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Finite element modelling with contact and friction for plane strain sheet forming process

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Abstract
For the purpose of studying the sheet forming process of thin-walled structures a specific finite element method has been developed based on plane strain shell elements for cross-section analysis with elastic-plastic behaviour. The contact and friction boundary conditions are imposed by moving shaped tools using the exterior penalty method and cold roll-forming is considered.

INTRODUCTION

As we are concerned by sheet forming process and unloading problems in order to get residual stresses the elasto-plastic model is used for numerical calculations of sheet metal forming presented in the paper instead of the simpler rigid-plastic model. The standard incremental implicit approach based on incremental displacements as primary unknowns and updated Lagrangian description of motion is used for cross-section analysis. Even for roll forming process where a sheet metal is continuously and progressively formed into a product with required cross-section by passing through a series of forming rolls arranged in tandem, the section analysis represents an alternative procedure to the full 3D. analysis very C.P.U. time consuming when contact is involved.

CROSS-SECTION ANALYSIS

The basic governing equations are given by the principle of virtual work at time t+Δt referred to the current configuration at time t witch can be written in the following way [1]:

$$\int_V [\Delta S:\delta\varepsilon+(\sigma+\Delta S):(\nabla\Delta u.\delta u\nabla)]dv = R + \int_{st} \Delta t.\delta u \, ds_t + \int_{sc} \Delta f.\delta u \, ds_c \qquad (1)$$

where σ is the Cauchy stress tensor and ΔS the convected increment of Kirchhoff stress.

This permits us to calculate a certain state of equilibrium based on the knowledge of the previous equilibrium state together with the actual increments in the external loads on St or the prescribed displacements and contact forces on Sc.

To avoid error accumulation and to permit unloading the new equilibrium state is calculated by an iterative Newton's method where the term:

$$R = \int_{st} t.\delta u \, ds_t + \int_{sc} f.\delta u \, ds_c - \int_{v} \sigma : \delta\varepsilon \, dv \tag{2}$$

is often called the residual which is expected to vanish in the increment.

The finite element discretization has been made on plane strain shell element of Reissner-Mindlin type. Since independent rotational degrees of freedom are used only Co continuity is required and this by-passes the difficulties caused by the C1 requirement of the classic Kirchhoff theory. With these assumptions using linear shape functions and considering generalized plane strain conditions we derived a finite element with two nodes and one rotational and two translational degrees of freedom at each nodes :

$$u = \sum_{\alpha=1}^{2} N_\alpha(\xi) \; u_\alpha \tag{3}$$

$$w = \sum_{\alpha=1}^{2} N_\alpha(\xi) \; w_\alpha \tag{4}$$

$$\theta = \sum_{\alpha=1}^{2} N_\alpha(\xi) \; \theta_\alpha \tag{5}$$

This element which allows for combined in-plane and bending behaviour is divided into a number of layers in order to capture the spread of plasticity over the thickness. For integration over the sheet thickness ten layers are used although the number of layers can be extended up to 20 in particular for element on small tools radii.

Since moderately thick sheets will be analysis the usual assumption of plane stress state in the thickness direction can be justified here. Then the thinning effect due to large plastic strain is taken into account by an update procedure assuming plastic incompressibility in every time step iteration.

It is well known that the so-called Reissner Mindlin thick shell element family in direct application to thin plate situations can induce locking. In the thin plate limit Kirchhoff's assumptions of vanishing transverse shear strain must be satisfied. A procedure is to impose a shear strain field which satisfies the limit thin plate condition. For our two nodes element this is achieved by evaluating the shear strain γ

at the element mid-point. This leads to an identical expression for the substitute shear stiffness matrix obtained by an one point selective integration.

