FE modeling of pavement joints for dynamic analysis
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ABSTRACT
A finite element (FE) algorithm is developed to analyze the dynamic response of multiple, jointed concrete pavements to moving aircraft loads. A contact element model is developed to accurately represent the dowel pavement joints based on the Lagrangian multiplier method. The accuracy of the FE algorithm developed is verified by the available experimental and analytical solutions. A parametric study is conducted to investigate the effects of various parameters on the dynamic response of pavements.

INTRODUCTION
The early development of pavement analysis methods is primarily based on the closed-form solutions obtained from the static analysis of infinitely long pavements. The effects of discontinuities due to joints are neglected for convenience. With the advent of computers, the finite element method (FEM) has been used in recent years to analyze the discontinuous pavements. Huang and Wang [3] developed a solution scheme based on the FEM to analyze a two slab system connected with dowel bars at the joint. The effect of dowel joints was taken into account by specifying the joint efficiency across a joint. In the subsequent development, the imaginary shear transfer (vertical) springs were used along the joint between two adjacent slabs to indirectly account for the load transfer devices. Such a model has been widely used for
aggregate interlock and keyed joints (Tabatabaie et al. [8], Larralde [4]) as well as for dowel joints (Chou [2]) because of their simplicity and more realistic approach than specifying joint load transfer efficiency. In some other studies (Tabatabaie et al. [8], Larralde [4]), the dowel joints have been modeled by deep beam elements. The length of the beam element is taken as the width of joint opening. The relative deformation between the dowel bar and the concrete is represented by a vertical spring that extends between the surrounding concrete and the dowel bar.

The aforementioned studies consider the pavement loads are static by neglecting the dynamic interaction between moving vehicles/aircraft and the pavement. The majority of the literature available on the dynamic analysis of pavements treats the pavement system as an infinitely long beam/plate (Kerr, [5]). A modified FE approach presented by Kukreti et al. [6] considered the dynamic response of jointed pavements to moving aircraft loads. In this study, the transverse and longitudinal joints are modeled by equivalent vertical springs. The moment transfer efficiency of the dowel joints is assumed to be zero.

**SCOPE**

In this study, a procedure based on the FEM is presented to analyze the dynamic response of jointed concrete pavements to moving aircraft loads. A special joint element is developed to accurately model the dowel pavement joints, based on the contact theory. This joint element considers the effects of dowel looseness and dowel embedment.

Fig. 1 shows the FE representation of various pavement components. The concrete pavement is modeled by four-noded rectangular thin plate elements and the underlying soil medium is represented by continuous
Winkler springs and dashpots. Doweled joints are considered for pavement transverse joints. The dowel bars are represented by massless plane frame elements. The dowel-pavement interaction effects are represented by employing contact elements between the dowel bar and the pavement. The aggregate interlock or keyways are assumed in the longitudinal joints and represented by the vertical spring elements. The dynamic aircraft-pavement interaction is accounted for by modeling the aircraft by masses supported by spring-dashpot systems which represent the landing gear of the aircraft. It is assumed that the aircraft travels along a straight line with a specified initial horizontal velocity and acceleration.

FINITE ELEMENT ALGORITHM FOR THE DYNAMIC ANALYSIS OF JOINTED RIGID PAVEMENTS

Pavement Resting on an Elastic Foundation

By following the standard variational principles, the matrix equation governing the plate resting on an elastic foundation can be expressed in the form (Ref. 1):

\[
[k]\{e\} = \{Q\}
\]

where \([k]\) is the system stiffness matrix, \(\{e\}\) contains nodal displacements and rotations and \(\{Q\}\) is the force vector. The force vector \(\{Q\}\) can be expressed by the following equation:

\[
\{Q\} = \sum \int_A [N]^T q(x,y) \, dA
\]

where \([N]\) is the shape functions, \(q(x,y)\) is the dynamic force acting on the pavement due to moving aircraft and \(\sum\) represents summation of quantities for all elements.

