Modelling of layered engineering surface contacts
K. Mao, T. Bell, Y. Sun
The Wolfson Institute for Surface Engineering, School of Metallurgy and Materials, University of Birmingham, Birmingham B15 2TT, UK

Abstract

The ever increasing demands in surface and subsurface properties of engineering components have led in the past few years to the rapid development of advanced surface coating techniques. A method for calculating the layer and subsurface stresses arising from the two-dimensional, dry, frictional contact of two elastic bodies with a rigidly bonded surface layer is presented. The results show that the substrate stress distributions between friction and frictionless contact are noticeable different.

1 Introduction

Conventional Hertz theory is restricted to frictionless smooth surfaces and perfectly elastic solids. Significant progress in the field of non-layer surface contact over the last few decades has been associated largely with the removal of these restrictions [2]. A proper treatment of friction at the interface of bodies in contact has enabled the elastic theory to be extended to both slipping and rolling contact in a realistic way. At the same time developments of the theories of plasticity and linear viscoelasticity have enabled the stresses and deformations at the contact of inelastic bodies to be examined. More recently, with the
development of numerical techniques, progress has been made in the study of real rough surface contact taking into account roughness parameters [3].

However, in the case of layered surface contact, the understanding of their contact mechanics is still in the initial stage. The modelling of layered surface contact is of great analytical and practical importance in many engineering applications where the load bearing capacity, friction and wear characteristics can be significantly improved by depositing thin layers of hard wear resistant materials, such as nitrides and carbides, onto the surfaces. In the more realistic case of an elastic layer on an elastic substrate, achievements had been made by Gupta and Walowit [4] assuming frictionless contact based on Sneddon [5]'s theory. They derived the mathematical formulation to the problem of a layered elastic surface indented by an elastic and a rigid indenter. Relatively little work has been done on the sliding contact between layered surfaces. Recently, Elsharkawy and Hamrock [6] have presented a dry smooth sliding contact model for two elastic cylinders coated with multiple layers.

2 The Contact Model

This model is at present on the basis of the following major assumptions, some of which may be relaxed as the work proceeds: (1) The structure of the contact bodies, is considered to be composed of one or more elastically homogeneous surface layers rigidly bounded to each other and in turn to the substrate. Each component of the system, ie each layer and the substrate, is taken as having its own distinct mechanical properties. (2) The system is taken as being in contact with another elastic body, which may be elastic halfspace or another multilayered body. (3) The contact is assumed to give rise to a state of plain strain, so that the system is considered in two dimensions only. This assumption holds well for line contacts, such as those which arise in gears and roller bearing. (4) Strains are assumed small, leading to the usual linear elastic theory assumptions. (5) The contact is considered to be dry, ie the present of a lubricant is neglected.

Figure 1 shows an elastic half-space coated with one elastic layer. The layer is completely bonded to the elastic half-space it covers. A common method used in the numerical solution of contact problems is to divide the contact boundary into elements, over which it is assumed that the pressure is uniform [3]. For a detailed description of the solution procedure of the contact model, it can be referred to the work of Mao, Sun and Bell [7]. The research presented here takes the surface pressure distribution from a contact model developed by the authors, which simulates rough layered body contact, and calculate the stresses in the layer and substrate due to this surface loading.
3 Expressions for the stresses

Using the coordinate system shown in figure 1, the stresses $\sigma_x$, $\sigma_y$ and $\tau_{xy}$, in the x and y directions respectively, in any component $i$ are given by ($j=\sqrt{-1}$)

$$
\sigma_{x,i} = -\frac{1}{2\pi} \int_{-\infty}^{\infty} \omega^2 G_i e^{-j\omega y} \frac{dG_i}{dx_i} d\omega
$$

$$
\sigma_{y,i} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \omega^2 \frac{d^2 G_i}{dx_i^2} e^{-j\omega y} d\omega
$$

$$
\tau_{xy,i} = \frac{1}{2\pi} \int_{-\infty}^{\infty} j\omega \frac{dG_i}{dx_i} e^{-j\omega y} d\omega
$$

where $i = 1$ for layer and $i = 2$ for substrate, $G_i$ is the Fourier transform of the Airy stress function, and it is given by the following expression in which the constants $A_i$, $B_i$, $C_i$, $D_i$ (the stress function coefficients) are general functions of $\omega$, and are determined from the boundary conditions.

