Simulation of a small-scale steady state test
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Abstract
This paper describes the use of the finite element method to simulate the Small-Scale Steady State (S4) test for the dynamic crack propagation in a gas pipeline. In this test, a containment cage is placed outside the pipe to limit the deformation. A moving boundary condition model to simulate the pipe wall contact with the containment cage behind the crack tip has been developed. Using the results obtained from these calculations, we can examine the S4 test and determine dynamic fracture resistance of material. This in turn can be used to predict the critical pressure in a full-scale pipe. Results of this type will be of considerable practical importance to the gas industry.

1. Introduction.
Of critical importance in dynamic fracture mechanics is the calculation of the crack driving force, $G$, for a propagating crack and the experimental determination of dynamic fracture resistance of the material, $G_d$. The particular application here is that of a cracked pressurized pipeline. There have been a number of instances where cracks have propagated in a rapid manner for a few kilometers in gas transmission pipelines. Clearly it is important that these catastrophic rupture events must be prevented. This problem is recognised as one of the most important issues of dynamic fracture mechanics. This paper addresses the problem of rapid crack propagation (RCP) in polyethylene (PE) pipe in light of a growing need for fracture control in these pipes.

Since the full-scale test for the PE gas pipe, due to its sheer scale, expense and long turn-round time, is not easy for routine testing or quality control, attention has turned to the design of laboratory tests using short (5-7 diameters) specimens. The Small-Scale Steady State (S4) test has been developed by Leevers and his colleagues [1] in order to assess the fracture characteristics of PE gas distribution pipes during rapid axial crack propagation.

A unique feature of the S4 test is that a series of containment rings around the pipe will limit the extent of the pipe wall deformation. As the
crack propagates, the opening pipe wall behind the crack tip will impact against the rings. Thus, it is important to analyse the ring/pipe interaction as this has a significant effect on the computed crack driving force $G$.

The work described here concerns a computational procedure to simulate the S4 test and includes the development of a model for the contact condition between the cracked pipe and the rigid rings. This computational model permits a better understanding of the S4 test and, more importantly, can be used to determine the fracture resistance of the material, $G_d$. Since $G_d$ cannot be obtained directly from the experiment, numerical simulation is necessary to estimate this quantity. Once $G_d$ is known, simulations of the full scale conditions can be used to predict the critical pressure for crack propagation in pipelines. The analysis capability to simulate the S4 test can be used at the distribution pipeline design stage to prevent rapid fracture. This requires that the crack driving force, $G$ (a function of geometry and loading), is less than the fracture resistance, $G_d$ (a material property).

2. The Small-Scale Steady State test.

In the S4 test, initiation results from impact of a chisel projectile with the pipe and the resulting crack propagates rapidly along the axial direction (see Figure 1). Decompression by backflow of gas is restricted by a series of internal baffles which divide the pipe interior into a set of short chambers. These baffles prevent the axial decompression wave travelling ahead of the crack tip. Thus, the crack tip pressure remains at the original line pressure for the S4 test. The advantage of this is that the steady state crack propagation condition can be achieved in a much shorter distance than in a full scale test.

Flaring of the pipe walls behind the running crack tip is restricted by an external containment cage, consisting of a series of rings having a small clearance of 5 to 15 mm from the outside surface of the pipe. A typical pipe diameter is 250 mm. The cage limits the extent of the pipe deformation, helping to maintain internal pressure by reducing the rate of radial gas escape. Thus it plays an important role in the S4 test and has a significant effect on the crack driving force in the pipe.
3. A computation model to simulate S4 test.

3.1. Existing pipe fracture analysis capability.

It is recognised that the major challenge is to perform a computational analysis of the test and to correctly simulate the process of fluid/structure interaction, which is critical in the calculation of the crack driving force. This is accomplished using a unique finite element code PFRAC (Pipeline Fracture Analysis Code) which has been developed to analyse the complete fluid/structure/fracture interaction events that take place in the fractured gas pipeline. The key features of this code are given in Table 1. The code provides (i) a detailed description of the fluid dynamic aspects of the problem, (ii) modeling of shell structure deformation, (iii) mechanisms for rapid crack advance, and (iv) the algorithms that couple the fluid dynamics, structural mechanics and fracture mechanics models [2]. Validation has been accomplished through comparison with instrumented full scale tests. The initial development of the PFRAC model was for an unconstrained pipe. The modifications described here detail a recent enhancement to consider the constraining rings. This involves the impact of the opening pipe against the rings. This is an important factor in calculating the crack driving force $G$ for a given set of S4 test conditions.

3.2. PFRAC enhancements.

In the computational model of the S4 test simulation, the pipe is modeled in PFRAC using the shell elements. The cross section of the pipe and rings is shown in Figure 2. During crack propagation, the deformation of the pipe wall will be limited by the rings. These steel rings are assumed rigid in comparison to the more flexible polyethylene. As such the pipe wall stops when it impacts these rings. Since the rings effectively represent a rigid barrier, they are not modelled explicitly in the PFRAC.