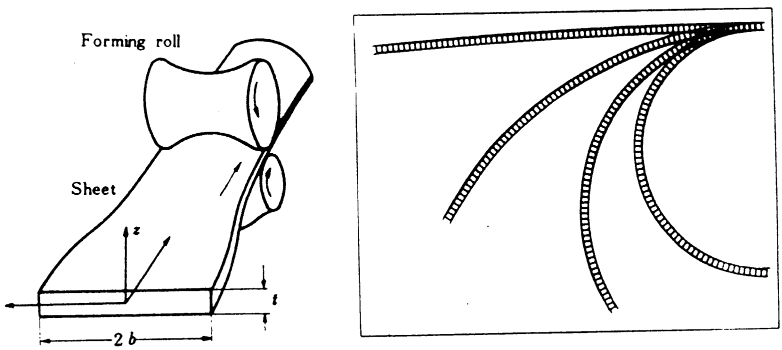
BOUNDARY CONDITIONS

In the present formulation of sheet-metal forming process it is assumed that the tool parts are completely rigid bodies whose relative movements are forced on a deformable elastic-plastic solid. It is worth noting that the moving tools not only translate as rigid bodies but can change their shapes continuously with time for roll-forming process (see figure 1 for example). Then we can use parametric functions of space and coordinates varying with time to describe the rigid moving surfaces :

$$X(t) = \sum_{k=1}^{n} N_k(\xi)\ x_k(t) \tag{6}$$

$$Z(t) = \sum_{k=1}^{n} N_k(\xi)\ z_k(t) \tag{7}$$

where n = 2 or 3 for linear or quadratic shape functions.



Fig(1) Roll-forming of circular cross-section

In an incremental approach of time step dt we determine an appropriate prescribed displacement where the non-penetration inequality is used to just bring a or some material points to be incontact with the rigid surface :

$$du_n - dg_n + d_n \leq 0 \qquad \text{on } S_c \tag{8}$$

where du_n is the increment of the normal displacement constrained by the given rigid surface motion dg_n. To resolve this boundary constraint we apply the exterior penalty method:

$$df_n = -k_n (du_n - dg_n) \quad \text{on } S_c \tag{9}$$

with a very large penalty parameter $k_n > 0$.

The point will stay in contact as long as the resultant contact pressure f_n updated by its material increment df_n remains negative. We assume that there exists a Coulomb friction law so that no relative motion is observed if:

$$|f_t| < \mu \, |f_n| \tag{10}$$

Thus a same relation that (9) holds between the tangential increment of the friction force and the tangential increment of the displacement:

$$df_t = -k_t (du_t - dg_t) \quad \text{on } S_c \tag{11}$$

When the state of stress of a contact point is such that:

$$|f_t| = \mu \, |f_n| \quad \text{and} \quad |df_t| = \mu \, |df_n| \tag{12}$$

the point is said to be sliding and (11) gives the consistent tangential penalty parameter as :

$$k_t = \mu \, | \, df_n | \, / \, |(\, du_t - dg_t)| \tag{13}$$

The contact condition and the Coulomb s' friction law are satisfied in a integrated sens over each individual contact element side (upper and lower surface) consistent with the level of finite element shape functions used. The friction forces can then be determined using the quantities available in the course of the iteration process and contribute via the penalty parameters to the applied loads and to the stiffness matrix of the system such that:

$$\Delta f^e_{i\beta} = \int_{Sce} [k_n dg_n n_i + k_t dg_{tj} (\delta_{ij} - n_i n_j)] \; N_\beta \; dS_{ce} \tag{14}$$

$$K^e_{ij\alpha\beta} = \int_{Sce} [k_n n_j n_i + k_t (\delta_{ij} - n_i n_j)] \; N_\alpha \; N_\beta \; dS_{ce} \tag{15}$$

From a mechanical or computational point of view it seems better in (14) and (15) to associate the local normal axes to the element side instead to the tool because displacements stresses and equilibrium are analyzed only for the deformable body. However this approach can give rise to problems especially if a rather coarse mesh is to be used but it is not really the case for our cross section analysis.

SOLUTION PROCEDURE

Contact and friction:

We assume that in the incremental solution the response at time t has been calculated and that (i-1) iterations have been performed to calculate the solution at time t+Δt . Then in each iteration the first step in our procedure is to update the pressure contact vectors with their material increment evaluated at the integration point of the contactor element sides The decision on whether a contactor integration point is releasing or is in sticking or sliding condition is based on considering the total and relative magnitudes of these updated pressure contact forces as follows :

* if $f_n \geq 0$ the point is assumed to have experienced tension release and in this case the penalty parameters and the normal and tangential contact pressure are set to zero.