$$
G_i = (A_i + B_i x_i) e^{-\omega |x_i|} + (C_i + D_i x_i) e^{\omega |x_i|}
$$

By applying the boundary conditions [7] to the model, it is found that $C_2 = D_2 = 0$ and the six simultaneous equation in Table 1 for the remaining stress function coefficients are obtained.

Table 1 Matrix equation for the stress function coefficients

$$
\begin{bmatrix}
\omega^2 & 0 & 0 & 0 & 0 & 0 \\
-|\omega| & 1 & |\omega| & 1 & 0 & 0 \\
\frac{1}{q} & \frac{1}{h} & q & hq & -1 & 0 \\
-|\omega| & \frac{1-|\omega|h}{q} & |\omega|q & q(1+|\omega|h) & |\omega| & -1 \\
-|\omega|f_{ji} & \frac{j_{ji}-|\omega|h_{ji}}{q} & |\omega|qf_{ji} & q(j_{ji}+|\omega|h_{ji}) & |\omega|\gamma_{ji} & -\gamma_{ji} \\
\frac{|\omega|k_{ji}}{q} & \frac{|\omega|h_{ji}k_{ji}-2}{q} & |\omega|qk_{ji} & q(|\omega|h_{ji}k_{ji}+2) & -\gamma |\omega|k_{ji} & 2\gamma
\end{bmatrix}

\begin{bmatrix}
A_1 \\
B_1 \\
C_1 \\
D_1 \\
A_2 \\
B_2
\end{bmatrix}

= \begin{bmatrix}
P \\
Q \\
0 \\
0 \\
0 \\
0
\end{bmatrix}

q = e^{j\omega h}, \nu' = \frac{2-\nu}{1-\nu}, \nu'' = \frac{\nu}{1-\nu}, k = 1+\nu'', j' = 1-\nu', j = 3-\nu'

P = \frac{2}{\omega} \sin \frac{\omega a}{2}, Q = \mu \frac{2}{\omega^2} \sin \frac{\omega a}{2}
As \( G \) is an even function, the range of the integrals may be reduced from \([-\infty, \infty]\) to \([0, \infty]\). By making the changes of variables \( \xi = x/h \), \( \zeta = y/h \) and \( s = \omega h \),

\[
A_1' = \frac{A_1}{h^2}, \quad B_1' = \frac{B_1}{h}, \quad C_1' = \frac{C_1}{h^2}, \quad D_1' = \frac{D_1}{h}, \quad A_2' = \frac{A_2}{h^2}, \quad B_2' = \frac{B_2}{h}
\]

and noting that the integral is in the positive region, the simultaneous equations of Table 1 may be rewritten as in Table 2. Thus

\[
\sigma_{x_i} = -\frac{1}{\pi h} \int_0^\infty s^2 \left[ \alpha_i e^{-s \xi} + \beta_i e^{s \xi} \right] \cos s \zeta \, ds
\]

\[
\sigma_{y_i} = -\frac{1}{\pi h} \int_0^\infty s \left\{ e^{-s \xi} (2B_i' - s \alpha_i) - e^{s \xi} (2D_i + s \beta_i) \right\} \cos s \zeta \, ds
\]

\[
\tau_{x_i} = \frac{1}{\pi h} \int_0^\infty s \left\{ e^{-s \xi} (B_i' - s \alpha_i) + e^{s \xi} (D_i + s \beta_i) \right\} \sin s \zeta \, ds
\]

where

\[
\alpha_i = A_i' + B_i' \xi
\]

\[
\beta_i = C_i' + D_i' \xi
\]

