Simulating dynamic motion of ship in contact with elastic mooring elements by a nonlinear algorithm

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ABSTRACT

A nonlinear algorithm for simulating dynamic motion of ship in contact with elastic mooring elements was developed. During small time steps all parameters in ship-water-spring system can be taken as linear. Force balance in the system is described by linear second order differential equations for six degrees of freedom. Nonlinearities are taken into account by re-evaluating all affecting forces and mass, damping and stiffness matrices at every time step. Ship is described as a rigid body with three symmetry planes. Water effects are taken into account as added mass and damping coefficients.

Solution consists of two parts, which are summed up. In the first part steady, slowly changing and aperiodic forces are taken into account. Such forces are wind, current, ice and second order wave forces. The second part of solution is for first order wave forces.

The program can be used to calculate mooring forces and energies caused by ships. Various kinds of affecting parameters can be included at the same time without building up a heavy element model of ship and water around it. So far, on the other hand, the principle of superposition is used. In some situations, for example, in simulating combined effect of waves and current more sophisticated theory should be employed.

INTRODUCTION

A ship, water around it, mooring lines and fenders and a berth form a nonlinear vibration system, which is affected by irregular forces during and after a ship's berthing operation. Wave loads on fenders and mooring ropes
are highly dependent on dynamic properties of the whole system. Deterministic equations for wind, current and ice forces do not include kinetic energies and time history. Hydraulic modelling studies can be very expensive due to great variety of affecting factors.

Simulation program of ship motions can be a useful tool for a designer to evaluate effects of vessel sizes and types, local weather conditions, and variety of mooring lines and fender types in assessment and design of terminals. The algorithm based on harmonic response functions of rigid ship body is quick and flexible compared with heavy element model of ship and water around it, and need of computational capacity is much lower.

SIMULATION ALGORITHM

The algorithm is based on solutions to linear second order differential equations of motion for six degrees of freedom. They are solved by a time-stepping procedure with all external and reaction forces evaluated at every time step. During simulation two right-handed Cartesian coordinate systems are used. The main coordinate system is fixed in space. The ship coordinate system is translating with the ship with the origin at the center of gravity of the body. It can also rotate about the z-axis, but not about the axes of x and y. All forces and matrices are defined in the main coordinate system and then transformed into the ship coordinate system where the equations of motion are solved.

\[ M_k \ddot{q}_k + C_k \dot{q}_k + K_k q_k = F \]  \hspace{1cm} (1)

where \( k = 1 \) or \( 2 \) (solution phase), \( M, C \) and \( K \) are mass, damping and stiffness matrices in this order, and \( q \) is a vector of general coordinates. The force vector \( F \) may include steady, slowly changing, harmonic and aperiodic forces.
Solution for steady, slowly changing and anharmonic forces

The solution is the sum of homogeneous and particular solutions. In particular solution the instantaneous position of equilibrium is taken into account. It can be also a function of time, when there are no restoring forces in the direction of motion. Homogeneous solution is written in harmonic or exponential form depending on the type of eigenvalues (complex or real) [11]. When summed up with particular solution it has to satisfy initial conditions.

\[ q_{1i}(\Delta t) = q_{1hi}(\Delta t) + q_{1pi}(\Delta t) \]

\[ q_{1hi}(\Delta t) = \sum_{j=1}^{N} Q_{1hij} \cos(\omega_j \Delta t - \alpha_{1ij}) \]

\[ q_{1hi}(\Delta t) = \sum_{j=1}^{N} Q_{1hij} e^{r_j \Delta t} \]

\[ q_{1pi}(\Delta t) = Q_{1pi} \Delta t \]

where \( i = 1,2,...,6 \) direction of motion, \( N = 6 \) degrees of freedom, \( Q \) is a constant, \( r \) and \( \omega \) are the real and imaginary parts of the eigenvalue. \( \Delta t \) is a time step and \( \alpha \) is a phase angle. The force vector \( F \) in equation (1) for this solution phase is the sum of following forces:

- \( F_A \) = wind force
- \( F_C \) = current force
- \( F_{W2} \) = second order wave force
- \( F_I \) = ice force (crushing or drifting)
- \( F_R \) = reaction and restoring forces
- \( F_H \) = cushion effect near solid quaywall
- \( F_T \) = other forces, i.e. tugboat effects

The new position of the ship in main coordinates is evaluated when small deflection increments arise during one time step are firstly transformed into the main coordinate system and then added to the previous position coordinate values. The length of a single time step depends on the maximum deflection allowed to arise on fenders and mooring ropes during one step. The greater the nonlinearity of the mooring system is the smaller the deflections may be. However, there are a reasonable limits for the time value due to round-off-errors and correspondency reasons on the other side.

Particular solution for first order wave forces

The harmonic solution formula in this case is
where \( N_w \) is number of wave components, \( Q_2 \) is an amplitude coefficient for waves, \( \omega \) is a frequency of a wave component, \( \epsilon \) is a basic phase angle and \( \alpha_2 \) is the phase angle between motion and force. Irregular waves are described as a sum of regular harmonic wave components. In the same way, the solution for this phase is the sum of responses for every wave component.

**Matrices**

Hydrodynamic effects are taken into account as damping, added mass and restoring coefficients [8]. Ship's body is assumed to have three symmetry planes in the ship coordinate system in spite of rotations about the x- and y-axes. The added mass, damping and stiffness matrices are different in the two solution cases. The stiffness matrix \( K_1 \) is re-evaluated on account of instantaneous coefficients, but in the case of \( K_2 \) the mean values from the previous phase of same sign are reasonable to be used. Added mass and damping matrices are dependent on the frequencies of motions. In the first case the natural frequencies of the system can be used, but in the second case the definition is more complicated. If there are many wave components with many different frequencies, one possibility is to use the simulated value of the previous phase of ship motion as a feed-back.

**Forces**

The equation for wind and current forces \( F_A \) and \( F_C \) are based on dynamic pressure, in which shape of structure is taken into account with drag coefficients [2, 3, 4, 5]

\[
F_{Li}(\varphi) = \frac{1}{2} \rho c_i(\varphi) v^2 A_i
\]

where

- \( L = A \) or \( C \)
- \( i = 1, 2 \) and \( 6 \) for current force
- \( i = 1, 2, 4 \) and \( 6 \) for wind force
- \( \rho \) density of air or water
- \( v \) velocity of wind or current
- \( c \) drag coefficient
- \( \varphi \) drift angle
- \( A \) wind or hull area, multiplied by moment arm in degrees of freedom of \( 4 \) and \( 6 \)

The equation for wind and current driving an icefloat has the form [6]
\[ F_{ld} = 1/2 \rho \ c \ v^2 \ A \]  

where \( \rho \) density of air or water  
\( c \) drag coefficient for ice/wind or ice/water  
\( v \) velocity of wind at 10 m altitude, or velocity of current at -0.5 m depth under ice  
\( A \) area of an icefloat

If an icefloat is crushing against ship's hull the force is [7]

\[ F_{fc} = \sigma \ h \ b \]  

where \( \sigma \) effective strength of ice  
\( h \) ice thickness  
\( b = b(s) \) breadth of crushing area depending on penetrated distance and shape of hull

Simulation of an icefloat impact on ship’s hull is based on the law of energy conservation [1, 7], and has the form

\[ W_l = W_o - W_{fc} - W_k - W_p + W_{id} \]  

where \( W_l \) kinetic energy of ice at the time of \( t \)  
\( W_o \) " " beginning  
\( W_{fc} \) energy needed for crushing  
\( W_k \) the part of energy transferred to kinetic energy of ship and surrounding water  
\( W_{id} \) increase of kinetic energy of ice during the time \( t \) caused by wind and current