The effect of the rings is only to limit the deformation of the pipe wall and not to arrest crack. The internal gas pressure $p$ is equivalent to the original line pressure $p_0$ since decompression does not occur. Thus, the model is based
on the assumption that steady state fracture propagation is achieved for the pipe during the S4 test. As shown in Figure 2, the Y-axis is a symmetry axis since straight crack propagation generally occurs. The simulation condition assumes that the pipe and ring are concentric. The point Q is restrained against vertical motion. Vector, scalar and geometrical analyses are used to solve the contact problem.

4. Contact conditions between pipe and ring.

The contact algorithm of the pipe wall impact against rings is described here. At each time step, the initial PFRAC calculation assumes that the rings are not present. The procedure then is to advance down the pipe in the axial direction checking impact with each ring in turn. Impact takes place when the new pipe wall location is beyond the ring location. For a given ring, the computation starts at the bottom of the pipe, which is point Q as shown in Figure 2, then progresses circumferentially to establish the locations of the points \( \cdots, J, I, \cdots \) until the top point of the wall is reached.

The radius of the pipe wall is \( R_p \) at a given point in time and the radius of the ring is \( R_r \) (fixed). When crack propagates, the radius \( R_p \) will change its magnitude due to the wall deformation. At any time \( t \), a generic node point on the boundary of the pipe is at a radius \( R_p \). At the end of the next time step, the point will have moved and it is necessary to determine if it comes in contact the ring. If the calculational result from PFRAC is

\[
R_p \leq R_r
\]  

the point \( J(x_j, y_j) \) will be inside the ring. This is a normal calculation where the pipe wall does not touch the ring. Thus there is no need to implement the contact algorithm in this instance. However, if the deformation is such that

\[
R_p > R_r
\]  

the implication is that the pipe wall impact against the ring. Because of the rigid rings restraint, the pipe can not extend beyond the \( R_r \). When this situation occurs the contact condition at point \( J(x_j, y_j) \) is
Table 2: Computational procedure for pipe wall impact with rigid rings.

\[ R_p = R_r \]  \hspace{1cm} (3)

where the rigid ring limits the deformation of the crack pipe. Point \( J \) is brought back along a radial line to the wall location. This modified location is given by the vector \( \vec{c} \) where \( R_p = |\vec{c}| \) as shown in Figure 2. Thus the contact condition has been established.

There is also a significant effect on the opening velocity and acceleration of the wall. According to the equations of the motion, the velocity, \( \dot{u} \), can be expressed by the differential of the displacement, \( u \), over a time increment, \( \Delta t \), which is given by

\[ \dot{u} = \frac{u_t - u_{t-\Delta t}}{\Delta t} \]  \hspace{1cm} (4)

where superposed dot indicates the differential of time. Similar analysis for the acceleration, \( \ddot{u} \), gives

\[ \ddot{u} = \frac{\dot{u}_t - \dot{u}_{t-\Delta t}}{\Delta t} \]  \hspace{1cm} (5)

Because of the pipe opening displacements constraint imposed by rings, the velocities and accelerations of the pipe also have to be corrected based on the modified displacement.

From on the above discussion, the moving boundary condition model to simulate the crack pipe wall contact with the rings has been developed. The flow chart for this numerical procedure is illustrated in Table 2. This contact algorithm and computational process have been successfully incorporated into PFRAC. The geometric expressions describing the opening of the pipe wall behind crack tip are given in appendix A.
5. Results and analysis.

A number of instrumented S4 test have been conducted in the U.S. on 0.25 m diameter pipe [3]. This instrumentation included timing wires to determine crack velocity and pressure transducers to measure the pressure drop behind the crack tip. In many cases, the velocity is relatively constant; as expected in steady state propagation. The decay length is the length for the pressure to drop from the initial line pressure, $p_0$, to zero.

Computational simulations of the S4 tests have carried out. In the model the rigid rings are evenly spaced along the axial direction of the pipe with a 15 mm clearance from the pipe. For test ID.10.11.3, an average crack velocity of 150 m/s and a decay length of 4.0D have been measured. The computed deformations for the simulation of this test are plotted in Figure 3. These profiles illustrate the opening deformations of the pipe wall at the top of the pipe (point S in Figure 2) at various times during RCP. The comparisons of these deformations have been made for the pipe (a) with rings and (b) without rings.

The ultimate objective of the S4 test simulation carried out here is to evaluate the material fracture toughness for the pipe material, $G_d$. The approach to evaluate $G_d$ follows

1. From the instrumented S4 test data, the pressure, average crack velocity $v_{S4}$ and decay length $D_{LS4}$ can be measured.
2. The steady state crack driving force $G_d$ is calculated by the S4 test simulation model. This represents $G_d$ at the corresponding velocity $v_{S4}$.