Contact Element Model for Dowel Joints

In this study, a contact element is developed based on the Lagrange multiplier method (Nour-Omid et al. [7]) to idealize the pavement-dowel bar contact problem. In the contact element algorithm, one end of the dowel bar is assumed to be fully embedded into the pavement. The other side is considered free to move vertically and/or slide depending on the level of pavement-dowel interaction. The contact forces developed between the pavement and the dowel bar where it is fully embedded, are the normal force, tangential
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Free End (painted or oiled)  Fixed End (No relative movement between dowel and pavement)

(a) Conventional Representation of a Doweled Joint

Plate Element

(b) Finite Element Representation of a Doweled Joint

force and the moment about the x axis. The contact forces developed on the free end of the dowel bar are the normal and tangential forces. The tangential forces are assumed to satisfy the Coulomb frictional law.

In the finite element discretization, the dowel bar is idealized by the two noded massless plane frame elements including the effects of shear deformation. Contact elements are employed at all possible contact points between the pavement and the dowel bar (See Fig. 2).

In the finite element formulation, the equilibrium state of the pavement system is obtained by minimizing the total potential energy. Appropriate constraint conditions are added to the potential energy to account for the contact state. The equilibrium equation for the total system that satisfies the contact conditions can be obtained in the following form:

\[
\begin{bmatrix}
[K]_{pav} & [B]_{ij} & 0 \\
[B]_{ji} & [B]_{jj} & [B]_{jk} \\
0 & [B]_{kj} & [K]_{dow}
\end{bmatrix}
\begin{bmatrix}
\{q\}_{i}^{pav} \\
\{q\}_{j}^{pav} \\
\{q\}_{k}^{dow}
\end{bmatrix}
= \begin{bmatrix}
\{R\}_{i} \\
\{R\}_{j} \\
\{R\}_{k}
\end{bmatrix}
\]  \tag{3}

where \([K]_{pav}\) is the stiffness matrix of the pavement subgrade system and \([K]_{dow}\) is the stiffness matrix of the dowel bar. The matrix \([B]\) in Equation 3 represents the contact kinematics and its specific terms can be obtained by considering for different contact conditions. Further details of the formulation can be obtained in Ref. 1.

Dynamic Aircraft-Pavement Interaction

The dynamic interaction force \(q(x,y)\) in Equation 2 can be expressed in the form (Ref. 1)

\[
q(x,y) = (M_i g - M_i \ddot{u}) \delta (x - \xi , y - \eta) + mg - mw - cg \dot{w} \tag{4}
\]
where $M_i$ is the aircraft mass, $u$ is the vertical displacement of the suspended mass, $mg$ is the plate weight, $mw$ is the inertia force due to plate mass, $c w$ is the force due to foundation damping, $\delta$ is the dirac-delta function and $(\xi, \eta_i)$ is the position of the $i$ th aircraft wheel load. The over dot (.) denotes derivative with respect to time $t$. In view of Equations 2 and 4 and expressing the plate deflection in terms of nodal variables, Equation 1 can be transformed into the following form:

$$[M] \frac{d^2\{e\}}{dt^2} + [c] \frac{d\{e\}}{dt} + [k]\{e\} = \{\ddot{w}\} + [\bar{N}(\xi, \eta_i)]^TM_i g$$

$$- [\bar{N}(\xi, \eta_i)]^T M_i \ddot{u}_i$$

(5)

where $[M]$, $[c]$ and $[\bar{w}]$ are the mass matrix, damping matrix and pavement weight vector.

