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Vibration of axially moving narrow membrane travelling between two roll-supports having finite radius

H. Koivurova

Department of Mechanical Engineering, University of Oulu, SF-90570 Oulu, Finland

ABSTRACT

The dynamic behaviour of a band moving between two support pulleys having finite radius is investigated. In practical applications it has been noticed that in a band-pulley-system boundary of contact region travel due to the interaction between band and pulleys. To understand how the interaction affect on dynamic characteristics of the system, a nonlinear finite element model of axially moving band with two finite radius roll-support has been developed. Numerical results for band-pulley-system under harmonic boundary excitation are considered.

INTRODUCTION

Axially moving material problems consider dynamic response, vibration and stability of structural members which are in a state of translation. Examples of engineering systems employing axially moving materials include magnetic tapes, belt and chain drives, wide paper sheets and webs, pipes containing flowing fluid etc. Recent developments in the research on axially moving materials are reviewed in reference [1].

The dynamic response of systems including axial movement differs from the corresponding traditional system. Because of the continuous longitudinal travel of material, the equations of motion contains two additional inertia terms caused by Coriolis acceleration and by centripetal acceleration. Therefore, the free vibration is characterized by differing up- and downstream wave speeds, by natural frequencies that decrease monotonically with increasing translation speed and by complex, speed dependent eigenfunctions.

Models for the dynamics of axially moving materials consider the transverse response of the element as it translates with prescribed speed between supporting wheels, pulleys or guides. During operation, the element is subjected to tension forces which originate from the support and drive mechanisms. Traditionally, the interaction of the travelling material with the support system has not been considered [2] and the supporting wheels has been idealized as 'simple' supports in fixed points. However, in some practical applications (paper sheet in a paper mill) the boundary of contact zone have been detected to shift due to interaction between finite rolls and travelling material. The variation of the length of span and the movement of the boundary of contact region lead us to an open boundary problem. If the roll supports are included in the system examined, problem change to the contact problem of interacting band and rolls.

The purpose of this paper is to investigate dynamic characteristics of axially moving membrane with finite radius cylindrical end supports. The problem is strongly nonlinear. A model based on the finite element method is presented for determining effects on interaction between supports and axially moving membrane. The model is formulated to include the nonlinear terms arising from large amplitude oscillations as well as variation in tension along the membrane. The study is limited to x,z plane and the longitudinal velocity of the sheet is assumed to be constant. The interaction between band and roll supports is modelled by penalty function contact algorithm.

The accuracy of the computational model was verified using available experimental results and a rather good agreement was observed. To have an insight into the character of the band-pulley-system, the response due to harmonic boundary excitation have been examined.

EQUATIONS OF MOTION

The physical model being considered, as shown in Fig. 1, is a continuous band of length l passing over two pulleys of radius r at a constant axial transport velocity v and initial tension T_s.

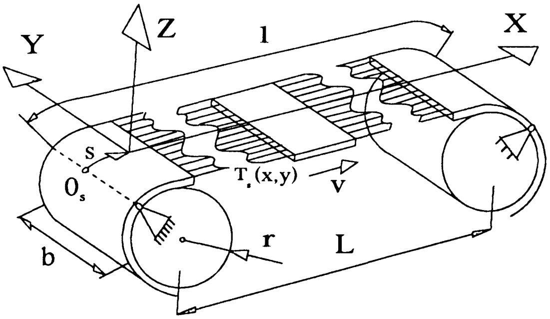

Figure 1. Model of moving membrane.

There is two different possible approach to formulate the equations of motion. First we could consider a straight span with varying length as an open boundary problem. The equation of motions could be derived through the Hamilton's principles [7]. Because of the movement of boundary points the influx and the outflux of energy can't be omitted [3] and the Hamilton's principles consist term of virtual work of non-conservative forces acting on the system.

Secondly we can contemplate a system where the boundary points are chosen on the rigid support cylinders so that the displacement of the convective band on the boundary is prevented. If the friction between band and rolls are ignored the system is conservative [7] and the Hamilton's principles can be stated

$$\delta \int_{t_1}^{t_2} (T - U)\,dt = 0 \tag{1}$$

The equations for the moving narrow band are developed based upon the following hypotheses:

1. Only free, undamped vibrations are considered.
2. The band is rigidly supported at its ends, which permits a conservative analysis.
3. The motion of the membrane occur in x,z plane.
4. Only longitudinal oscillations caused by transverse vibrations are considered.
5. The effect of gravity is neglected.

