Numerical analysis of bi-material interface notch crack behaviour

B.Bachir Bouiadjra, M.Belhouari, M.Benguediab & B.Serier
L.E.C.M, Departement of Mechanical Engineering, University of Sidi Bel Abbes, BP.89, Cité Ben M’hidi, 2000, Algeria.

Abstract

The finite element method is extended to the analysis of the behaviour of an interface crack in bi-material specimen with a central hole. First, only the notch effect is considered, the field of stress and variation of the stress concentration factor as a function of the Young’s modulus ratio are determined. Secondly, the notch interface crack behaviour is investigated, the variations of the stress intensity factor versus the Young’s modulus ratio and crack length are shown as well as the distribution of stresses in the plate and along the interface.

1 Introduction

Fracture along an interface between phases plays a major role in limiting the toughness and ductility of multi-phase materials [1]. The fracture of bi-materials have a singular behaviour due to the difference between elastic properties of the bounded materials and to the presence of defects at the interface, and can intervenes by:
- brittle or ductile manner in the volume of one of two materials (cohesive fracture),
- propagation of an interface crack (adhesive fracture),
- mixed propagation : interface crack can, under certain conditions, deviates in the one of the massive materials.
Since the study of Williams [1], many efforts have been made in order to understand the unusual characteristics of bi-material interface fracturing [2-7]. In this investigation, finite element calculations were carried out to study the
simultaneous effects of notch and interface presence on the behaviour of bi-material interface fracture. In the first part, these effects are studied without the presence of crack. The variations of stress concentration factor as a function of young’s modulus ratio of the two bounded materials are presented with the stress distribution around the notch and along the interface. In the second part, the behaviour of notch interface crack is studied. Variations of the interfacial stress intensity factors are presented as a function of young’s modulus ratio and crack length as well as the stress distributions from the crack tip and along the interface.

2 Mathematical formulation

To determine the stress fields near the interface crack tip, let’s consider a bi-material assembly with interface crack, (Figure 1) characterized by the parameters $E_1, v_1$ for the material 1 and $E_2, v_2$ for the material 2.

![Bi-material Interface crack.](image)

Figure 1: Bi-material Interface crack.

A fundamental difference between the analysis and interpretation of stress intensity factor for interface crack compared with the crack in homogeneous materials is the possibility of oscillatory singularities [8]. An important result for the presentation of interface fields is the observation made by Dundurs [9] that a wide class of plane problems of elasticity contain two (rather than three) independent variables. These two non dimensional combinations of the four elastic constants are Dunder’s [9] parameters $\alpha$ and $\beta$ which are defined by:

$$\alpha = \frac{E_1 - E_2}{E_1 + E_2}$$

(1)

with

$$\bar{E}_j = \begin{cases} E & \text{in plane stress} \\ \frac{E}{1 - v_j} & \text{in plane strain} \end{cases} \quad j = 1, 2$$

and
With \( \beta = \frac{\mu_1 (1 - 2k_2) - \mu_2 (1 - 2k_1)}{2 \mu_1 (1 - k_2) + \mu_2 (1 - k_1)} \) 

\( k_j = \begin{cases} 
\frac{3 - v_j}{1 + v_j} & \text{in plane stress} \\
3 - 4v_j & \text{in plane strain} 
\end{cases} \quad j = 1, 2 \)

Where \( v \) and \( \mu \) are Poisson’s ratio and shear modulus, respectively. 
\( \alpha \) and \( \beta \) are zero when the two materials have the same elastic features, they change sign when the materials are reversed (1 situated at \( y < 0 \) and 2 at \( y > 0 \)).

The differences between the elastic properties of the bounded materials lead to a distribution of normal and shearing stresses in the interface. According to Williams [1], the field of stress is a function of:

\[ \sigma = f(r^{\lambda - 1}) \]

Where \( r \) is the distance from the crack tip and \( \lambda \) is an eigenvalue having the expression:

\[ \lambda = n + \frac{1}{2} + i\varepsilon \]

\( n \) is an integer, \( i^2 = -1 \) and \( \varepsilon \) is a parameter of bi-material defined by:

\[ \varepsilon = \frac{1}{2\pi} \ln \left( \frac{1 - \beta}{1 + \beta} \right) \]

In the case of bi-material systems, \( \lambda \), which is a parameter based on the stiffness properties of the two materials, can be lesser or greater than \( \frac{1}{2} \). The strength of the singularity given by \((\lambda - 1)\), can be lesser or greater than \(-\frac{1}{2}\) in bi-materials systems; \( \lambda \) is exactly equal to \( \frac{1}{2} \) when both materials are the same, and the singularity for such case is equal to \(-\frac{1}{2}\) [10].

The stress field near the crack tip is given by the following expression:

\[ \sigma_{ij}(\theta, \varepsilon) = \frac{1}{\sqrt{2\pi}} \left[ \begin{array}{c} \text{Re} \left\{ Kr^{i\varepsilon} f_{ij}^{I}(\theta, \varepsilon) + i \text{Im} \left\{ Kr^{i\varepsilon} f_{ij}^{II}(\theta, \varepsilon) \right\} \right\} \end{array} \right] \]

Where \( f_{ij}^{I}(\theta, \varepsilon) \) and \( f_{ij}^{II}(\theta, \varepsilon) \) are non-dimensional functions that depend only on \( \theta \) and \( \varepsilon \). The angular variation is normalized, so that \( f_{22}^{I}(0, \varepsilon) = 1 \) and \( f_{12}^{I}(0, \varepsilon) = 1 \).