Applying D'alembert's principle for the aircraft suspension system gives

$$M_i \ddot{u}_i + c_i \dot{u}_i + k_i u_i = M_i g + c_i [\bar{N}(\xi, \eta_i)] \frac{d\{e\}}{dt}$$

$$+ c_i [\bar{N}(\xi, \eta_i)] \dot{\{e\}} + k_i [\bar{N}(\xi, \eta_i)] \{e\}$$

(6)

where $c_i$ represents the damping coefficient of the suspension system, $k_i$ is the spring constant of the suspension system and $t$ in $[\bar{N}(\xi, \eta_i)]_t$ denotes the derivative with respect to $t$. Equations 5 and 6 are the two sets of required coupled equations from which the nodal displacement vector $\{e\}$ and the displacements of aircraft mass $u_i$ can be obtained. A solution scheme adopted in this study utilizes the Newmark-β method to carry out the numerical integration scheme. Also, the time variable $t$ in Equations 5 and 6 is changed by using the position of the suspended masses $(\xi)$ as a pseudo-time variable for convenience. The dynamic force deflection equation for the pavement and the suspended aircraft mass given in Equations (5) and (6), respectively, can be transformed and combined into the following matrix form:

$$\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \{\ddot{e}\}_j = \begin{bmatrix} [R_1] \\ [R_2] \end{bmatrix}_j$$

(7)

The above equation is solved by utilizing an iterative procedure. Further details of the formulation are given by Alvappillai [1].
NUMERICAL RESULTS

Verification

The accuracy of the contact algorithm presented is verified by comparing the finite element results with the analytical solutions for a contact problem involving uniformly loaded plate with sagged supports. In this problem, two opposite edges of the plate are simply supported while the support at other two edges are sagged so that there is a gap between the plate and the support in the unloaded state (Fig. 3(a)). The amount of sag (δ) is assumed as uniform so that the sagged support lie in a plane parallel to the plate edges. The plate is deformed by a uniformly distributed load of constant magnitude (q). The plate will start to touch the sagged support only when the loading has reached to a certain level. If the loading is increased further, the contact between the plate and the sagged support will spread or advance. The analytical solutions presented by Dundurs et. al. (1974) for the above problem are used in this study for comparison.

For the finite element analysis, a sagged supported steel plate with the following geometric and material properties is assumed; Plate dimensions; a = b = 24 in., Plate thickness (h) = 1 in., Amount of sag (δ) = 0.1 in., Young's Modulus = 3x10^7 psi, Poisson's ratio = 0.3

In the finite element analysis, the plate is discretized by thin plate elements. The sagged support is represented by a series of plane frame elements lying below and parallel to the plate edges, as shown in Fig. 3(b). The vertical degree of freedoms of all the nodes of the plane frame elements are locked to

![Fig. 3 Sagged Supported Plate Used in Verification](image-url)
account for rigid support. Contact elements are employed between the nodal points of the plane frame elements and the sagged supported plate edges. The initial gap for all the contact element are assumed to be 0.1 in.

The finite element results obtained for the deflections of the plate edge that have the sagged support are compared with analytical solutions in Fig. 4 for different level of contacts (2a/c = 0.1, 0.2 and 0.3). Fig. 4 shows an excellent agreement between the finite element and the analytical results.

The verification of contact algorithm could not be performed directly in the area of pavement analysis due to unavailability of experimental or analytical studies. However, the applicability of the algorithm in the dynamic analysis of pavements are verified by utilizing available analytical and experimental results and the results are presented in Ref. 1.

Parametric Study

A parametric study conducted to investigate the behavior of a multiple, jointed pavement system to a moving aircraft. The pavement system used for the analysis consists of three discrete slabs in the longitudinal direction and two slabs in the transverse direction. Keyed joint and doweled joints are assumed for the longitudinal and transverse joints, respectively. The aircraft considered in the analysis is the B-727 model which has twin landing gear with a maximum gross weight of 169 kips. It is assumed that the main landing gear carries 90% of the total aircraft weight and the remaining 10% by the nose gear. Therefore, the each gear in the main twin assembly carries approximately 77 kips. For simplicity, only the main landing gear represented by two suspended moving masses, is considered in the analysis. The following geometric and material properties for the pavement-subgrade system are assumed.