The transverse velocity of a particle of membrane is

$$\dot{w} = w_t + v w_s \tag{2}$$

where s denotes the initial arc length measured from point O_s. The longitudinal velocity of a particle is composed of the transportation velocity and the local velocity caused by variation in the longitudinal displacement.

$$\dot{u} = u_t + v(1 + u_s) \tag{3}$$

The kinetic energy is

$$T = \frac{1}{2} \int_{-b/2}^{b/2} \int_0^l m\{[u_t + v(1 + u_x)]^2 + (w_t + v w_x)^2\}\,ds\,dy \tag{4}$$

where m denotes the mass per unit area of the sheet and l indicate total length of the band (see figure 1). The potential energy of the band between the extreme supports is

$$U = \int_{-b/2}^{b/2} \int_0^l [T_s \varepsilon + \frac{1}{2} EA\,\varepsilon]\,ds\,dy \tag{5}$$

where the non-linear strain measure is approximately

$$\varepsilon = u_s + \frac{1}{2} w_s^2 \tag{6}$$

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Thus, substituting expressions (4) and (5) for T and U in equation (1) we obtain in standard manner the equation of motion

$$
\begin{aligned}
m(u_{,tt} + 2vu_{,st} + v^2u_{,ss}) - EA(u_{,s} + \frac{1}{2} w_{,s})_{,s} &= 0 \\
m(w_{,tt} + 2vw_{,st} + v^2w_{,ss}) - T_s w_{,ss} - EA(u_{,s} + \frac{1}{2} w_{,s})_{,s} &= f(s,y,t)
\end{aligned}
\tag{7}
$$

where f is transverse contact force per unit area.

NUMERICAL SOLUTION FOR THE RESPONSE

An exact solution of equations (7) cannot be obtained in general, because the contact forces are unknown. Also the other possible approach, the open boundary problem, would be impossible to solve analytically. Thus, finite element discretization is pursued. This is done by implementing the convective terms and the influence of the initial tension from equations (7) as additional force to finite element code DYNA3D [6].

$$
\begin{aligned}
mu_{,tt} &= P_s - F_s - 2mvu_{,st} - mv^2u_{,ss} \\
mw_{,tt} &= P_{\hat{z}} - F_{\hat{z}} - 2mvw_{,st} - (mv^2 - T_s)w_{,ss}
\end{aligned}
\tag{8}
$$

where

P = accounts for external and body force
F = stress divergence vector
$\hat{z}$ = transverse direction

DYNA3D is an explicit finite element program having capability to handle both the nonlinear material behaviour and large displacement response of three-dimensional structures. The interaction between roll-supports and membrane are modelled by using penalty contact algorithm of DYNA3D. The algorithm consists of placing normal interface springs between all penetrating nodes and contact surface. Stiffness factor of interface spring is computed based on the thickness and bulk modulus of the element in which it resides. The magnitude of interface force is proportional to the amount of penetration.

The effect of variation of tension in equation (7) is included in implicitly by normal response of shell element.

The convective terms in equation (8) are discretized in a same manner as in reference [7]. The membrane between supports is subdivided into two-dimensional elements. The configuration is approximated by

$$
\begin{cases}
u(s,y,t) = \mathbf{N}\mathbf{u} \\
w(s,y,t) = \mathbf{N}\mathbf{w}
\end{cases}
\tag{9}
$$

where N(s,y) is shape function matrix (row vector) and **w**(t) and **u**(t) are nodal displacement vectors. Now by substituting expressions (4) and (5) into (1) and by replacing u and w by (9) the approximative variational statement is

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obtained. Integrating by parts to eliminate the time derivative of the variation leads to the equations of motion and the equation (8) can be written as a set of equations

$$\begin{aligned} \boldsymbol{M}\ddot{\boldsymbol{u}} &= \boldsymbol{P}_s + \boldsymbol{F}_s - \boldsymbol{G}\dot{\boldsymbol{u}} - \boldsymbol{K}_s\boldsymbol{u} \\ \boldsymbol{M}\ddot{\boldsymbol{w}} &= \boldsymbol{P}_z + \boldsymbol{F}_z - \boldsymbol{G}\dot{\boldsymbol{w}} - \boldsymbol{K}_z\boldsymbol{w} \end{aligned} \tag{10}$$