A key step in this analysis procedure is to calculate $G_d$ from S4 test since $G_d$ is the critical material property in the pipe fracture event. Because $G_d$ could not be directly measured from the S4 test, it only can be determined from the S4 test simulation. Figure 4 illustrates the calculated results of crack driving force $G$ with crack propagation distance in a S4 test simulation for a SDR11 pipe at a pressure of 0.311 MPa and a temperature of 10°C. The steady state condition is achieved. This relative steady value is referred to as $G_d$. 

Figure 3: Profiles of the opening deformation of the pipe wall centre line, with rings and without rings, at various times during RCP.
Once $G_d$ is known, simulations of full scale conditions can be used to predict the likelihood of RCP in field operating conditions. Note that the critical pressure in the S4 test, $p_{cS4}$, is not equivalent to the critical pressure $p_c$ in the full-scale pipe because decompression is prevented. The value of pressure in full-scale pipe calculation which gives $G = G_d$ is the critical pressure. This issue is discussed in a follow up paper [4].

Conclusions.
1. The contact conditions to simulate the pipe/ring interaction have been successfully incorporated into PFRAC and this allows simulation of the S4 test for crack propagation in a PE pipe.

2. Analyses of the S4 test can be used to evaluate the material fracture toughness for the pipe material, $G_d$. Once $G_d$ is known, simulations of full scale conditions can be used to predict the likelihood of RCP in field operating conditions.

Acknowledgement
The authors wish to thank M. F. Kanninen, S. C. Grigory and L. Kim for making S4 test data available.

References


Appendix A

The geometric expressions governing the opening pipe wall behind the crack tip are developed here. In Figure 2, without loss of generality, the point $O$ is the origin, the coordinate centre of the pipe and the ring. Point $I(x_i, y_i)$ is a position of the pipe section after deformation at time $t$. The location of $I$ has been found by moving around the circumference from point $Q$ on the bottom. The length $|\bar{b}|$, which is the circumferential length of one element is also known. During crack propagation, the circumferential stresses of the pipe behind the crack tip are negligible and the circumferential length of the pipe is relatively unchanged so that the element length, $|\bar{b}|$, is effectively the original length. The objective is to find the position $J(x_j, y_j)$. Two equations are required to solve for the unknown coordinates $x_j$ and $y_j$. According to the definition of the vector we can write

$$\vec{a} = (a_1, a_2), \quad \vec{b} = (b_1, b_2), \quad \vec{c} = (c_1, c_2).$$

The direction cosines and sines of the vectors are

$$\cos \theta_a = \frac{a_2}{|\vec{a}|}, \quad \sin \theta_a = \frac{a_1}{|\vec{a}|};$$
$$\cos \theta_c = \frac{c_2}{|\vec{c}|}, \quad \sin \theta_c = \frac{c_1}{|\vec{c}|}.$$  (7)

According to geometrical relation in Figure 2 and the law of cosines, the angle $\beta$ is

$$\beta = \cos^{-1}\left[\frac{|\vec{a}|^2 + |\vec{c}|^2 - |\vec{b}|^2}{2|\vec{a}||\vec{c}|}\right]$$  (8)

From the geometrical model in Figure 2, angle $\beta$ is defined as

$$\beta = \theta_a - \theta_c$$  (9)

Substituting equations (7) and (8) into (9), then

$$\cos^{-1}\left[\frac{|\vec{a}|^2 + |\vec{c}|^2 - |\vec{b}|^2}{2|\vec{a}||\vec{c}|}\right] = \sin^{-1}\left(\frac{a_1}{|\vec{a}|}\right) - \sin^{-1}\left(\frac{c_1}{|\vec{c}|}\right)$$  (10)

At contact point, $(x_j, y_j)$ is the same as $(c_1, c_2)$. Due to equation (10) the unknown value $x_j$ is obtained as

$$x_j = |\vec{c}| \sin[\sin^{-1}\left(\frac{a_1}{|\vec{a}|}\right) - \cos^{-1}\left(\frac{|\vec{a}|^2 + |\vec{c}|^2 - |\vec{b}|^2}{2|\vec{a}||\vec{c}|}\right)]$$  (11)

In order to find the other unknown value $y_j$, substituting $x_j$ into equation (7), the calculation is

$$\theta_c = \sin^{-1}\left(\frac{c_1}{|\vec{c}|}\right) \quad \text{and} \quad y_j = |\vec{c}| \cos \theta_c$$  (12)

The unknown value $y_j$ is obtained as

$$y_j = |\vec{c}| \cos[\sin^{-1}\left(\frac{c_1}{|\vec{c}|}\right)]$$  (13)

The equations (11) and (13) are used to solve for two unknown values $x_j$ and $y_j$. Therefore, the geometric formulas for the deformed crack pipe wall have been established and knowing point $I$, point $J$ can be found.