On friction behavior of rubber on concrete surfaces

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Abstract

The friction behavior of rubber materials on various contact surfaces is strongly affected by the contact pressure and the relative sliding velocity. Based on a great number of experiments of rubber block moving on concrete surfaces, a friction law is presented in this paper. It is characterized by a dependence on contact pressure and sliding velocity. The identification of the required parameters of this friction law was done by means of a least-square method followed by re-analyses of the experiments. Numerical examples are used to investigate the friction behavior of rubber materials on wet concrete.

1 Introduction

The improvement of the performance of tires is still mainly done empirically by means of time-consuming and expensive test series which are also strongly influenced by the atmospheric conditions. With the development of powerful computer and computational techniques, great effort has been done to establish suitable material models and contact algorithms in order to apply numerical simulations within the process of tire-development. For the optimization of the load transfer mechanism, the frictional behavior of rubber blocks on various road surfaces has to be captured correctly. Therefore, a friction law based on experimental results is necessary for a realistic numerical simulation.

As pointed out in [5], the friction behavior of rubber materials is directly related to contact pressure and relative sliding velocity between the contact interfaces. For a better understanding of the mechanism and for determination of the dependence of the traction forces on the normal pressure and
the sliding velocity a comprehensive experimental investigation by means of the so-called Linear Friction Tester (LFT) was carried out [1], [2].

The sliding movement of a highly deformable body like a rubber block on a friction surface is a rather complicated mechanism. Depending on the characteristics of the surface two major mechanisms are responsible for the development of traction forces [3]. On non-deformable, smooth surfaces, e.g., concrete surfaces, the load transfer capacity between the rubber block and the surface is determined by a frictional process. The frictional effect can be split into an adhesive and a hysteretic part depending on the smoothness of the surface [4]. Further dependencies are the contact pressure between the rubber block and the sliding surface, and the sliding velocity of the rubber block.

In the paper, the friction model which describes the effect of contact pressure and relative sliding velocity on the coefficient of friction is presented. The calibration of the required model parameters is done by means of re-analyses of experiments with rubber blocks moving on concrete surfaces.

2 Friction law and finite element formulation

2.1 Elasto-plastic friction law

Two states of frictional contact can be distinguished: the sticking state, in which no relative tangential movement in the contact interface occurs, and the sliding state associated with a relative tangential motion of the contacting bodies. In the so-called regularized formulation of frictional contact [6], the total tangential slip $g_T$ is split into a recoverable portion $g_T^{EL}$ and an irrecoverable portion $g_T^{IN}$. $g_T^{EL}$ is associated with elastic micro-deformations of the asperities in the contact zone. Since the associated deformations vanish once the loading is removed, it can be associated with the sticking state. The elastic constitutive behavior of the asperities on the micro-level can be obtained from experiments. The simplest possible model is an isotropic linear elastic constitutive law for the tangential contact stress, i.e.

$$\tau_T = c_T g_T^{EL},$$

with

$$g_T^{EL} := g_T - g_T^{IN}. \tag{2}$$

$c_T$ denotes the elastic tangential stiffness modulus.

The tangential plastic slip, $g_T^{IN}$, is governed by a constitutive evolution equation obtained, in a similar fashion as in the theory of non-associative plasticity, as

$$g_t^{IN} = \dot{\lambda} \frac{\partial f_s}{\partial \tau_T}. \tag{3}$$
For a given plastic slip function $f_s$,

$$f_s(p, v, \tau_T) \leq 0,$$

(4)

where $p$ is the Cauchy contact pressure and $v$ is the relative sliding velocity. The loading/unloading conditions concerning the tangential slip can be expressed as

$$\dot{\lambda} \geq 0, \quad f_s \leq 0, \quad f_s \dot{\lambda} = 0.$$

(5)

Based on the results of a comprehensive experimental study [2], the contact pressure and sliding velocity dependent friction law within a framework equivalent to plasticity theory is formulated as

$$f_s(p, v, \tau_T) = -\frac{\alpha |p|^n - \beta |p|^m}{a + b |v|^m + c |v|^{2m}} + ||\tau_T|| \leq 0.$$

(6)

where $\alpha, \beta, n$ are contact pressure dependent parameters, and $a, b, c$ and $m$ are connected to the relative sliding velocity. For the given plastic slip function (6), the evolution equation of tangential plastic slip is obtained as

$$g_T^{IN} = \dot{\lambda} \frac{\partial f_s}{\partial \tau_T} = \dot{\lambda} \frac{\tau_T}{||\tau_T||}.$$

(7)

The coefficient of friction at the sliding state is computed as

$$\mu(p, v) = \frac{\alpha |p|^{n-1} + \beta}{a + b |v|^m + c |v|^{2m}}.$$

(8)

### 2.2 Finite element formulation

The determination of the plastic slip component within the finite element formulation is done analogous to the plasticity theory by means of a return mapping algorithm and implicit Euler-integration.