where **G** is additional gyroscopic inertia matrix

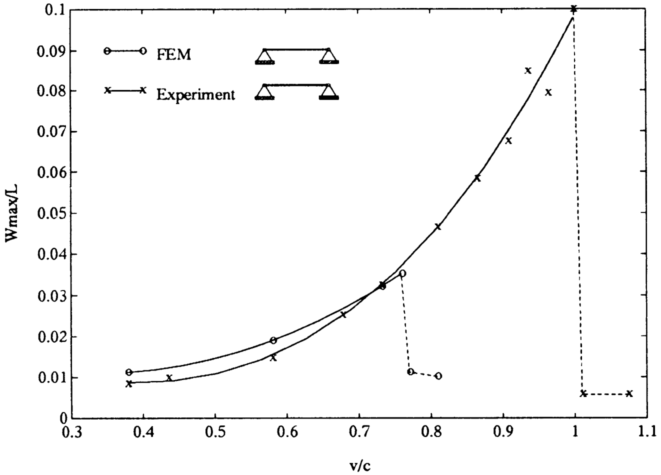
$$\boldsymbol{G} = \int_{-b/2}^{b/2}\int_0^l m\,v\,(\boldsymbol{N}^T\boldsymbol{N}_{,s} - \boldsymbol{N}_{,s}^T\boldsymbol{N})\,ds\,dy \tag{11}$$

and $\mathbf{K}_s$ and $\mathbf{K}_z$ are additional stiffness matrices

$$\begin{aligned} \boldsymbol{K}_s &= \int_{-b/2}^{b/2}\int_0^l (-m\,v^2)\boldsymbol{N}_{,s}^T\boldsymbol{N}_{,s}\,ds\,dy \\ \boldsymbol{K}_z &= \int_{-b/2}^{b/2}\int_0^l (T_s - m\,v^2)\boldsymbol{N}_{,s}^T\boldsymbol{N}_{,s}\,ds\,dy \end{aligned} \tag{12}$$

RESULTS AND DISCUSSION

To verify the overall accuracy of approach adopted a comparison with experimental results of Ames, Lee and Zaiser [4] has been made. In the experiment they excited a straight string with harmonic boundary excitation and observed the variations in the string configuration as axial velocity *v* changed. The frequency f = 30 Hz and the non-dimensional amplitude $W_0/L = 0.005$. The approximate solution of maximum amplitude obtained from FEM by 40 elements and results of the experiment are shown in figure 2.



$W_0/L = 0.005$
$\omega = 189$ 1/s
$T_x = 4.5$ N
E = 0.55 GPa
A = 7.9 mm^2
m = 3.1 g/m
r = 0

$$c = \sqrt{\frac{T_x}{m}}$$

Figure 2. Amplitude-velocity response.

To have an insight into the character of the band moving between two support pulley having finite radius, the response of harmonic boundary excitation have been computed. Now the amplitude of excitation and the axial velocity is kept fixed. The computation is carried on until a steady state is obtained. Figure 3 shows a graph of the computed sheet response during a half cycle. In general, response is smooth and the movement of limiting points of contact can be clearly seen.

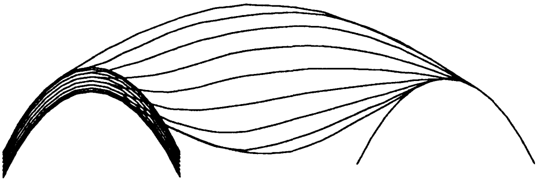


Figure 3. Calculated sheet configuration during half period of vibration.

In figure 4 the displacement history of midpoint of a sheet is compared with 'simple' supports to finite roll-supports.

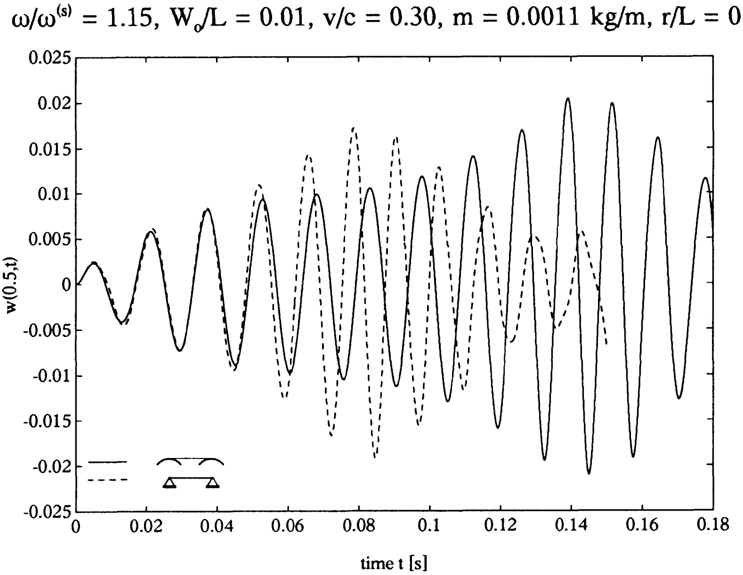


Figure 4. Transverse displacement histories of a midpoint of a sheet.