* if $f_n \leq 0$ and $|f_t| < \mu\ |f_n|$ the point continues to stick or sticking contact conditions must be assumed by setting the penalty parameters to a large positive number which may be function of the local stiffness of the body.

* if $f_n \leq 0$ and $|f_t| \geq \mu\ |f_n|$ then the state of the point must be updated to sliding with :

$$k_t = \mu\ |df_n|\ /\ |(du_t - dg_t)| \tag{16}$$

$$\text{sign} = df_t/|df_t| \quad \text{and} \quad f_t^i = f_t^{i-1} + \text{sign}\ \mu|df_n| \tag{17}$$

where f_t^{i-1} must satisfy the previous sliding condition (iteration i-1).

Constitutive relation:

The procedure include the anisotropic Hill's model of initial cross anisotropy with isotropic and cinematic hardening. The extension to the complete large strain case requires an objective formulation of these classical small strain constitutive relations. This is simply achieved by using a rotating frame formalism where the stress strain and state variables tensors are replaced by their rotated tensors by an objective rotation R :

$$\bar{\sigma} = R^T . \sigma . R \quad , \quad \bar{\varepsilon} = R^T . \varepsilon . R \ , \quad \ldots \tag{18}$$

Since elastic strains are assumed to be small it is convenient to ensure the objectivity of the hypoelastic law by writing it in the same rotating frame.In order to integrate the elasto-plastic relations the rotating frame formalism simply requires following the rotation R for all elements and making the appropriate rotation but taken in the state at the middle of the total displacement increment to evaluate the rotated tensors of strain and stress.We use the elastic predictor-radial corrector implicit algorithm in the rotating frame to integrate the constitutive relations and many authors have demonstrated the overall superiority of this method over others proposed algorithms.

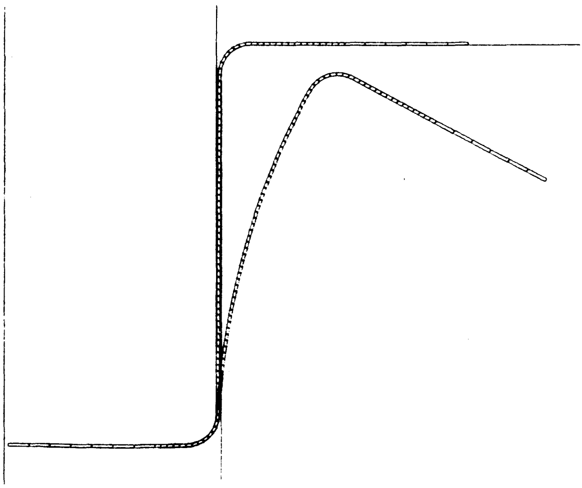
Notice that according to the rotating frame formalism the values are calculated based solely on the converged values at the end of the previous increment. The non-converged values at the previous global iteration within the current step play no explicit role in the stress evaluation. This scheme is path-independent and avoids numerical unloadings.The last aspect demands that in an efficient solution procedure the elasto-plastic stiffness matrix must be consistent with the implicit integration procedure of the constitutive rate equations in order to improve the rate of convergence. A derivation of this consistent modulus can be found in [1].

NUMERICAL EXAMPLES

In order to analyze a springback sensitive situation the numerical model discussed above has been used to simulate experimental plane strain deep drawings carried with blank-holder pressure. As an example the figure (2) shows the deformed mesh and the resulting springback where the blank-holder normal force is 48000 N and the punch travel H is 100 mm .The material is a sheet of mild steel of thickness 0.78 mm width = 150 mm and where the parameters of the Hill'criterion are as follows:

$$\text{Anisotropy } r=1.1 \qquad \sigma = 556\ (0.00355 + \varepsilon)^{0.221}$$

The die and punch (100 mm width) have the same radius of 7 mm and the lubricant coefficient of friction is μ = 0.17.