#### 2.2.1 Elastic prediction

Considering the time interval $[t_n, t_{n+1}]$ and assuming that no plastic slip occurs, the elastic prediction $\tau_T^{trial}$ in increment $(n + 1)$ is computed as

$$\tau_T^{trial} = cT (g_T^{n+1} - g_T^{IN, trial}) = cT (g_T^{n+1} - g_T^{IN, trial}) = \tau_T + cT \Delta g_T.$$

(9)

With the relative sliding velocity defined as

$$|v| = \frac{||\Delta g_T||}{\Delta t},$$

(10)

the plastic slip function yields

$$f_s^{trial}(p_{n+1}, v, \tau_T^{trial}) = -\frac{\alpha |p_{n+1}|^{n} - \beta |p_{n+1}|^{m} + c |v|^{2m} + ||\tau_T^{trial}||}{a + b |v|^m + c |v|^{2m}}.$$

(11)
### 2.2.2. Correction – Return mapping step

In the case of $f_{n+1}^{\text{trial}}(p_{n+1}, v, \tau_{T,n+1}^{\text{trial}}) < 0$ (STICK), there is no inelastic (plastic) slip. Therefore,

$$\Delta g_{T}^{IN} = 0$$  \hspace{1cm} (12)

and

$$\tau_{T,n+1} = \tau_{T,n+1}^{\text{trial}}.$$  \hspace{1cm} (13)

In the case of $f_{n+1}^{\text{trial}}(p_{n+1}, v, \tau_{T,n+1}^{\text{trial}}) \geq 0$ (SLIP), a correction step (return-mapping step) has to be carried out. Integration of evolution equation of plastic slip in eqn (7) yields

$$g_{T,n+1}^{IN} = g_{T,n}^{IN} + \Delta \lambda \mathbf{n}_{T,n+1},$$  \hspace{1cm} (14)

where

$$\mathbf{n}_{T,n+1} = \frac{\tau_{T,n+1}}{||\tau_{T,n+1}||} = \frac{\tau_{T,n+1}^{\text{trial}}}{||\tau_{T,n+1}^{\text{trial}}||}.$$  \hspace{1cm} (15)

The real tangential contact stress is given as

$$\tau_{T,n+1} = c_T[(g_{T,n} + \Delta g_T) - (g_{T,n}^{IN} + \Delta g_{T}^{IN})],$$

$$\tau_{T,n+1} = \tau_{T,n+1}^{\text{trial}} - \Delta \lambda c_T \mathbf{n}_{T,n+1}.\hspace{1cm} (16)$$

The consistency parameter $\Delta \lambda$ may be determined by means of substitution of eqn (16) into the condition $f_{n+1} = 0$ in eqn (6) as

$$\Delta \lambda = \frac{f_{n+1}^{\text{trial}}(p_{n+1}, v, \tau_{T,n+1}^{\text{trial}})}{c_T}.\hspace{1cm} (17)$$

Therefore, at the end of the increment $(n+1)$, $\tau_{T,n+1}$ and $\Delta g_{T}^{IN}$, are obtained as:

$$\tau_{T,n+1} = \tau_{T,n+1}^{\text{trial}} - f_{n+1}^{\text{trial}}(p_{n+1}, v, \tau_{T,n+1}^{\text{trial}}) \frac{\tau_{T,n+1}^{\text{trial}}}{||\tau_{T,n+1}^{\text{trial}}||}.$$  \hspace{1cm} (18)

$$\Delta g_{T}^{IN} = \Delta \lambda \mathbf{n}_{T,n+1} = \frac{f_{n+1}^{\text{trial}}(p_{n+1}, v, \tau_{T,n+1}^{\text{trial}})}{c_T} \frac{\tau_{T,n+1}^{\text{trial}}}{||\tau_{T,n+1}^{\text{trial}}||}.$$  \hspace{1cm} (19)

### 2.2.3 Consistent algorithm tangent stiffness

In general, the Newton-Iteration scheme is used to solve the non-linear equation set. The consistent algorithmic tangent stiffness (consistent contact matrix) may be determined from the following linearized expressions

$$D\tau_{T,n+1} \Delta p \quad \text{and} \quad D\tau_{T,n+1} \Delta g_{T}.$$  \hspace{1cm} (20)

