Finite element analysis of indentation experiments
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ABSTRACT

There are only a few methods suitable for a quantitative characterization of the mechanical properties of surfaces and surface coatings; the indentation testing is one of those methods. In order to improve the indentation test and to gather more information on the complete deformation process within the specimen finite element calculations were carried out using special contact elements to represent the contact conditions between indentor and surface.

It is shown that the indentation process depends on a variety of parameters given by the test system, i.e. the material to be investigated as well as the test device. A separation of the parameter dependencies can only be achieved by a combination of both, experimental and numerical investigations.

Examplarily it is shown that the numerical calculations are in very good agreement with analytical solutions and experimental results.

INTRODUCTION

Within the last decades the importance of the characterization and determination of the mechanical properties of surfaces, surface coatings, and small sized elements has steadily increased. Nevertheless there are only few methods practicable to this purpose. One of the methods which seem to be most effective is the depth sensing indentation experiment. In this ex-
periment an externally loaded diamond indentor penetrates into the surface of the specimen while the depth of indentation $d$ is continuously measured as a function of the applied load $F$. The resulting 'indentation curve' is widely used to calculate further material properties such as hardness or Young's modulus. It is shown elsewhere [1, 2] that the indentation process depends on a huge variety of parameters; first of all there are the characteristics of the material:

- elastic properties,
- elastoplastic properties,
- viscoplastic properties,
- coating thickness and interface properties if coatings are concerned.

Secondly a group of characteristics of the test device has an important influence especially if very small indentations needed for the characterization of coatings are prepared:

- geometry and geometrical imperfections of the indentor,
- compliance of the test system, and the
- friction between indentor and specimen.

Because of the multitude of parameters finite element calculations were carried out in order to investigate the parameter dependencies of the indentation process.

FINITE ELEMENT MODEL

Geometry and boundary conditions
The indentation experiment has been simulated using the commercial finite element code ADINA [3]. Specimen as well as indentor are modeled with isoparametric elements with 8 or 20 nodes in the cases of two- or threedimensional calculations, respectively. The geometry of the model is dependent on the experimental setup; most commonly threefaced or squarebased diamond pyramids of the Berkovich- or Vickers type are used. Therefore it is necessary to use a threedimensional model of the process. However, for several sensitivity studies it is sufficient to perform twodimensional axisymmetric calculations of the indentation with a cone. From detailed two- and threedimensional investigations on the influence of the indentor tip
geometry including its geometrical imperfections [1, 4, 5] it can be seen, that these influences become very significant at small indentation depths (d \leq 200 \text{ nm}) necessary for the characterization of coatings.

Since the boundary conditions, namely, the contact area between indentor and surface, are continuously changing with the applied load, the ADINA contact elements are used, which allow to model both, frictionless contact and contact with Coulomb friction. A detailed description of the contact algorithm of the ADINA program can be found in Refs. [3 and 6].

**Material behaviour**

The simulation of the material behaviour was based on the assumption that the diamond indentor behaves in a linear elastic manner. The material characteristics of the specimen are described by linear elastic and elastoplastic material models with von Mises yield condition. Some additional investigations using a more complex viscoplastic material model are reported in literature [7]. In all cases, deviations from isotropy have been neglected although in reality orthotropic behaviour may be prevailing. Since large deformations and strains have to be expected, a Lagrange formulation of the problem, i.e. the Updated Lagrange option of the ADINA code, is used.

As an example a part of a two-dimensional axisymmetric finite element mesh is shown in Fig. 1; with respect to the experiment a blunted indentor tip is modelled with a tip radius of 1.0 \text{ \textmu m}.

Fig. 1: Undeformed two dimensional finite element mesh; zoom of the indentation area. Indentor tip blunted, tip radius: 1.0 \text{ \textmu m}. 

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The contact problem of the indentation process can be solved analytically if several simplifying assumptions are made. First of all the problem is only solvable for linear elastic material behaviour, e.g. the infinite elastic half space. The problem of a concentrated point force acting vertically on the elastic half space was solved by Boussinesq [8]. A more realistic situation of a conical indentor was investigated by Sneddon [9]; applying the theory of Hankel transforms Sneddon determines the equation for the indentation depth \( d \) as a function of the applied load \( F \):

\[
d = \left( \frac{F \pi (1 - \nu^2)}{2E \tan (\alpha / 2)} \right)^{1/2},
\]

where \( E \) = Young’s modulus and \( \nu \) = Poisson’s ratio, \( \alpha \) = indentor angle (comp. Fig. 2).

