30° and 60°.

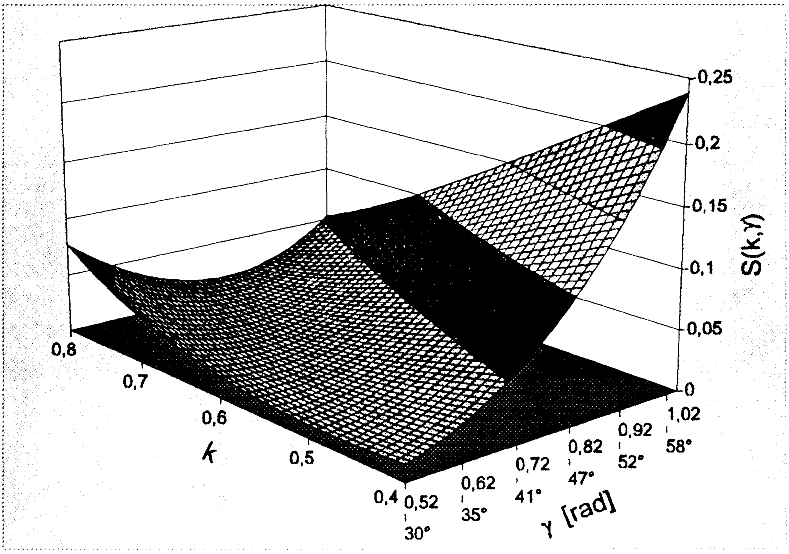

Figure 9: Diagram of the values of $S(k, \gamma)$.

The diagram clearly shows that the function that represents the surfaces has a saddle-backed shape. There is a unique k value that corresponds to a given γ value and minimises the surface. In particular, for an angle γ equal to 45° we find a k value of 0.76, even if the practical values used by manufacturers are about 0.6.

It is also possible to indicate an empirical approximate expression of $k = f(\gamma)$ that minimises $S(k, \gamma)$. The analysis of figure 10 that shows the couples of optimal values of k and γ, permits us to deduce a linear relationship between this parameters; k in particular is nearly equal to the value of γ, given in radians.

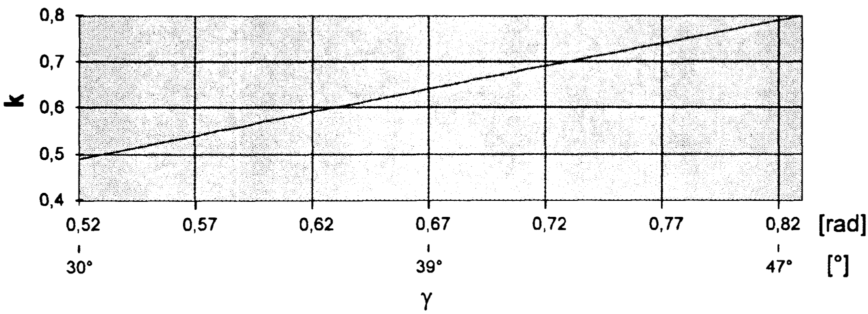

Figure 10: Optimum k-γ values relationship.

Conclusions

The present paper analyses the three-screw pumps from the point of view of profile generation. In particular it takes into account the problem of instantaneous flow rate generation by means of analytical considerations.

The conclusion is that this kind of pump has a theoretical profile that does not allow any reflux. However, if we consider the actual way of building the screw, it is necessary to blunt the edges of the three screws to reduce friction and this may cause clearances that may produce reflux.

In order to minimise this possible refluxes, the paper shows the dependence of this phenomenon by the characteristic parameters and explains how to optimise the profile geometry.

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