Fig(2) Plane strain deep-drawing and spring-back

The comparison with the experimental geometric shape of the sheet after springback is in a very good agreement witch provides a numerical verification of the proposed section analysis. The side wall curl which is a result of the bending moment created by the tensile and compressive stresses

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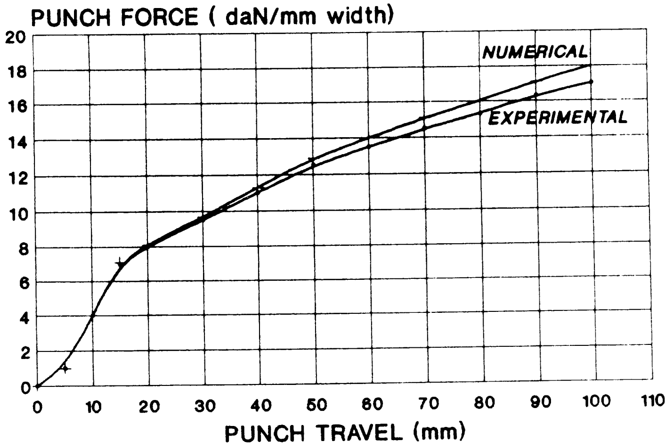
throughout the sheet thickness is clearly obtained (see figure (2)) and figure (3) shows the calculated and experimental punch force versus punch travel .



Fig(3) Punch force versus punch travel

Conceptionally the roll forming process is a 3D. bending work of sheet metal which develops complicated 3D. curved surfaces with smooth or sharp corners (stiffeners).Since the position of the neutral layer at the cold-bent parts (corners) affects the size of the average longitudinal residual stress and needs a refined calculation , the already proposed cross section analysis is used. A given cross section of the sheet is numerically deformed continuously by tools from the i-th to the (i+1)-th roll-stand profile (see figure 1 for a circular tube and the following figure 4 for more complex profile).

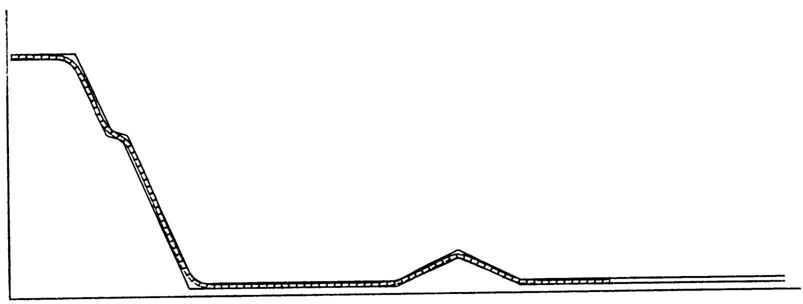


Fig.(4) Complex sheet profile

In order to take account of the transitions of the longitudinal strains we use a less refined mesh of the 3D.

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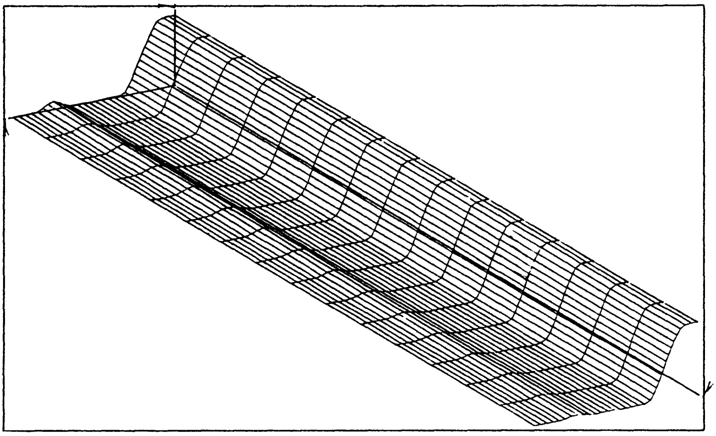
curved surface between two roll-stand (figure 5 for example) with 4 nodes quadrilateral Mindlin shell elements which are under-integrated by 1x1 Gauss-point and hourglass control.



Fig(5) Example of 3D. deformed mesh between roll-stands

By the above mentioned method of analysis numerical calculations were made about forming process for the product with a circular arc cross-sectional profile and compared with experimental results carried out by Kiuchi and al. [5].

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