Wave force effects are divided to first order wave forces depending on single waves, and to second order drift forces depending on wave group properties on a longer time period. The strip theory is used get total force on ship. Hence first order wave forces are summed up of \( N_w \) regular wave force components on \( N_S \) cross-sectional elements over the hull length.

\[ F_{W/l} (t) = \sum_{j}^{N_w} \sum_{k}^{N_s} (A_{jk} \cos \omega_j t + B_{jk} \sin \omega_j t) \Delta s_k \]  

where \( \Delta s \) is the length of section element, \( A \) and \( B \) are elemental force constants consisting of inertial, damping and displacement components [8]. The ISSC-formula is used to describe spectral densities of natural waves and to determine parameters of single wave components [9]. In formulating the equation for the second order wave forces the works of Løken, Olsen [10] and
Faltinsen [4] (see [1]) have been applied

\[ F_{W2i}(t) = \sum_{j=1}^{N_w} \sum_{m=1}^{N_w} \sum_{k=1}^{N_i} (a_j a_m \cos((\omega_m - \omega_j)t - (\epsilon_m - \epsilon_j)) T_{ik} \Delta s_k \]  

(9)

where \( i = 1, 2 \) and 6, \( a \) = amplitude and \( T \) is a transfer coefficient, which depends on shape of hull and drift angle. Using the asymptotic theory presented by Faltinsen et al. [4] the coefficient can be written as

\[ T_{ik} = \frac{1}{2} \rho g \frac{\sin^2(\theta_k - \varphi_w)}{n_{ik}} n_{1k} \]

(10)

\[ n_{1k} = \sin \theta_k, \quad n_{2k} = \cos \theta_k, \quad n_{6k} = x \cos \theta_k - y \sin \theta_k \]

where \( \theta \) is the angle between x-axis and the tangent of hull surface in horizontal direction. \( \varphi_w \) is the direction of wave propagation compared with x-axis.

The water between a berthing ship and a solid quaywall squeezes, which introduces a cushion effect causing an extra force on the ship. To get this effect with to the simulation algorithm, a simple theory was applied. If the ship is assumed to be long enough, the water flow around the ship can be neglected. Based on the Bernoulli equation of pressure there is a balance between the amount of water running down under the ship and rise of water level. After some derivations [1] the lateral force caused by unbalance between the hydrostatic pressures on opposite sides of hull can be written as

\[ F_H = -\frac{1}{2} \rho g \sum_{k=1}^{N_i} z_{Hk}(z_{Hk} + 2D) \Delta s_k \]

(11)

\[ z_{Hk} = v_k \left| v_k \right| \frac{D^2}{(2g d)} \]

where \( z_{Hk} \) is instantaneous hydraulic level, \( D \) is a draft, \( d \) is the distance between hull and quaywall, and \( v \) is velocity of ship section in the direction of positive Y-axis in the main coordinate system.

Fig. 2, The water between a berthing ship and a solid quaywall squeezes.

RESULTS

A few examples of the use of PRESS in fendering problems are included here.
Figure 3 shows the effect of fender spring rate on the maximum stroke of the fender on centric impact in model tests [12]. The results of PRESS-simulation are shown in the same figure, and they agree very well. The theoretical line and some experimental results, made by Saurin [13], about the effect of eccentricity of impact after transverse motion, can be seen in figure 4. The simulated points show also the effect of fender stiffness to the amount of energy to be absorbed by the fender. This is, because added masses and damping coefficients are dependent on vibration frequencies and thus on fender spring rates. Fig. 6 shows the peak amplitudes of a tanker model, moving in waves and loading the fenders [14]. The same situations are simulated, and the results can be seen in figure 5. The tendency of the points...
is the same - amplitudes are increasing with increasing period of waves - but no clear peaks can be seen, possibly due to differences in the mooring datas.

REFERENCES