**Case 1:** $f_{n+1}^{\text{trial}}(p_{n+1}, v, \tau_{T,n+1}^{\text{trial}}) < 0$ (STICK):

$$\tau_{T,n+1} = \tau_{T,n} + c_T (g_{T,n+1} - g_{T,n}).$$  \hspace{1cm} (21)
Hence, the derivatives of the tangential contact stress $\tau_{T,n-1}$ with respect to $p_{n+1}$ and $g_{T,n+1}$ are

$$\frac{\partial \tau_{T,n-1}}{\partial p_{n+1}} = 0,$$  \hspace{1cm} (22)

and

$$\frac{\partial \tau_{T,n-1}}{\partial g_{T,n+1}} = c_T I.$$  \hspace{1cm} (23)

**Case 2:** $f_{n+1}^{\text{trial}}(p_{n+1}, \mathbf{v}, \mathbf{\tau}_{T,n+1}^{\text{trial}}) \geq 0$ (SLIP):

From eqn (19), the tangential contact force is computed as

$$\tau_{T,n+1} = \tau_{T,n+1}^{\text{trial}} - f_{n+1}^{\text{trial}}(p_{n+1}, \mathbf{v}, \mathbf{\tau}_{T,n+1}^{\text{trial}}) \frac{\mathbf{\tau}_{T,n-1}^{\text{trial}}}{\|\mathbf{\tau}_{T,n-1}^{\text{trial}}\|} = \frac{a|p_{n+1}|^n + \beta|p_{n+1}|}{a + b|\mathbf{v}|^m + c|\mathbf{v}|^2} \frac{\mathbf{\tau}_{T,n+1}^{\text{trial}}}{\|\mathbf{\tau}_{T,n+1}^{\text{trial}}\|}$$  \hspace{1cm} (24)

Making use of eqn (15), the above equation may be rewritten as

$$\tau_{T,n+1} = C_p C_v \mathbf{n}_{T,n+1}$$  \hspace{1cm} (25)

with

$$C_p = \alpha |p_{n+1}|^n + \beta |p_{n+1}|$$  \hspace{1cm} (26)

$$C_v = \frac{|\mathbf{v}|^2}{a|\mathbf{v}|^m + b|\mathbf{v}| + c}.$$  \hspace{1cm} (27)

The derivative of the tangential contact stress with respect to $p_{n+1}$ leads to

$$\frac{\partial \tau_{T,n+1}}{\partial p_{n+1}} = \frac{\partial \tau_{T,n+1}^{\text{trial}}}{\partial |p_{n+1}|} \frac{\partial |p_{n+1}|}{\partial p_{n+1}} = (\alpha n |p_{n+1}|^{n-1} + \beta) C_v \mathbf{n}_{T,n+1}$$  \hspace{1cm} (29)

and

$$\frac{\partial |p_{n+1}|}{\partial p_{n+1}} = \text{sign}(p_{n+1}) = -1$$  \hspace{1cm} for  \hspace{0.5cm} p_{n+1} < 0 \hspace{1cm} (30)

it follows

$$\frac{\partial \tau_{T,n+1}}{\partial p_{n+1}} = (\alpha n |p_{n+1}|^{n-1} + \beta) C_v \text{sign}(p_{n+1}) \mathbf{n}_{T,n+1}.$$  \hspace{1cm} (31)

The derivative of the tangential contact stress with respect to tangential slip is computed as

$$\frac{\partial \tau_{T,n+1}}{\partial g_{T,n+1}} = C_p \frac{\partial C_v}{\partial |\mathbf{v}|} \mathbf{n}_{T,n+1} \otimes \frac{\partial |\mathbf{v}|}{\partial g_{T,n+1}} + C_p C_v \frac{\partial \mathbf{n}_{T,n+1}}{\partial g_{T,n+1}}.$$  \hspace{1cm} (32)
With the definition of \( C_v \), its derivative to \( |v| \) reads
\[
\frac{\partial C_v}{\partial |v|} = m|v|^{2m-1} \left( b|v|^m + 2c \right) \left( a|v|^{2m} + b|v|^m + c \right)^2.
\] (33)

The derivative of the relative sliding velocity \( |v| \) with respect to the tangential slip is
\[
\frac{\partial |v|}{\partial g_{T,n+1}} = \frac{\partial |v|}{\partial \Delta g_T} \frac{\partial \Delta g_T}{\partial g_{T,n+1}} = \frac{1}{\Delta t} \frac{\Delta g_T}{||\Delta g_T||}.
\] (34)