Fig. 4 Plate Edge Deflection for Different Levels of Contact

Fig. 5 Effect of Pavement thickness
Pavement-Subgrade Properties
Six 300 in. by 300 in. (25ft. by 25 ft.) slabs
Pavement thickness = 12 in., Young's Modulus (E) = 3.6x10^6 psi, Poisson's ratio (v) = 0.15, Mass density (ρ) = 0.0002174 lb.s^2/in.^4, Modulus of subgrade reaction (k) = 300 pci, Subgrade damping = 5%

Joint Properties
(1) Transverse Doweled Joint:
Dowel bar diameter (d) = 1.0 in., spacing (s) = 15 in., Dowel Looseness (γ) = 0.0005 in., Dowel length = 36 in., Joint width = 0.25 in., Young's modulus of dowel bar = 29x10^6 psi, Shear modulus of dowel bar = 11x10^6 psi,
(2) Longitudinal Joint
Longitudinal keyed joint is represented by a very stiff vertical spring element.

Aircraft Properties
Spring stiffness of the aircraft suspension = 1x10^5 lb/in., Damping of aircraft suspension = 0.5%, Aircraft velocity = 100 mph

The results of the parametric study are presented for static, moving force and moving mass loading conditions in a dimensionless form. The effects of pavement thickness (h/l) and relative subgrade stiffness on the maximum pavement deflection (wD/Pl^2) are shown in Figs. 5 and 6, respectively. As expected, the pavement deflection decreases with increasing pavement thickness and the increasing subgrade stiffness. The moving force and moving mass solutions give less deflection for this particular case in which the aircraft is assumed to travel at a constant speed of 100 mph.

Fig. 7 shows the effect of dowel looseness (γ) on the dowel joint efficiency. It is observed that the
joint efficiency decreases approximately linearly with increasing dowel looseness. About 99% of the joint efficiency is achieved when the dowel has no looseness. The joint efficiency decreases to 80%, 72% and 70% in case of static, moving force and moving mass loading conditions, respectively, when the dowel looseness increases from 0 to 0.001 in. (γ/w = 0.04, where w is the joint width). This suggests that the dowel-pavement interaction effects due to dowel looseness are more pronounced in dynamic loadings than the static case.

The variation of the maximum pavement deflections with dowel looseness is presented in Fig. 8. As it is observed, the deflection increases with increasing dowel looseness, approximately in a linear manner. When γ/w increases from 0 to 0.04 (dowel looseness from 0 to 0.001 in.), the maximum deflections increases by about 6%.

The effect of aircraft velocity (v/v_{cr}) on the maximum pavement deflection represented by the dynamic magnification factor (w/w_{s}) is depicted in Fig. 9. The critical aircraft velocity, v_{cr}, can be given by the following expression:

\[ v_{cr} = \left[ \frac{4kD}{(ρh)^{2}} \right]^{1/4} \]

where D = plate rigidity = Eh^3/12(1−ν^2). It is observed that the moving mass solution gives larger deflection than the static solution for small velocity ratios (v/v_{cr} < 0.05). A maximum dynamic magnification factor of about 1.05 is observed at a velocity ratio of 0.05. The moving force and moving mass solutions give smaller deflection than the static for large the velocity ratios.

CONCLUSIONS

An algorithm based on the finite element method is presented to analyze the dynamic response of multiple,
jointed concrete pavements to moving aircraft loads. This model considers the dowel-pavement interaction effects and includes the effects of dowel looseness and dowel embedment on the pavement response. The parametric study conducted indicates the importance of dowel-pavement interaction effects in the pavement response. The increasing pavement deflection is obtained when the dowel-pavement interaction effects are considered. Dowel looseness caused by repeated loads and other factors decreases the joint efficiency of the pavement joints. It is observed that the increase in dowel looseness from 0 to 0.005 causes decease in joint efficiency from 99% to 70% in case of moving mass idealization.

REFERENCES


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