\( K \) is the complex stress intensity factor defined by:

\[ K = K_1 + iK_2 \]

\( K_1 \) and \( K_2 \) are stress intensity factors for the interface crack compatible with classical SIF for homogeneous materials [5]. They depend linearly on the applied
loads and on the shape of the interface. When $\varepsilon \neq 0$, $K_1$ and $K_2$ can not be interpreted as stress intensity factors related directly to the normal and shearing stresses.

3 Analysis and Results

Our calculations have been made by the finite element code Franc2D, developed by Pr Ingraffea [12] and his team at Cornell University. The present work is divided into two parts: in the first, we study the notch effect with the presence of to the bi-material interface and in the second part, we analyze the bi-material interface notch crack behaviour.

3.1 Notch effect without presence of the crack

Let's consider a thin plate with central hole with length $H$, wide $W$ and hole radius $r$ such as: $W/H = 0.5$ and $r/W = 0.2$. The plate is constituted by two elastic materials (Figure 2), having the characteristics $E_1$, $\nu_1$ for the material 1 and $E_2$, $\nu_2$ for the material 2. We fix $\nu_1 = \nu_2 = 0.3$ and we vary $E_1$ and $E_2$ in order to study their effect. The plate is subjected to a traction load in the $y$ direction with a constant applied stress $\sigma_0 = 1$MPa. The bulk is meshed by eight nodes isoparametric quadrilateral elements with an appropriate refinement around the notch. Typical finite element mesh is shown in figure 2-b.

![Model of bi-material plate with central hole.](image)

Figure 3 represents the variation of the stress concentration factor $k_t = \sigma_{\text{max}}/\sigma_0$ (characterizing the notch) as a function of Young's modulus ratio of the bounded materials $R = E_2 / E_1$. For $R = 1$, we can observe that the value of $k_t$ is in the
Order of 3 which is in agreement with values given in the literature [11]. In figure 3, we can also observe that, if the ratio R increases, the factor $k_y$ decreases. The difference in $k_y$ between $R=1$ and $R=10$ is in the order of 7% and 16% between $R=1$ and $R=100$. To explain this effect, we represent in figure 4 the normal stress contours ($\sigma_n$) in the plate for $R=1,5$ and 10. We can note for $R=1$, that the distribution of normal stresses is symmetric through the interface and the notch. This symmetry disappears when the ratio R is greater than 1. The hardest material is subjected to a more elevated compressive stresses visible near the lower point of the notch.

These results are confirmed in figure 5 where we represent the distribution of normal stresses around the notch for $R=1,5$ and 10. We can firstly note, for $R=1$, that the distribution of normal stresses around the notch is nearly sinusoidal, which annuls the effect of compression stresses on all sides of the notch. We also note for $R=5$ and $R=10$, that the compression stresses are more concentrated below the notch.
In figure 6, we represent the distribution of normal stresses from the notch tip and along the interface for different ratios $R$. The stresses are concentrated at the notch, and go by decreasing until becoming nearly constant along the interface.

3.2 Behaviour of bi-material interface crack emanating from a notch

We consider the same bi-material plate with interface notch crack having the total length a figure 7-a. We mesh with isoparametric quadrilateral elements for the bulk and specialized quarter point elements for the crack tip region. Typical finite element mesh is shown in figure 7-b.
In figure 8, we represent the variation of the normalized stress intensity factor in mode I (opening) $K_I/K_0$ ($K_0=\sigma_0 (\pi a)^{1/2}$) as a function of normalized crack length $a/w$ for different ratios $R$. It can be seen that normalized SIF mode I increases with the crack length and the ratio $R$. It means that, on one hand, the variation versus the crack length of the mode I SIF is the same as that in homogeneous and isotropic materials, and, in the other hand the growth of the toughness of one material with respect to the other leads in a consequent increase in the mode I SIF.
An inverse conclusion can be made on the evolution of mode II normalized SIF $K_{II}/K_0$. In figure 9, we note that $K_{II}$ decreases when both $a/w$ and $R$ decreases but it absolute value increases. It means that $K_{II}$ have the same evolution as $K_I$ but in negative sense. This is confirmed by the results of figure 10 where we plot the variation of both $K_I$ and $K_{II}$ versus the ratio $R$ for a constant value of $a$.

![Figure 9: Variation of normalised $K_{II}$ versus normalised crack length.](image)

In order to have a better understanding of these phenomena, the normal stress contour near the crack tip for $r=1.5$ and 10 are presented in figure 11 (a, b and c). It's clearly shown that, for $R=1$ (same material), the normal compressive stresses are symmetric on all sides of crack lips. This explains the zero value of $K_{II}$. In the cases of $R=5$ and $R=10$, the non-symmetry of compressive stresses on the crack lips is more important when $R$ increases. This explains the increasing value of $K_{II}$ versus $R$.

![Figure 10: Variation of normalised $K_I$ and $K_{II}$ versus young's modulus ratio.](image)
Figure 11: Contour display of normal stresses near the crack tip.

In figure 12, we represent the normal stress distribution from the crack tip and along the interface. The normal stress is maximum at the crack tip, and decreases and becomes nearly constant when the considered point is away from the crack tip. We also note that the normal stress along the interface increases when the ratio R decreases.

Figure 12: Distribution of the normal stresses from the crack tip and along the interface.
4 Conclusion

The present work has been made aiming to analyze the behaviour of notch interface crack in a bi-material plate. The obtained results leads us to deduce the following conclusions:

- The stress distribution is non-symmetric from one material to another. The hardest material is subjected to more compressive stresses.
- The difference between the elastic properties of the two materials provokes a mixed mode opening of the crack.
- The mode I stress intensity factor $K_I$ increases with the crack length and the young’s modulus ratio.
- The mode II stress intensity factor $K_{II}$ decreases with the crack length and the young’s modulus ratio but its absolute value increases.
- The normal stresses at the interface are nearly constant when the distance from the considered point and the crack tip increases.

References