### Table 2  Matrix equation for the derived stress function coefficients

\[
\begin{bmatrix}
  s^2 & 0 & s^2 & 0 & 0 & 0 \\
  -s & 1 & s & 1 & 0 & 0 \\
  1 & 1 & r^2 & r^2 & -t & 0 \\
  -s & (1 - s) & st^2 & t^2 (1 + s) & st & -t \\
  -sf_1 & j_1 - sf_1 & stf_1 & t^2 (j_1 + sf_1) & stf_2 & -\gamma f_2 \\
  k_1 & k_1 - \frac{2}{s} & k_1 t^2 & t^2 \left( k_1 + \frac{2}{s} \right) & -\gamma k_2 & \frac{2 \gamma t}{s}
\end{bmatrix}
\begin{bmatrix}
  A_i' \\
  B_i' \\
  C_i' \\
  D_i' \\
  A_2' \\
  B_2'
\end{bmatrix} = \begin{bmatrix}
  P \\
  Q \\
  0 \\
  0 \\
  0 \\
  0
\end{bmatrix}
\]

\[
t = e^t, \quad \nu' = \frac{2 - \nu}{1 - \nu}, \quad \nu'' = \frac{\nu}{1 - \nu}, \quad \gamma = \frac{(1 - \nu^2)E_1}{(1 - \nu_i^2)E_2},
\]

\[
f = 1 - \nu', \quad j = 3 - \nu', \quad k = 1 + \nu'', \quad P = \frac{2h}{s} \sin \frac{sa}{2h}, \quad Q = \frac{2h}{s^2} \sin \frac{sa}{2h}
\]
4 Results

A computer code written in Fortran 77 has been developed to simulate the specific contact problem based on the developed model. Computations were performed on a Digital DECstation 5000/120 with 16 Mbytes of RAM.

To confirm that the model accurately simulates a smooth dry frictionless elastic contact, the case of a layer with material properties being identical and varying from those of the substrate was investigated. The results show good agreement with those of Cole and Sayle [9] and Gupta and Walowit [4].

To investigate the effects of friction, layer thickness and elastic modulus on the subsurface stress distribution of layered surface, one case was considered in the present study to demonstrate the significance of the analysis developed herein. That is the half-space is coated with a 3 μm hard layer, titanium nitride (TiN). Table 3 shows the material properties of the cylinder, layers and substrate and these values are taken from Elsharlawy and Hamrock [5].

<table>
<thead>
<tr>
<th>material</th>
<th>Modulus of elasticity (GPa)</th>
<th>Poisson’s Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>cylinder</td>
<td>200</td>
<td>0.3</td>
</tr>
<tr>
<td>Substrate</td>
<td>200</td>
<td>0.3</td>
</tr>
<tr>
<td>TiN coating</td>
<td>640</td>
<td>0.2</td>
</tr>
</tbody>
</table>

Fig. 2 shows the subsurface stress distributions (Von Mises) for the TiN coating case without friction. From this figure, it can be found that the subsurface stress distribution is asymmetrical and the maximum stress happened under the surface. However, as the friction is introduced, the subsurface stress distribution changes dramatically as shown in Fig. 3. From this figure, it can be seen that the maximum Von Mises stress increases compared to frictionless case and its position moves up to the top of the subsurface. The scale of Figs. 2 and 3 is four times of the half contact width in horizontal direction and two times of the half contact width in vertical direction.

Conclusions

A method for calculating the subsurface stress distributions arising from the two-dimensional dry frictional contact of layered elastic solids is described. The sample results presented show how the subsurface stress distributions change
with coefficient of friction. That is the introduce of friction will increase the subsurface stress greatly and change the position of maximum stress.

Acknowledgements

The authors gratefully acknowledge the support of BRITE/EURAM project BE-4242 who sponsored this work.

References

Fig. 1 Coordinates and notations for one layered elastic solid

Fig. 2 Substrate Von Mises stress distribution (GPa) for Fig. 1 model under a load of 0.1 N/μm, coefficient of friction = 0
Fig. 3 Substrate Von Mises stress distribution (GPa) for Fig.1 model under a load of 0.1 N/μm, coefficient of friction = 0.6