Making use of eqn (15), the last derivative in eqn (32) can be expressed as
\[
\frac{\partial n_{T,n+1}}{\partial g_{T,n+1}} = \frac{1}{||\tau_{T,n+1}^{\text{trial}}||} \frac{\partial \tau_{T,n+1}^{\text{trial}}}{\partial g_{T,n+1}} = \frac{\tau_{T,n+1}^{\text{trial}}}{||\tau_{T,n+1}^{\text{trial}}||^2} \frac{\partial ||\tau_{T,n+1}^{\text{trial}}||}{\partial g_{T,n+1}}.
\] (35)

Note that \( \tau_{T,n+1}^{\text{trial}} \) may be written as \( c_T (g_{T,n}^{el} + \Delta g_T) \), then,
\[
\frac{\partial \tau_{T,n+1}^{\text{trial}}}{\partial g_{T,n+1}} = \frac{\partial (c_T (g_{T,n}^{el} + \Delta g_T))}{\partial g_{T,n+1}} = c_T \mathbf{I}.
\] (36)
\[
\frac{\partial ||\tau_{T,n+1}^{\text{trial}}||}{\partial g_{T,n+1}} = \frac{\tau_{T,n+1}^{\text{trial}}}{||\tau_{T,n+1}^{\text{trial}}||} \cdot \frac{\partial \tau_{T,n+1}^{\text{trial}}}{\partial g_{T,n+1}}.
\] (37)

Hence,
\[
\frac{\partial n_{T,n+1}}{\partial g_{T,n+1}} = \frac{c_T}{||\tau_{T,n+1}^{\text{trial}}||} \mathbf{I} - \frac{c_T}{||\tau_{T,n+1}^{\text{trial}}||} \frac{\tau_{T,n+1}^{\text{trial}}}{||\tau_{T,n+1}^{\text{trial}}||^2} \otimes \frac{\partial \tau_{T,n+1}^{\text{trial}}}{\partial g_{T,n+1}}
\]
\[
= \frac{c_T}{||\tau_{T,n+1}^{\text{trial}}||} (\mathbf{I} - n_{T,n+1} \otimes n_{T,n+1}).
\] (38)

Therefore, the derivative of the tangential contact stress with respect to the tangential slip follows as
\[
\frac{\partial \tau_{T,n+1}}{\partial g_{T,n+1}} = C_p \frac{\partial C_v}{\partial |v|} \frac{1}{\Delta t} n_{T,n+1} \otimes \frac{\Delta g_T}{||\Delta g_T||} + \frac{c_T}{||\tau_{T,n+1}^{\text{trial}}||} (\mathbf{I} - n_{T,n+1} \otimes n_{T,n+1}).
\] (39)

3 Parameter identification

Based on a comprehensive experimental data basis, the identification of parameters required by the proposed friction law is performed by means of a least-square method.
For a given contact pressure $p_j$ and a relative sliding velocity $v_i$, the coefficient of friction may be computed by eqn (8) as

$$
\mu_{j,i} = \frac{\alpha_i |p_j|^{n-1} + \beta_i}{a + b|v_i|^m + c|v_i|^{2m}} = f_{j,i}.
$$

(40)

$\mu_{j,i}$ and $f_{j,i}$ denote the computational and experimental results of the friction coefficient, respectively. Application of the least-square method

$$
S = \sum_{j,i} (\mu_{j,i} - f_{j,i})^2 \Rightarrow \min.
$$

(41)

leads to the conditions

$$
\frac{\partial S}{\partial \alpha} = 0, \quad \frac{\partial S}{\partial \beta} = 0,
$$

(42)

and

$$
\frac{\partial S}{\partial a} = 0, \quad \frac{\partial S}{\partial b} = 0, \quad \frac{\partial S}{\partial c} = 0,
$$

(43)

from which the parameters $\alpha$, $\beta$, $a$, $b$, and $c$ can be determined iteratively. The applied solution scheme is given as follows:

1. For all values $m$ and $n$ in the interval of interest, the following steps are performed within two nested loops.

2. First assume the initial values $a = 1.0$, $b = 0.0$ and $c = 0.0$, from eqn (42) the initial values of $\alpha_i$ and $\beta_i$ may be determined by the following equations

$$
\alpha_i \sum_j |p_j|^{2(n-1)} + \beta_i \sum_j |p_j|^{n-1} = \sum_j f_{j,i} |p_j|^{n-1},
$$

$$
\alpha_i \sum_j |p_j|^{n-1} + \beta_i \sum_j |p_j|^{2(n-1)} = \sum_j f_{j,i}. \quad (44)
$$

3. The parameters $a$, $b$ and $c$ can be obtained in general by eqn (43) and the calculated initial values for $\alpha_i$ and $\beta_i$. This leads to the equations

$$
\alpha_i \sum_{i,j} |p_j|^{2(n-1)} + \beta_i \sum_{i,j} |p_j|^{n-1} = \sum_{i,j} f_{j,i} |p_j|^{n-1},
$$

$$
\frac{1}{a + b|v_i|^m + c|v_i|^{2m}} = \sum_{i,j} f_{j,i}. \quad (45)
$$

for each sliding velocity $v_i$.