Fig. 2: Coordinate system and symbols used in the text.

Fig. 3: Load displacement curve of the elastic indentation by a cone.
In addition, Sneddon receives the components of the displacement vector at the surface of the specimen (see e.g. formulas listed in [1]). In Figs. 3 and 4 the results of the finite element calculation ($E = 210000$ MPa, $\nu = 0.3$) are compared to the analytical results. It can clearly be seen that the agreement between analytical and numerical results is very satisfactory. Therefore the finite element model is used for the more complex and realistic case of elastoplastic material behaviour.

![Graph of Displacements $\delta_r(r)$ and $\delta_z(r)$ at the surface of the specimen.](image)

**Fig. 4**: Displacements $\delta_r(r)$ and $\delta_z(r)$ at the surface of the specimen.

**THE PROBLEM OF FRICTION BETWEEN INDENTOR AND SPECIMEN**

As mentioned above, the ADINA contact elements allow to model the friction between the indentor and the specimen. For this purpose the nodal forces in the contact region are separated into tangential and normal forces, followed by the calculation of the contact conditions, i.e. sticking contact, sliding contact, or lifting off (tension release).

The application of the contact elements to the indentation problem seems to be more effective than the usage of gap elements because according to the expected large deformations all stresses and strains can be calculated with respect to the deformed structure. In addition, the handling of these elements is very comfortable combined with a low cost of computing time.

As shown in Fig. 5, where the indentation curves of steel are plotted for three friction coefficients $\mu_R$, the influence of the friction usually cannot be neglected. Only if systems with reasonable small friction coefficients are investigated (e.g. a-C:H coatings [1]) the numerical error due to the frictionless simulation of the problem will be very
small. It seems to be important to point out that in the case of large $\mu_R$ (e.g. $\mu_R = 0.5$) the indentation curve shows - probably characteristic - several steps.

![Indentation Curve](image)

**Fig. 5:** Calculated indentation curves for steel; tip radius: $r = 0.1 \, \mu m$; parameter: friction coefficient $\mu_R$.

The differences in the deformation beneath the indentor can be seen in Fig. 6. The deformed zone of the FE-mesh is plotted for the cases of $\mu_R = 0.0$ (a) and $\mu_R = 0.5$ (b). If the friction is small the surface nodes of the specimen can move to the edge of the indentation causing the 'piling up' of the (ductile) material. Otherwise the sliding of the surface nodes is restrained and, consequently, the piling up remains significantly smaller. From this result one can conclude that it is necessary to investigate the friction parameter in experimental setups, which, of course, depends on both, the environmental conditions (humidity, lubricants, ...) and the preparation of the surface of the specimen.

![Deformed Zone](image)

**Fig. 6:** Deformed zone of the finite element mesh; tip radius: $r = 0.1 \, \mu m$;

a) $\mu = 0.0$  

b) $\mu = 0.5$. 
COMPARISON TO EXPERIMENTAL RESULTS

Last but not least one example serves to show the correlation between experimental and numerical results. In Fig. 7 the calculated and experimentally obtained indentation curves for silicon are plotted. Despite a deviation in the unloading part due to the initiation of cracks in the experiment [1] both results match rather well. Some more examples of this correlation are reported in literature [1, 2, 7].

![Indentation curves for silicon: experiment (Shimadzu DUH-200) and finite element calculation.](image)

Fig. 7: Indentation curves for silicon: experiment (Shimadzu DUH-200) and finite element calculation.

Consequently, the solution of the inverse problem, i.e. varying the numerical input data (material properties) in order to fit the experimental indentation curve, will provide a tool to determine these material properties by indentation testing. Of course, this can only be achieved if the machine parameters of the experimental setup are well known and correctly modeled.

SUMMARY

The finite element method (FEM) was used to calculate the depth sensing indentation experiment with very high accuracy and in good agreement to analytical as well as experimental results.
The FEM is a powerful tool for performing sensitivity studies on the influence of the parameters involved in the indentation process. The combination of numerical and experimental effort provides the possibility to separate the material response from the influence of the machine parameters.

ACKNOWLEDGEMENT

The author gratefully acknowledges the financial support by the Deutsche Forschungsgemeinschaft (DFG) under the grant So 104/9.

REFERENCES