In figure 5 and 6 maximum amplitude W_{max}/L is presented as a function of non-dimensional excitation frequency $\omega/\omega^{(m)}$, where

$$\omega^{(m)} = \frac{\pi}{L}\sqrt{\frac{T_x}{m}}\,\left(1 - \left(\frac{v}{c}\right)^2\right) \qquad (13)$$

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is the first natural (angular) frequency of the sheet. The results show that the period of vibration decreases rapidly with increasing amplitude of vibration in both cases. The influence of finite cylindrical supports increase slightly the period of vibration and maximum amplitude.

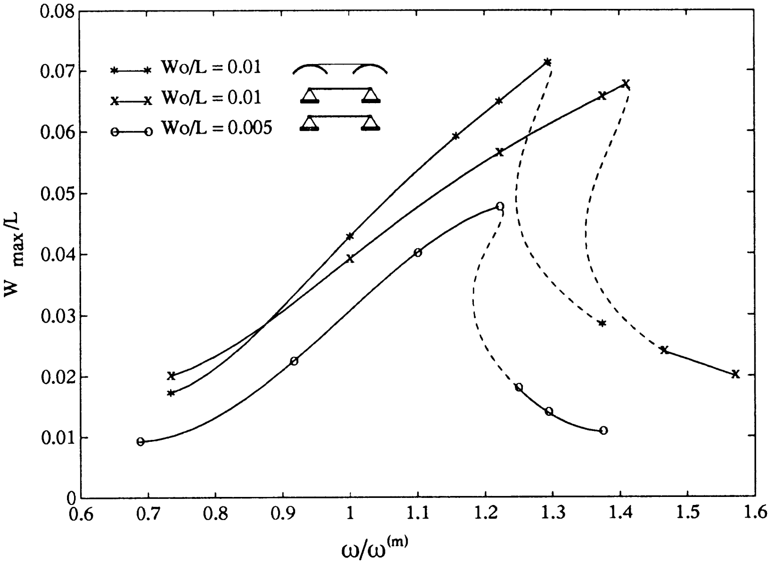

Figure 5. Response-frequency curves for axial moving band at v/c = 0.3.

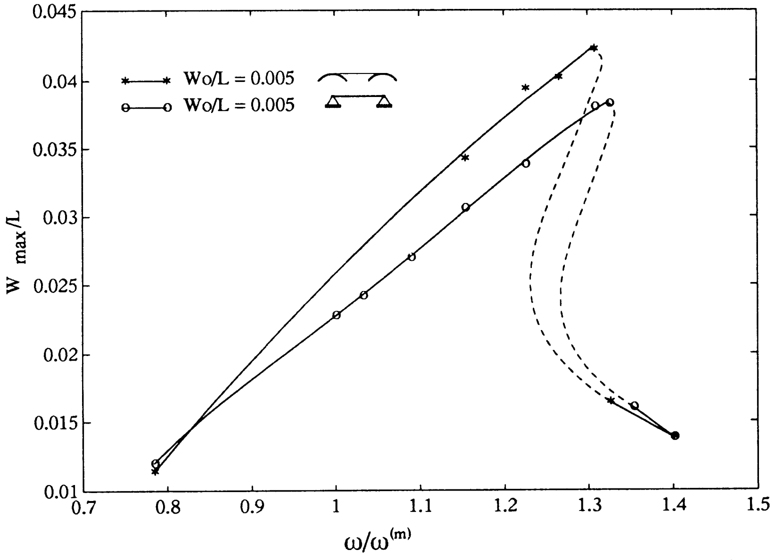

Figure 6. Response-frequency curves for axial moving band at v/c = 0.7.

SUMMARY AND CONCLUSIONS

The dynamic response of a band system moving between two finite radius cylindrical supports has been considered and a nonlinear model based on finite element method is developed. The nonlinear terms arising from large amplitude, tension variation and effect of interaction between band and supports.

The results show that the finite radius pulleys have distinct influence on the behaviour of axially moving sheet. The boundary of contact region moves clearly. This weaken the nonlinear harding phenomena, lower vibration frequency and stengthen maximum amplitude of the axially moving membrane.

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