4. Substitution of the obtained values of $a$, $b$ and $c$ into the eqn (42) leads to

$$
\alpha_i \sum_{i,j} \frac{|p_j|^{2(n-1)}}{t_i} + \beta_i \sum_{i,j} \frac{|p_j|^{n-1}}{t_i} = \sum_{i,j} \frac{f_{j,i} |p_j|^{n-1}}{t_i}, \quad (46)
$$

with

$$
t_i = \frac{1}{a + b|v_i|^m + c|v_i|^{2m}}. \quad (47)
$$

for solving the parameters $\alpha_i$ and $\beta_i$. 


5. The third step and the fourth step are repeated until the required accuracy or the maximum number of iterations is reached. The accuracy is defined as the difference of the error \( S \) of two following iterations.

6. For all values \( m \) and \( n \) in the interval of interest, the parameters \( \alpha, \beta, a, b, c \) and the error \( S \) are computed. Then, the according values of \( m \) and \( n \) with the minimal error \( S \) are selected as the results of the parameter identification.

4 Numerical example

The proposed friction law and the described method for the required identification of the required parameters are used in a numerical investigation concerning the frictional behavior of a rubber block under different pressures and at different sliding velocities. The rubber material is modeled by an Ogden model

\[
W = \sum_{n=1}^{3} \frac{\gamma_n}{\alpha_n} (\lambda_1^{\alpha_n} + \lambda_2^{\alpha_n} + \lambda_3^{\alpha_n} - 3),
\]

where \( \lambda_n \) (n=1,2,3) is the principal stretch ratio. The material parameters used in the numerical analyses are: \( K = 6.03610E + 5 \text{ N/mm}^2 \), \( \gamma_1 = -5.23829E + 0 \text{ N/mm}^2 \), \( \alpha_1 = -1.02881E - 1 \), \( \gamma_2 = 3.14432E + 0 \text{ N/mm}^2 \), \( \alpha_2 = 2.05761E - 2 \), \( \gamma_3 = -5.24418E - 10 \text{ N/mm}^2 \), and \( \alpha_3 = -2.24691E + 1 \).

![Illustration of FE-mesh](image-url)

Figure 1: Illustration of FE-mesh

The geometry and finite element mesh used in this numerical investigation is shown in Fig. 1. The finite element mesh of the rubber block consists of 208 4-node plane strain elements and 241 nodes. The upper and lower contact body are rubber material and concrete, respectively. The rigid concrete body is fixed at the four corner nodes. At first, the top side of rubber block was subjected to an uniform pressure. In order to simulate a
rigid pressure plate, additional constraint conditions ensuring equal vertical displacements at all nodes on the top side of rubber block were introduced. Subsequently, the rubber block was moved from left to right. The applied pressures were 1 bar, 2 bar, 4 bar and 8 bar (1 bar = 0.1 N/mm²), respectively. The sliding velocities are 1 mm/s, 10 mm/s, 100 mm/s and 1000 mm/s, respectively.

The curve of friction coefficient over contact pressure obtained from the experiments and the numerical simulations at 100 mm/s is illustrated in Figure 2. It shows that the friction coefficients obtained by the numerical simulations are in good agreement with the experimental results. The distribution of $\sigma_y$ on the deformed configuration at stable slip state, under 4 bar at 100 mm/s, are depicted in Figure 3.

![Figure 2: Coefficients of friction for various pressures obtained by experiments and numerical analyses, respectively, at sliding velocity 100 mm/s.](image)

5 Conclusions

The friction behavior of rubber on wet concrete surfaces is strongly related to the contact pressure and the relative sliding velocity. Based on experimental results obtained by the authors and available in the literature, a friction law characterized by separated contact pressure function and relative sliding velocity function was presented in this paper. The parameters of the friction law were identified by means of numerical re-analyses of experiments with a rubber block moving on concrete surfaces. The results of a numerical analysis demonstrates that the friction law yields to reasonable and practicable results.
Figure 3: Distribution of $\sigma_y$ at stable slip state on a wet concrete surface (relative slip velocity: 100 mm/s, pressure: 4 bar)

References


